# Math for College Algebra B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the <u>B.E.S.T. Standards for Mathematics webpage</u> of the Florida Department of Education's website and will continue to undergo edits as needed.

## Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

#### Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

#### Benchmark

#### focal point for instruction within lesson or task

This section includes the benchmark as identified in the <u>B.E.S.T. Standards for Mathematics</u>. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

## **Connecting Benchmarks/Horizontal Alignment**

in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

#### Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

#### Vertical Alignment

#### across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

### Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:



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- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

#### Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

## Strategies to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Mathematics Standards. For more information about tiered instruction, please see the Effective Tiered Instruction for Mathematics: ALL Means ALL document.

#### Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

#### **Instructional Items**

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



# Mathematical Thinking and Reasoning Standards MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

## Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

# MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

#### **Clarifications:**

Teachers who encourage students to participate actively in effortful learning both individually and with others:



- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

# MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

#### **Clarifications:**

Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

## MA.K12.MTR. 3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

## **Clarifications:**

Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

## MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:



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- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

## Clarifications:

Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

# MA.K12.MTR.8.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

## **Clarifications:**

Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

## MA.K12.MTR.912.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.



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- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

## **Clarifications:**

Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

## MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

# Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.



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## Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.	<ul> <li>Student noves</li> <li>Student asks questions to self, others and teacher when necessary.</li> <li>Student stays engaged in the task and helps others during the completion of the task.</li> <li>Student analyzes the task in a way that makes sense to themselves.</li> <li>Student builds perseverance in self by staying engaged and modifying methods as they solve a problem.</li> </ul>	<ul> <li>Teacher builds a classroom community by allowing students to build their own set of "norms."</li> <li>Teacher creates a culture in which students are encouraged to ask questions, including questioning the accuracy within a real-world context.</li> <li>Teacher chooses differentiated, challenging tasks that fit the students' needs to help build perseverance in students.</li> <li>Teacher builds community of learners by encouraging students and recognizing their effort in staying engaged in the task and celebrating errors as an opportunity for learning.</li> </ul>
MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.	<ul> <li>Student chooses their preferred method of representation.</li> <li>Student represents a problem in more than one way and is able to make connections between the representations.</li> </ul>	<ul> <li>Teacher plans ahead to allow students to choose their tools.</li> <li>While sharing student work, teacher purposefully shows various representations to make connections between different strategies or methods.</li> <li>Teacher helps make connections for students between different representations (i.e., table, equation or written description).</li> </ul>
MA.K12.MTR.3.1 Complete tasks with mathematical fluency.	• Student uses feedback from teacher and peers to improve efficiency.	• Teacher provides opportunity for students to reflect on the method they used, determining if there is a more efficient way depending on the context.



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MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.	<ul> <li>Student effectively justifies their reasoning for their methods.</li> <li>Student can identify errors within their own work and create possible explanations.</li> <li>When working in small groups, student recognizes errors of their peers and offers suggestions.</li> <li>Student communicates mathematical vocabulary efficiently to others.</li> </ul>	<ul> <li>Teacher purposefully groups students together to provide opportunities for discussion.</li> <li>Teacher chooses sequential representation of methods to help students explain their reasoning.</li> </ul>
MA.K12.MTR.8.1 Use patterns and structure to help understand and connect mathematical concepts.	<ul> <li>Student determines what information is needed and logically follows a plan to solve problems piece by piece.</li> <li>Student is able to make connections from previous knowledge.</li> </ul>	<ul> <li>Teacher allows for students to engage with information to connect current understanding to new methods.</li> <li>Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept.</li> <li>Teacher provides opportunities for students to develop their own steps in solving a problem.</li> </ul>
MA.K12.MTR.912.1 Assess the reasonableness of solutions.	<ul> <li>Student provides explanation of results.</li> <li>Student continually checks their calculations.</li> <li>Student estimates a solution before performing calculations.</li> </ul>	<ul> <li>Teacher encourages students to check and revise solutions and provide explanations for results.</li> <li>Teacher allows opportunities for students to verify their solutions by providing justifications to self and others.</li> </ul>
MA.K12.MTR.7.1 Apply mathematics to real-world contexts.	<ul> <li>Student relates their real-world experience to the context provided by the teacher during instruction.</li> <li>Student performs investigations to determine if a scenario can represent a real-world context.</li> </ul>	• Teacher provides real-world context in mathematical problems to support students in making connections using models and investigations.



# Algebra I Areas of Emphasis

In Algebra I, instructional time will emphasize five areas:

- performing operations with polynomials and radicals, and extending the Laws of Exponents to include rational exponents;
- (2) extending understanding of functions to linear, quadratic and exponential functions and using them to model and analyze real-world relationships;
- (3) solving quadratic equations in one variable and systems of linear equations and inequalities in two variables;
- (4) building functions, identifying their key features and representing them in various ways; and
- (5) representing and interpreting categorical and numerical data with one and two variables.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following:

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of the major mathematical topics to all stakeholders.
- Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

		Linear	Solving	Building	
	Operations with	Quadratic	Equations and	Functions,	Representing
Number Sense	Polynomials and	Quadratic	Systems of	Identifying Key	and Interpreting
and Operations	Radicals and Laws	and Exponential Functions	Linear	Features and	Categorical and
-	of Exponents		Equations	Various	Numerical Data
			and Inequalities	Representations	
MA.912.NSO.1.1	Х	Х	Х	Х	n/a
MA.912.NSO.1.2	Х	n/a	n/a	Х	n/a
MA.912.NSO.1.4	Х	n/a	Х	n/a	n/a

