

Mathematical Thinking and Reasoning Standards

MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction, but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:

Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:

Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:

Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:

Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:

Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:

Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
<p>MA.K12.MTR.1.1 <i>Actively participate in effortful learning both individually and collectively.</i></p>	<ul style="list-style-type: none"> • Student asks questions to self, others and teacher when necessary. • Student stays engaged in the task and helps others during the completion of the task. • Student analyzes the task in a way that makes sense to themselves. • Student builds perseverance in self by staying engaged and modifying methods as they solve a problem. 	<ul style="list-style-type: none"> • Teacher builds a classroom community by allowing students to build their own set of “norms.” • Teacher creates a culture in which students are encouraged to ask questions, including questioning the accuracy within a real-world context. • Teacher chooses differentiated, challenging tasks that fit the students’ needs to help build perseverance in students. • Teacher builds community of learners by encouraging students and recognizing their effort in staying engaged in the task and celebrating errors as an opportunity for learning.
<p>MA.K12.MTR.2.1 <i>Demonstrate understanding by representing problems in multiple ways.</i></p>	<ul style="list-style-type: none"> • Student chooses their preferred method of representation. • Student represents a problem in more than one way and is able to make connections between the representations. 	<ul style="list-style-type: none"> • Teacher plans ahead to allow students to choose their tools. • While sharing student work, teacher purposefully shows various representations to make connections between different strategies or methods. • Teacher helps make connections for students between different representations (i.e., table, equation or written description).
<p>MA.K12.MTR.3.1 <i>Complete tasks with mathematical fluency.</i></p>	<ul style="list-style-type: none"> • Student uses feedback from teacher and peers to improve efficiency. 	<ul style="list-style-type: none"> • Teacher provides opportunity for students to reflect on the method they used, determining if there is a more efficient way depending on the context.

MTR	Student Moves	Teacher Moves
<p>MA.K12.MTR.4.1 <i>Engage in discussions that reflect on the mathematical thinking of self and others.</i></p>	<ul style="list-style-type: none"> • Student effectively justifies their reasoning for their methods. • Student can identify errors within their own work and create possible explanations. • When working in small groups, student recognizes errors of their peers and offers suggestions. • Student communicates mathematical vocabulary efficiently to others. 	<ul style="list-style-type: none"> • Teacher purposefully groups students together to provide opportunities for discussion. • Teacher chooses sequential representation of methods to help students explain their reasoning.
<p>MA.K12.MTR.5.1 <i>Use patterns and structure to help understand and connect mathematical concepts.</i></p>	<ul style="list-style-type: none"> • Student determines what information is needed and logically follows a plan to solve problems piece by piece. • Student is able to make connections from previous knowledge. 	<ul style="list-style-type: none"> • Teacher allows for students to engage with information to connect current understanding to new methods. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. • Teacher provides opportunities for students to develop their own steps in solving a problem.
<p>MA.K12.MTR.6.1 <i>Assess the reasonableness of solutions.</i></p>	<ul style="list-style-type: none"> • Student provides explanation of results. • Student continually checks their calculations. • Student estimates a solution before performing calculations. 	<ul style="list-style-type: none"> • Teacher encourages students to check and revise solutions and provide explanations for results. • Teacher allows opportunities for students to verify their solutions by providing justifications to self and others.
<p>MA.K12.MTR.7.1 <i>Apply mathematics to real-world contexts.</i></p>	<ul style="list-style-type: none"> • Student relates their real-world experience to the context provided by the teacher during instruction. • Student performs investigations to determine if a scenario can represent a real-world context. 	<ul style="list-style-type: none"> • Teacher provides real-world context in mathematical problems to support students in making connections using models and investigations.