Algebraic Reasoning	Operations with Polynomials and Radicals and	Linear, Quadratic and	Solving Equations and Systems of	Building Functions, Identifying Key	Representing and Interpreting
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	Laws of Exponents	Exponential Functions	Linear Equations and Inequalities	Features and Various Representations	Categorical and Numerical Data
MA.912.AR.1.1	Х	Х	Х	X	Х
MA.912.AR.1.2	n/a	n/a	Х	n/a	n/a
MA.912.AR.1.3	Х	n/a	n/a	n/a	n/a
MA.912.AR.1.4	Х	n/a	n/a	n/a	n/a
MA.912.AR.1.7	Х	Х	Х	Х	n/a
MA.912.AR.2.1	n/a	n/a	Х	n/a	n/a
MA.912.AR.2.2	n/a	Х	n/a	n/a	Х
MA.912.AR.2.3	n/a	Х	n/a	n/a	n/a
MA.912.AR.2.4	n/a	Х	n/a	Х	n/a
MA.912.AR.2.5	n/a	Х	n/a	Х	Х
MA.912.AR.2.6	n/a	n/a	Х	n/a	n/a
MA.912.AR.2.7	n/a	n/a	Х	n/a	n/a
MA.912.AR.2.8	n/a	n/a	Х	n/a	n/a
MA.912.AR.3.1	Х	n/a	Х	n/a	n/a
<u>MA.912.AR.3.4</u>	n/a	Х	n/a	n/a	n/a
<u>MA.912.AR.3.5</u>	n/a	Х	n/a	n/a	n/a
<u>MA.912.AR.3.6</u>	n/a	Х	n/a	Х	n/a
<u>MA.912.AR.3.7</u>	n/a	Х	n/a	Х	n/a
<u>MA.912.AR.3.8</u>	n/a	Х	n/a	Х	n/a
<u>MA.912.AR.4.1</u>	n/a	Х	n/a	Х	n/a
<u>MA.912.AR.4.3</u>	n/a	Х	n/a	Х	n/a
<u>MA.912.AR.8.3</u>	n/a	Х	n/a	n/a	n/a
MA.912.AR.8.4	n/a	Х	n/a	Х	n/a
MA.912.AR.8.6	n/a	Х	n/a	X	n/a
<u>MA.912.AR.9.1</u>	n/a	n/a	X	n/a	n/a
MA.912.AR.9.4	n/a	n/a	Х	n/a	n/a
<u>MA.912.AR.9.6</u>	n/a	n/a	Х	n/a	n/a

Functions	Operations with Polynomials and Radicals and Laws of Exponents	Linear, Quadratic and Exponential Functions	Solving Equations and Systems of Linear Equations and Inequalities	Building Functions, Identifying Key Features and Various Representations	Representing and Interpreting Categorical and Numerical Data
MA.912.F.1.1	n/a	Х	n/a	Х	Х
MA.912.F.1.2	Х	n/a	n/a	Х	n/a
MA.912.F.1.3	n/a	n/a	n/a	Х	n/a
MA.912.F.1.5	n/a	Х	n/a	Х	n/a



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MA.912.F.1.6	n/a	Х	n/a	Х	n/a
MA.912.F.1.8	n/a	Х	n/a	Х	Х
MA.912.F.2.1	n/a	Х	n/a	Х	n/a

	Financial Literacy	Operations with Polynomials and Radicals and Laws of Exponents	Linear, Quadratic and Exponential Functions	Solving Equations and Systems of Linear Equations and Inequalities	Building Functions, Identifying Key Features and Various Representations	Representing and Interpreting Categorical and Numerical Data
M	A.912.FL.3.2	n/a	Х	n/a	n/a	n/a
M	A.912.FL.3.4	n/a	Х	n/a	Х	n/a

Data Analysis & Probability	Operations with Polynomials and Radicals and Laws of Exponents	Linear, Quadratic and Exponential Functions	Solving Equations and Systems of Linear Equations and Inequalities	Building Functions, Identifying Key Features and Various Representations	Representing and Interpreting Categorical and Numerical Data
MA.912.DP.1.1	n/a	n/a	n/a	n/a	Х
MA.912.DP.1.2	n/a	n/a	n/a	n/a	Х
MA.912.DP.1.3	n/a	n/a	n/a	n/a	Х
MA.912.DP.1.4	n/a	n/a	n/a	n/a	Х
MA.912.DP.2.4	n/a	Х	Х	Х	Х
MA.912.DP.2.6	n/a	n/a	n/a	n/a	Х
MA.912.DP.3.1	n/a	n/a	n/a	n/a	Х



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## Number Sense and Operations

MA.912.NSO.1 Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.

## MA.912.NSO.1.1

## Benchmark

## MA.912.NSO.1.1 Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

#### Benchmark Clarifications:

Clarification 1: Instruction includes the use of technology when appropriate.

Clarification 2: Refer to the K-12 Formulas (Appendix E) for the Laws of Exponents.

Clarification 3: Instruction includes converting between expressions involving rational exponents and expressions involving radicals.

Clarification 4: Within the Mathematics for Data and Financial Literacy course, it is not the expectation to generate equivalent numerical expressions

## Connecting Benchmarks/Horizontal Alignment

MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7 ٠

MA.912.F.1.2, MA.912.F.1.6 ٠

# Terms from the K-12 Glossary

- Base •
- . Exponent
- Expression

## Vertical Alignment **Previous Benchmarks**

Next Benchmarks

• MA.8.NSO.1.3

# Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rationalnumber exponents to generate and evaluate numerical expressions (MA.912.NSO.1.1). In Mathematics for College Algebra, students extend the Laws of Exponents to properties of logarithms.

- Instruction includes using the terms Laws of Exponents and properties of exponents ٠ interchangeably.
- Instruction includes a continuation from Algebra I of working with patterns and the connection to mathematical operations and the inverse relationship between powers and radicals (MTR.5.1).
  - For example,  $\sqrt[3]{4^2} \cdot \sqrt{4} = 4^{\frac{2}{3}} \cdot 4^{\frac{1}{2}}$  is equivalent to the equation  $\sqrt[3]{4^2} \cdot \sqrt{4} = 4^{\frac{7}{6}}$ , which is equivalent to  $\sqrt[3]{4^2} \cdot \sqrt{4} = 4^{\frac{6}{6}}$ .



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- Students should make the connection of the root being equivalent to a unit fraction exponent (*MTR.4.1*).
  - For example,  $\sqrt[3]{8} = \sqrt[3]{2^3}$  is equivalent to the equation  $\sqrt[3]{8} = (2^3)^{\frac{1}{3}}$ , which is equivalent to the equation  $\sqrt[3]{8} = 2^{\frac{3}{1} \cdot \frac{1}{3}}$ , which is equivalent to the equation  $\sqrt[3]{8} = 2^1$ , which is equivalent to the equation  $\sqrt[3]{8} = 2$ .
- Students should be able to fluently apply the Laws of Exponents in both directions.
   For example, students should recognize that 2<sup>6</sup> is the quantity (2<sup>3</sup>)<sup>2</sup>.
- Instruction includes students being able to fluently evaluate numerical expressions with and without the use of technology.

#### Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may try to perform operations on bases as well as exponents.
- Students may multiply the base by the exponent instead of understanding that the exponent is the number of times the base occurs as a factor.
- Students may not truly understand exponents that are zero or negative.

# Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

- Part A. Natasha believes that any number to the zero power is one, and zero to any power is zero. Is her thinking correct? What would  $0^0$  be equivalent to?
- Part B. Louis enters  $0^{-2}$  on his calculator and receives an ERROR message. Explain to Louis why he received this message.

#### Instructional Task 2 (MTR.3.1, MTR.4.1)

Evaluate the following without using the use of technology. Answers should only have a single power and can expressed as a fraction or decimal.

Part A. 
$$\frac{7 \cdot 8^{2022}}{3 \cdot 8^{2024}}$$
  
Part B.  $\frac{(11^{74})^{10}}{(11^{82})^9}$ 

Instructional Task 3 (MTR.4.1, MTR.5.1)

Part A. Given  $f(x) = 1.5^x$ , complete the table below without the use of a calculator. Share with a partner how you determine the value of f(x) for each.

f(x)	()								
x		4	3	2	1	0	-1	-2	-3
		<b>.</b> .	<i>c</i> , ,		<b>. .</b> .				

Part B. Graph the function *f* in the domain of  $-3 \le x \le 4$  using technology. How does this compare to your table of values in Part A?

Part C. Using your graph from Part B, estimate to one decimal place, the value of x when f(x) = 2.

# Instructional Items

Instructional Item 1

Verify whether the equation below is true or false.



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 $(4\cdot 9)^5 = 4^5\cdot 9^5$ 

Instructional Item 2

Evaluate the numerical expression  $\left(-\frac{729}{64}\right)^{-\frac{2}{3}}$ .

Instructional Item 3

Choose all of the expressions that are equivalent to  $7^{\frac{5}{12}}$ 

a. 
$$(49\frac{1}{3})(7\frac{-1}{4})$$
  
b.  $(7\frac{2}{3})(7\frac{-1}{4})$   
c.  $7(7\frac{-1}{4})$   
d.  $\sqrt[5]{712}$   
e.  $\sqrt[12]{75}$ 

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## MA.912.NSO.1.2

# Benchmark

MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of
exponents.
<i>Example:</i> The expression $1.5^{3t+2}$ is equivalent to the expression $2.25(1.5)^{3t}$ which is equivalent
to $2.25(3.375)^t$ .
Connecting Benchmarks/Horizontal Alignment
• MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7
• MA.912.F.1.2, MA.912.F.1.6
Terms from the K-12 Glossary

- Base
- Expression
- Exponent

# Vertical Alignment

**Next Benchmarks** 

Previous BenchmarksMA.8.AR.1.1

Purpose and Instructional Strategies

In grade 8, students generated equivalent algebraic expressions using the Laws of Exponents with integer exponents. In Algebra I, students expanded this work to include rational-number exponents. In Math for College Algebra, students extend the Laws of Exponents to algebraic expressions with logarithms.

- Instruction of this benchmark helps build the foundation for operations with exponents and logarithms throughout the course.
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations and the inverse relationship between powers and radicals (*MTR.5.1*).



- For example, students should recognize that  $\sqrt{2a^3} \cdot \sqrt{8a^2b^5} = 4a^2b^3\sqrt{a}$  is equivalent to  $\sqrt{2a^3} \cdot \sqrt{8a^2b^5} = 4a^{\frac{5}{2}}b^3$ .
- Students should be able to fluently apply the Laws of Exponents in both directions. • • For example, students should recognize that  $a^6$  is the quantity  $(a^3)^2$ ; this may be
  - helpful when students are factoring a difference of squares.
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies.

## Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.

#### Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1) What value of x will create a true statement in the equation  $144^x = 1728$ ?

Instructional Task 2 (MTR.3.1, MTR.4.1)

Part A. Write the algebraic expression  $\left(\frac{6x^2}{9x^2y^{5}z^{-8}}\right)^3$ , as an equivalent expression where each

variable only appears once.

Part B. Compare your method of simplifying with a partner.

## Instructional Items

#### Instructional Item 1

Given the algebraic expression  $2.3^{2t-1}$ , create an equivalent expression.

Instructional Item 2

Use the properties of exponents to create an equivalent expression for the given expression shown below with no variables in the denominator.

 $(4x^{-0.5})^3 \div (9x^{\frac{1}{3}})^{-1.5}$ 

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## MA.912.NSO.1.3

#### Benchmark

## MA.912.NSO.1.3 Generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents.

Benchmark Clarifications:

Clarification 1: Within the Algebra 2 course, radicands are limited to monomial algebraic expressions.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.2
- MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7



- MA.912.AR.7.1
- MA.912.F.1.2, MA.912.F.1.6

#### Terms from the K-12 Glossary

- Base
- Exponent
- Expression
- Rational Number

#### Vertical Alignment

**Previous Benchmarks** 

Next Benchmarks

• MA.8.AR.1.1

## Purpose and Instructional Strategies

In Algebra I, students generated equivalent algebraic expressions with rational-number exponents and performed operations with numerical expressions involving square or cube roots. In Algebra 2, students generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents. In Math for College Algebra, students extend the Laws of Exponents to algebraic expressions involving radicals, with a focus on the n<sup>th</sup> root of numbers and rewrite radical expressions as expressions with rational exponents and vice versa, whose radicand may include an expression that with more than one term.

- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably, to explain the meaning of rational exponents, with a focus on both conversion and application of properties.
- Instruction emphasizes that principal square root and even index has two answers.
- Other vocabulary terms may include square root, cube root, nth root, index, radicand, radical form, rational exponent, and simplest form. For a square root, the index is known to be a 2, but is not explicitly written.



- Instruction includes student discovery of the patterns and the connection to mathematical operations (*MTR.5.1*).
  - Rational exponents are extended beyond just a simple fraction of  $(\frac{1}{n})$  to  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , where students should recognize that to evaluate expressions with rational exponents of form  $(\frac{m}{n})$ , the rational exponent can be written as  $(\frac{1}{n}) \cdot m$ .



- For example, given  $6^{\frac{3}{2}} = (6^{\frac{1}{2}})^3$ , ask students to explain why this is true.
- Students should recognize that a base raised to the fractional exponent  $\frac{1}{2}$  denotes a square root or raised to  $\frac{1}{3}$  denotes a cube root. This understanding will allow students to use commutative properties to rewrite the fractional exponent to something more easily solvable.
- For example,  $(8x)^{\frac{2}{3}} = (8x)^{\frac{1}{3}\cdot 2} = (2x^{\frac{1}{3}})^2 = 4x^{\frac{2}{3}}$  or  $(8x)^{\frac{2}{3}} = (8x)^{2\frac{1}{3}} = (64x^2)^{\frac{1}{3}} = 4x^{\frac{2}{3}}$ Create models of several radicals showing the relationship between the radical expression
- ٠ and the power expression (MTR.2.1). Because  $a^{\frac{1}{2}}$  is a number whose square is a,
- $\sqrt{a} = a^{\frac{1}{2}}$ . Similarly,  $\sqrt[3]{a} = a^{\frac{1}{3}}$ . In general,  $\sqrt[n]{a} = a^{\frac{1}{n}}$  for any integer *n* greater than 1. Instruction includes the reasoning behind this pattern. For example,  $\sqrt{a} = a^{x}$

$$(\sqrt{a})^{2} = (a^{x})^{2}$$
$$a^{1} = a^{2x}$$
$$1 = 2x$$
$$\frac{1}{2} = x$$

- Have students explore this concept further using the cube root and the nth root.
- Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying roots with variables. When taking an even root of an even power and the result is an odd power, the absolute value of the result must be used to ensure that the answer is nonnegative.

  - For example, for an odd index:  $\sqrt[n]{x^n} = x$  or  $\sqrt[3]{5^3} = 5$  and  $\sqrt[3]{(-5)^3} = -5$  For example, for an even index:  $\sqrt[n]{x^n} = |x|$  or  $\sqrt[4]{5^4} = 5$  and  $\sqrt[4]{(-5)^4} = 5$
  - Instruction includes decimals as exponents.
    - For example,  $x^{3.2} = x^{3\frac{2}{10}} = x^{3\frac{1}{5}} = x^{\frac{16}{5}}$ .
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies (MTR.2.1).
- Instruction includes allowing students to review how to use technology to evaluate expressions with radical and rational exponents and use the index to make predictions to find the correct number of roots.
- It is important to reinforce and activate the prior knowledge of simple calculations with radicals within this benchmark.
- There is often more than one approach to simplifying radical expressions. Spend time exploring the process with students and solicit their solutions and not just their answers. (MTR.4.1)

## Common Misconceptions or Errors



- Students may not understand the difference between an expression and an equation.
- For variable expressions in the radicand, students may try to take the square root of the exponent instead of dividing it into 2.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.
- Students may incorrectly simplify radicals with a negative radicand depending on whether the index is even or odd.
- Students may neglect to insert the absolute value when taking the even root of an even power and the result is an odd power.
- Students may not know how to do simple calculations with radicals; therefore, they may not take the square root of the perfect square factor, or they may suggest using a factor pair within a radical that does not contain a perfect square as a factor.
- Students may confuse radicands and coefficients and perform the operations on the wrong part of the expression.
  - For example, express  $(2p)^{\frac{1}{5}}$  in radical form. The correct answer is  $\sqrt[5]{2p}$  instead of  $2\sqrt[5]{p}$ .

## Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1, MTR.5.1)

Evaluate the expression  $\sqrt[3]{64}$ . Compare your strategy with a partner.

# Instructional Task 2 (MTR.2.1, MTR.3.1, MTR.5.1)

Part A. Without the use of technology, graph  $f(x) = \sqrt[3]{x}$  over the domain  $-1 \le x \le 1$ .

Part B. Without the use of technology, graph  $f(x) = x^{\frac{4}{3}}$  over the domain  $-1 \le x \le 1$ . Part C. Compare the graphs from Part A and Part B.

# Instructional Items

Instructional Item 1

Express the following as a radical  $(36x^4)^{0.5}$ .

Instructional Item 2

Express the following as a radical  $\left(\frac{1}{40}x^4\right)^{\frac{1}{2}}$ .

Instructional Item 3

Express the following as a radical  $(32x^6y^8)^{\frac{3}{4}}$ .

Instructional Item 4

Express  $5\sqrt[3]{abc}$  as an expression with exponents.

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive

## MA.912.NSO.1.6

Benchmark



## MA.912.NSO.1.6 Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents.

## Benchmark Clarifications:

Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

# **Connecting Benchmarks/Horizontal Alignment**

MA.912.AR.5.2, MA.912.AR.5.8, MA.912.AR.5.9

#### Terms from the K-12 Glossary

- ٠ Base
- Exponent
- Expression
- Inverse function

# Vertical Alignment

- **Previous Benchmarks** 
  - MA.8.NSO.1.3 • ٠
    - MA.912.AR.5.3

## Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rationalnumber exponents. In Mathematics for College Algebra, students extend their knowledge to the inverse of exponential expressions, resulting in logarithms, as well as their understanding of the properties of exponents to include the properties of logarithms as applied to numerical logarithmic expressions.

Next Benchmarks

• MA.912.C.2.3

- Instruction makes the connection between the properties of logarithms and the properties • of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of  $y = a^x$  and,  $x = \log_a y$  given a > 0 and  $a \neq 1$  (MTR.5.1). Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the "opposite" of the properties of the exponents.
- Other vocabulary terms may include common logarithm, natural logarithm, and change of base formula.
  - Common logarithm is  $\log_{10} a$ 0
  - Natural logarithm is  $\log_e a$ , which is denoted as *lna*. 0
  - Change of Base Formula is  $\log_a m = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log_a} = \frac{\ln m}{\ln a}$ . 0
- This chart may be used to compare the properties of logarithms to the properties of exponents.

Logarithms	Exponentials
$\log_a m + \log_a n = \log_a mn$	$a^m \cdot a^n = a^{m+n}$
$\log_a m - \log_a n = \log_a \frac{m}{n}$	$\frac{a^m}{a^n} = a^{m-n}$
$\log_a m^n = n \log_a m$	$(a^m)^n = a^{mn}$
$\log_a 1 = 0$	$a^{0} = 1$



- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition  $b^{\log_b x} = x$ . Therefore, students can take the log with base *a* of both sides of the equation to obtain  $\log_a b^{\log_b x} = \log_a x$ . Then, students can use the Power Property to rewrite the equation as  $(\log_b x)(\log_a b) = \log_a x$ . Students should notice that there the arguments for two of the logs are *b* and *x* with each the same base of *a*. So, one can divide both sides of the equation by  $\log_a b$  to isolate  $\log_b x$ , obtaining  $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$ . Therefore,  $\log_b x = \frac{\log_a x}{\log_a b}$ .
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties (*MTR.4.1*).
  - For example, present students with log<sub>1.03</sub> 1.092727 and ask them for its meaning, "What exponent should be applied to the base 1.03 to achieve the result of 1.092727?"
- Many examples in this course will require students to utilize the change of base formula to evaluate logarithms. Consider the example above.

$$\log_{1.03} 1.092727 = \frac{\log 1.092727}{\log 1.03}$$
 or  $\frac{\ln 1.092727}{\ln 1.03}$ 

- Evaluating this expression with a calculator requires the use of common logs or natural logs. Guide students to understand that common logs have a base of 10 while natural logs have a base of *e* and are easy to enter into a calculator. Have students use a calculator to evaluate the expression to find it is approximately equivalent to 3. Have students confirm this by checking the exponential equivalent expression  $1.03^3 = 1.092727$ .
- While students will see more complex logarithms in this course, it is appropriate to simplify the logarithms they work with initially in this benchmark until they reach an understanding of the properties of logarithms. Depending on the courses students have taken prior to Math for College Algebra, this may their first introduction to logarithms.
- Instruction includes the understanding that logarithms with base 10 are called common logarithms and written as  $\log x$  or  $\log_{10} x$ .
- Instruction includes the understanding that logarithms with base e are called natural logarithms and written as  $\ln x$  or  $\log_e x$ .
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate it or use the properties.
  - For example, present students with log<sub>2</sub> 64 and ask for its meaning, "the exponent required on the base 2 to obtain 64" (*MTR.2.1*).
- Students should understand how to use their one-to-one and inverse properties to evaluate expressions by making decisions on which base is best to use for solving.
  - One-to-One Properties

		$a^{\mathcal{Y}} = a^{\mathbf{x}}$	y = x				
		$log_a \mathbf{x} = log_a \mathbf{y}$	x = y				
Inverse Properties							
	Lo	Logarithmic Form		Exponential For	m		
		$log_a 1 = 0$		$a^0 = 1$			



0

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## Algebra I B1G-M

$log_a a = 1$	$a^1 = a$
$log_a a^x = x$	$a^x = a^x$
$a^{\log_{a^x}} = x$	$a^{xx} = a^x$

• Students should have practice combining the properties of logarithms to generate equivalent algebraic expressions by either simplifying (condensing) or expanding the expression.

Problem types include logarithms with different bases, including common logarithms and natural logarithms.

#### Common Misconceptions or Errors

- Students may see "log" as a variable rather than as an operation.
  - For example, they may see "log" as a common factor in the expression,  $\log 12 + \log 6$  and mistakenly write  $\log(12 + 6)$ .
  - For example, they may rewrite the expression  $\log(12 \cdot 6)$  and as  $\log 12 \cdot \log 6$ instead of  $\log 12 + \log 6$ . Similarly, they may incorrectly rewrite  $\log \left(\frac{12}{6}\right)$  as  $\frac{\log 12}{\log 6}$

instead of  $\log 12 - \log 6$ .

- For example, they may cancel the "log" from the numerator and the denominator in an expression.
- Students may incorrectly use the Change of Base Formula.
- Students may mistakenly evaluate  $\log_a 1$  as 1 instead of 0.
- When expanding a logarithmic expression, students may forget that the power on the logarithm becomes the coefficient.
  - For example,  $\log_5 16^2 = 2 \log_5 16$ . Or  $\log_2 16^3 = 3 \log_2 16 = 3 \cdot 4 = 12$ .
- Students may cancel the "log" from the numerator and the denominator in an expression. Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.

## Instructional Tasks

# Instructional Task 1 (MTR.3.1)

Recall that  $\log_b(x)$  is by definition the exponent which *b* must be raised to in order to yield x (b > 0).

Part A.

- Use this definition to compute  $\log_2(2^5)$ .
- Use this definition to compute  $\log_{10}(0.001)$ .
- Use this definition to compute  $\ln(e^3)$ .
- Explain why  $\log_b(b^y) = y$  where b > 0.

Part B.

- Evaluate  $10^{\log_{ln}(100)}$ .
- Evaluate  $2^{\log_2(\sqrt{2})}$ .
- Evaluate  $e^{\ln(89)}$ .

Explain why  $b^{\log_b x} = x$  where b > 0.

# Instructional Items

## Instructional Item 1 (MTR.3.1)

Use the properties of logarithms to rewrite each expression into lowest terms.



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- a.  $\log_2 2^5$
- b.  $\log_3 \frac{12}{5}$
- c.  $\log_5 3 + \log_5 13$ d.  $\log_5 150 \log_5 6$ e.  $3\ln e^7 \ln e^2$

# Instructional Item 2

What is the value of the logarithmic expression  $\log_2(2^6)$ ?

## Instructional Item 3 Evaluate the logarithmic expression $\log_4 64^{-5}$ .

## Instructional Item 4

Which of the following expressions are equivalent to  $\log_4 16$ ? Select all that apply.

- log 4 • log 16 log 16 ln 4 ln 4 ln 16 ln 16 ln 4 ln 4 log 16 log 16
- log 4

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered

# MA.912.NSO.1.7

# Benchmark

#### MA.912.NSO.1.7 Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.

## Benchmark Clarifications:

Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

## Connecting Benchmarks/Horizontal Alignment

• MA.912.AR.5.2, MA.912.AR.5.8, MA.912.AR.5.9

## Terms from the K-12 Glossary

• Inverse function

# Vertical Alignment

- **Previous Benchmarks** 
  - MA.8.NSO.1.3

Next Benchmarks • MA.912.C.2.8

## Purpose and Instructional Strategies



In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rationalnumber exponents. In Mathematics for College Algebra, students extend their knowledge to the inverse of exponential expressions, resulting in logarithms, as well as their understanding of the properties of exponents to include the properties of logarithms as applied to algebraic logarithmic expressions.

- Instruction makes the connection between the properties of logarithms and the properties of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of  $y = a^x$  and,  $x = \log_a y$  given a > 0 and  $a \neq 1$  (MTR.5.1). Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the "opposite" of the properties of the exponents.
- Other vocabulary terms may include common logarithm, natural logarithm, and change of base formula.
  - Common logarithm is  $\log_{10} a$
  - Natural logarithm is  $\log_e a$ , which is denoted as *lna*.
  - Change of Base Formula is  $\log_a m = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log a} = \frac{\ln m}{\ln a}$ 0
- This chart may be used to compare the properties of logarithms to the properties of exponents.

Logarithms	Exponentials
$\log_a m + \log_a n = \log_a mn$	$a^m \cdot a^n = a^{m+n}$
$\log_a m - \log_a n = \log_a \frac{m}{n}$	$\frac{a^m}{a^n} = a^{m-n}$
$\log_a m^n = n \log_a m$	$(a^m)^n = a^{mn}$
$\log_a 1 = 0$	$a^0 = 1$
$\log_a a = 1$	$a^1 = a$

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition  $b^{\log_b x} = x$ . Therefore, students can take the log with base a of both sides of the equation to obtain  $\log_a b^{\log_b x} = \log_a x$ . Then, students can use the the Power Property to rewrite the equation as  $(\log_b x)(\log_a b) = \log_a x$ . Students should notice that the arguments for two of the logs are b and x with each having the same base of a. So, one can divide both sides of the equation by  $\log_a b$  to isolate  $\log_b x$ , obtaining  $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}.$  Therefore,  $\log_b x = \frac{\log_a x}{\log_a b}$
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties (MTR.4.1).
  - For example, present students with log<sub>1.03</sub> 1.092727 and ask them for its meaning, "What exponent should be applied to the base 1.03 to achieve a result of 1.092727?"
- Students should have practice combining the properties of logarithms to generate equivalent algebraic expressions by either simplifying (condensing) or expanding the expression.
- Problem types include logarithms with different bases, including common logarithms and



natural logarithms.

#### Common Misconceptions or Errors

- Students tend to treat "log" as a variable rather than as an operation:
  - For example, they may see "log" as a common factor in the expression  $\log x +$  $\log y$  and mistakenly write  $\log(x + y)$ .
  - For example, they may distribute the "log" in the expression  $log(a \cdot b)$  and 0 rewrite it as  $\log a \cdot \log b$  instead of  $\log_a m + \log_a n$ . Similarly, they may incorrectly rewrite  $\log\left(\frac{a}{b}\right)$  as  $\frac{\log a}{\log b}$
  - For example, they may divide both sides of the equation log(7x 12) = 2log x0 by "log" to mistakenly obtain 7x - 12 = 2x.
- Students may cancel the "log" from the numerator and the denominator in an expression. • Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.
- When expanding a logarithmic expression, students may forget the power on the ٠ logarithm becomes the coefficient. They may also forget  $\log_a a = 1$ .
  - For example,  $\log 10 x^4 = \log_{10} 10 x^4 = \log_{10} 10 + \log_{10} x^4 = 1 + 4 \log x$ .
- Students may struggle to see the general structure and relationship between dividing and subtracting and multiplying and adding.
  - o For example, when expanding the logarithm of a quotient, students may write
  - $\ln \frac{3x^4}{y} = \frac{\ln 3x^4}{\ln y}$ . To address this misconception, remind students:  $\ln \frac{a}{b} = \ln a \ln b$ . For example, when expanding the logarithm of a product, remind students  $\log_3 2x = \log_3 2 + \log_3 x$ , not  $\log_3 2 \cdot \log_3 x$ .
- Students may incorrectly use the Change of Base Formula.
- Students may mistakenly evaluate  $\log_a 1$  as 1 instead of 0. •

# Instructional Tasks

## Instructional Task 1 (MTR.7.1)

The Loudness of Sound formula, measure in decibels (dB), is  $L = 10 \log I$ , where L is the loudness, and *I* is the intensity of sound.

- Part A. The formula  $\Delta L = L_2 L_1$ , describe the sound intensity level between two loudness,  $L_1$  and  $L_2$ . Write the formula in terms of the intensity of the sounds,  $I_2$ and  $I_1$ .
- Part B. Rewrite the formula for the sound intensity level,  $\Delta L$ , as a single logarithm.
- Part C. The table below shows the Ratios of Intensities and Corresponding Differences in Sound Intensity Levels. Using the Properties of Logarithms show that if a sound is 100 times as intense as another, it has a sound level about 20 dB higher.

$I_2/I_1$	$L_2 - L_1$
2.0	3.0 dB
5.0	7.0 dB
10.0	10.0 dB
100.0	20.0 dB
1000.0	30.0 dB



Part D. If the lowest or threshold intensity of sound a person with normal hearing can perceive is  $I_0 = 10^{-12} W/m^2$ , what is the intensity of sound at a pain level of 120 dB?

Instructional Task 2 (MTR.4.1, MTR.5.1)

Use the functions below to answer the following questions.

$$f(x) = 10^{0.2x}$$
$$h(x) = 5(\log x)$$

- Part A. Use technology to graph the functions f and h on the same coordinate plane.
- What do you notice? Part B. Determine f(2) and h(2). What do you notice?
- Part C. Discuss with a partner why logarithmic functions are said to be the inverse of exponential functions.

# Instructional Items

Instructional Item 1 Use the properties of logarithms to rewrite each expression into lowest terms. Part A.  $\log_4 4x^2$ Part B.  $\ln \frac{xy}{z}$ 

Instructional Item 2

Write each expression as a single logarithmic quantity: Part A.  $3 \ln x + 4 \ln y - 5 \ln z$ 

Part B. 
$$\frac{3}{2}\log_2 x^6 - \frac{3}{4}\log_2 x^8$$

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



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## Algebraic Reasoning

MA.912.AR.1 Interpret and rewrite algebraic expressions and equations in equivalent forms.

## MA.912.AR.1.2

## Benchmark

MA.912.AR.1.2	Rearrange equations or formulas	to isolate a quantity of interest.
Alaabra I Example:	The Ideal Gas I aw $PV = nPT$ can be re-	arranged as $T = \frac{PV}{P}$ to isolate temperatur

Algebra I Example: The Ideal Gas Law PV = nRT can be rearranged as  $T = \frac{1}{nR}$  to isolate temperature as the quantity of interest.

*Example: Given* the Compound Interest formula  $A = P(1 + \frac{r}{n})^{nt}$ , solve for P.

*Mathematics for Data and Financial Literacy Honors* Example: Given the Compound Interest formula  $A = P(1 + \frac{r}{n})^{nt}$ , solve for *t*.

Benchmark Clarifications:

*Clarification 1*: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.

*Clarification 2:* Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

## **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.3, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.2, MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.7.1
- MA.912.AR.8.1
- MA.912.AR.9.1

## Terms from the K-12 Glossary

• Equation

## Vertical Alignment

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Previous BenchmarksMA.912.AR.2.1

MA.912.AR.3.1

MA.912.AR.4.1

## Next Benchmarks

- MA.912.AR.6.1, MA.912.AR.6.6
  - MA.912.AR.7.3, MA.912.AR.7.4
- MA.912.AR.8.3

#### **Purpose and Instructional Strategies**

In Algebra I, students isolated a variable or quantity of interest in linear, absolute value and quadratic equations and formulas. In Mathematics for College Algebra, students highlight a variable or quantity of interest for other types of equations and formulas, including exponential, logarithmic, radical, and rational.



- Instruction includes making connections to inverse arithmetic operations (refer to Appendix D: Properties of Operations and Properties of Equality) and solving onevariable equations.
- Instruction includes making connections to inverse functions, such as, exponential and logarithm, square and square root, raising to the *n*th power and taking the *n*th root.
- Instruction includes justifying each step while rearranging an equation or formula.
  - For example, when arranging  $V = \frac{4}{3}\pi r^3$  for *r*, it may be helpful for students to highlight the quantity of interest with a highlighter, so students remain focused on that quantity for isolation purposes. It may also be helpful for students to identify factors or other parts of the equations.

## **Common Misconceptions or Errors**

- Students may not have mastered the inverse arithmetic operations.
- Students may not see that inverse operations can be performed with groups of numbers and variables treated as a single part of the formula (MA.912.AR.1.1).
- Students may be frustrated because they are not arriving at a numerical value as their solution. Remind students that they are rearranging variables that can be later evaluated as a numerical value.
- Having multiple variables and no values may confuse students and make it difficult for them to see the connections between rearranging a formula and solving one-variable equations.

# Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Given the equation  $ax^2 + bx + c = 0$ , solve for *x*.

Part B. Share your strategy with a partner. What do you notice about the new equation(s)?

#### Instructional Task 2 (MTR.7.1)

- Part A. Solve the formula for L, the length of the pendulum.
- Part B. A pendulum on Earth,  $g = 9.80 \frac{m}{s^{2}}$  has a period of 0.6 s, find its length.

Part C. Solve the formula for *g*, the acceleration of gravity.

Part D. If the same pendulum is moved to the moon, its period will be approximately 1.47 s. Calculate acceleration of gravity on the moon.

## Instructional Task 3 (MTR.7.1)

The total resistance,  $R_T$ , for a parallel combination of *n* resistors is found using the formula  $R_T = -\frac{1}{n}$ 

 $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}.$ 

- Part A. If the total resistance,  $R_T$ , for a parallel circuit with two resistors is  $1.71\Omega$  and one of the resistors has a resistance of  $3\Omega$ , what is the resistance of the second resistor?
- Part B. If we want to reduce the total resistance,  $R_T$ , of the parallel circuit to 1.5 $\Omega$  by adding a third resistor,  $R_3$ , what should be the resistance of  $R_3$ ?

#### Instructional Items



Last Updated: July 2024

#### Instructional Item 1

The formula for the velocity, v, of a satellite moving around Earth is  $v = \sqrt{\frac{6.67 \times 10^{-11} (m)}{r}}$ 

where m is the mass of the central body about which the satellite orbits, in this case Earth and r is the radius of orbit for the satellite. Solve for the radius r.

# Instructional Item 2

The lens equation  $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$  expresses the quantitative relationship among the distance,  $d_o$ , between an object and a lens, the distance,  $d_i$ , between the lens and the image position, and the focal length, f, of the lens. Solve for the distance between the lens and the image,  $d_i$ .

#### Instructional Item 3

The continuous compound interest formula,  $A = Pe^{rt}$ , where P represents the initial amount, A is the total amount, r is the rate of interest, and t represents time in years. The number e represents Euler's number is a mathematical constant. Solve for the time.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.1.3

## Benchmark

MA.912.AR.1.3	Add, subtract and multiply polynomial expressions with rational
	number coefficients.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes an understanding that when any of these operations are performed with polynomials the result is also a polynomial.

Clarification 2: Within the Algebra I course, polynomial expressions are limited to 3 or fewer terms.

## **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.1, MA.912.NSO.1.2
- MA.912.F.3.2

## Terms from the K-12 Glossary

- Area model
- Distributive property
- Polynomials

## Vertical Alignment

#### **Previous Benchmarks**

• MA.8.AR.1.2, MA.8.AR.1.3 MA.912.AR.1.7

## Next Benchmarks • MA.912.AR.1.6, MA.912.AR.1.8,

MA.912.AR.1.11 MA.912.AR.6.3

## **Purpose and Instructional Strategies**

In middle grades, students added, subtracted, and multiplied linear expressions. In Algebra I, students performed operations on polynomials limited to 3 or fewer terms. In Mathematics for



College Algebra, students extend this knowledge of polynomial expressions and perform operations on all polynomials.

- Instruction includes the connection to addition, subtraction, and multiplication of polynomials to develop the understanding of closure, and the connection to properties of exponents.
- Instruction includes making the connection to dividing polynomials and the understanding that division does not have closure.
- Reinforce like terms during instruction (using different colors can be a strategy to help identify them as unique from one another).
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model, properties of exponents and the distributive property (*MTR.2.1*).
  - o Area Model

The expression  $(2x^2 + 1.5x + 6)(3x + 4.2)$  is equivalent to  $6x^3 + 12.9x^2 + 24.3x + 25.2$  and can be modeled below:

	$2x^2$	1.5x	6
3 <i>x</i>	$6x^3$	$4.5x^{2}$	18 <i>x</i>
4.2	$8.4x^2$	6.3 <i>x</i>	25.2

• Multiplying Vertically

$$2x^{2} + 1.5x + 6$$

$$\underline{\times \quad 3x + 4.2}$$

$$8.4x^{2} + 6.3x + 25.2$$

$$\underline{6x^{3} + 4.5x^{2} + 18x}$$

$$6.3x^{3} + 12.9x^{2} + 24.3x + 25.2$$

o Multiplying horizontally

Using the distributive property:

$$(2x^3 - 4x^2 + 5x - 10) \cdot (x^2 + 10x - 3)$$
  
(2x<sup>3</sup>)(x<sup>2</sup> + 10x - 3) + (-4x<sup>2</sup>)(x<sup>2</sup> + 10x - 3) + (5x)(x<sup>2</sup> + 10x - 3)  
+ (-10)(x<sup>2</sup> + 10x - 3)

- Instruction should not rely upon the use of tricks or acronyms, like FOIL. (as FOIL only relates to two binomials)
- Students also can use special product cases such as perfect square trinomials and difference of squares.

Special Product Pattern Cases							
Perfect Square Trinomials	Perfect Square Trinomials	Difference of					
		Squares					
$(a+b)^2 = (a^2 + 2ab + b^2)$	$(a-b)^2 = (a^2 - 2ab + b^2)$	(a+b)(a-b)					
		$=(a^2-b^2)$					

#### Common Misconceptions or Errors

• Students may not understand the meaning of closure or the operations it applies to with



polynomials.

- Students may not understand like terms or the properties of exponents.
- Students might think that polynomials are limited to monomials, binomials, and trinomials with just one variable. Provide practice with polynomials with multiple variables and multiple terms.
- Students may not correctly apply the distributive property when multiplying polynomials, leaving out one or more terms. Encourage the use of the area model so that is easier for them to account for all the terms.
- Students may incorrectly distribute the exponent when expanding a binomial raised to a power. Remind the students that, the exponent indicates how many times the based is multiplied.

## **Instructional Tasks**

Instructional Task 1 (MTR.3.1, MTR.4.1)

- Part A. Determine the sum of  $3x^2 2x + 5$  and  $\frac{1}{6}x^2 + 7x + \frac{8}{7}$ . Explain the method used in determining the sum.
- Part B. Discuss whether the addition of polynomials will always result in another polynomial. Why or why not?
- Part C. Determine the difference of  $3x^2 2x + 5$  and  $x^2 0.25x + 1.24$ . Explain the method used in determining the difference.
- Part D. Discuss whether the subtraction of polynomials will always result in another polynomial. Why or why not?
- Part E. Determine the product of 2x + 5 and  $\frac{2}{9}x^2 \frac{11}{2}x + 1$ . Explain the method used in determining the product.
- Part F. Discuss whether the multiplication of polynomials will always result in another polynomial. Why or why not?
- Part G. Determine the quotient of  $9x^2 3x + 12$  and 3x. Explain the method used in determining the quotient.
- Part H. Discuss whether the division of polynomials will always result in another polynomial. Why or why not?

## **Instructional Items**

#### Instructional Item 1

Determine the sum/difference of the expression

$$\left(\frac{4}{5}a^2 - ab - 5\right) + \left(\frac{6}{5}a^2 + 2.5ab + 7\right) - (-5a^2 + 6ab + 4).$$

Instructional Item 2

Determine the product of the expression  $\left(\frac{1}{2}s+1\right)\left(\frac{1}{2}s-1\right)\left(\frac{1}{4}s^2+1\right)$ .

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

## MA.912.AR.1.5

#### Benchmark

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# MA.912.AR.1.5 Divide polynomial expressions using long division, synthetic division or algebraic manipulation.

**Connecting Benchmarks/Horizontal Alignment** 

MA.912.NSO.1.1, MA.912.NSO.1.2

## Terms from the K-12 Glossary

- Algorithm
- Divisor
- Polynomials

## Vertical Alignment

## **Previous Benchmarks**

- MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.4

## Next Benchmarks • MA.912.AR.1.6(Alg2/H) MA.912.AR.6.2(Alg 2H), MA.912.AR.6.3(PreCalc)

## **Purpose and Instructional Strategies**

In Algebra I, students perform operations on polynomials limited to 3 or fewer terms and divide polynomials by a monomial. In Mathematics for College Algebra, students extend this knowledge of polynomial expressions and perform operations on all polynomials.

- Instruction includes the connection to addition, subtraction, and multiplication of polynomials to develop the understanding of closure, and the connection to properties of exponents.
- Instruction includes making the connection to dividing polynomials and the understanding that division does not have closure.
- Instruction includes reintroducing proper vocabulary and terminology like dividend, divisor and quotient.

dividend		divisor		quotient		dividend	$\rightarrow$	6				
$\downarrow$		$\downarrow$		Ļ	or			-	=	3	$\leftarrow$	quotient
6	÷	2	=	3		divisor	$\rightarrow$	2				

- Instruction includes there are two methods for dividing: long division and synthetic. The connection to long division of whole numbers to develop the understanding that polynomial division follows a very similar procedure and that when the remainder is zero, the divisor and quotient are factors of the dividend (*MTR.5.1*).
- Long Division in this method, the higher degree is the dividend and the lower degree polynomial is the divisor. Students should know that they will not always get a 0 as a remainder. If it is a remainder other than 0, it should be written as <u>Remainder</u>.
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Last Updated: July 2024

# Algebra I B1G-M

# Long Division of Polynomials



- Synthetic division can only be used to divide a polynomial by a binomial of degree 1, such as x k. The terms in both the divisor and the dividend must be in descending order.
  - For example, divide  $-x^3 + 5x^2 + 6$  by x 3. In this example the k value is 3 and is the divisor.
    - Write the coefficients of the dividend in order of descending exponents, including a 0 for any missing terms. Write the *k*-value to the left of the vertical bar.

• Bring down the leading coefficient. Multiply the leading coefficient by the *k* value and write the product below the next leading coefficient and add.

• Multiply the sum (2 in this example) by the *k* value and write the product under the next leading coefficient. Repeat this process for the remaining coefficients.



- The last number in the bottom row is the remainder and the others are the coefficients of the quotient.
  - In this example, the quotient is  $-1x^2 + 2x + 6 + \frac{24}{x-3}$



- It is not the expectation of this benchmark that students perform division using a specific method or strategy. Students should be able to compare various methods and use the one most efficient depending on the task.
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model and properties of exponents.
  - Area Model
    - For example, when dividing  $2x^2 + 7x + 6$  by x + 2, students can create a table with the same number of rows as the number of terms in the divisor (as shown below).



Then, students can write the first term of the dividend in the upper left rectangle. Have students determine what the length must be if the area of that rectangle (shaded yellow) is  $2x^2$ .



Once students determine that its length is  $2x^2 \div x$  which is 2x, they can write the length of the rectangle.



Students should be able to see that they can calculate the area of the bottom left rectangle as 2x(2) = 4x.



Next, students can use this information to determine the area of the upper right rectangle (shaded green). Students should notice that the second term of the dividend is 7x, which means that when you add the like terms diagonally you need to get 7x. The area of this rectangle is 7x - 4x = 3x.



Lastly, students can calculate the length of the column, and then the area of the bottom right rectangle to determine the quotient of  $2x^2 + 7x + 6$  and x + 2 is 2x + 3.



## Common Misconceptions or Errors

- Students may forget to write the polynomials in standard form.
- Students may forget to insert a placeholder for polynomial divisors or dividends with missing terms, meaning those terms have a coefficient of zero.



- When using long division, students sometimes forget to multiply each monomial in the quotient by every term in the divisor. To address this misconception, remind students to use the distributive property to multiply by each term.
- Students may forget that synthetic division can only be used when you are dividing by a linear factor.
- When doing long division, students may add instead of subtracting the terms.
- When doing synthetic division, students may forget that the leading coefficient of the divisor should be 1.
- When doing synthetic division, students may forget to reverse the sign of the constant in the divisor.

## Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1, MTR.7.1) The volume of a rectangular prism is given by the polynomial  $3x^4 + 7x^3 - 12x^2 + 4x$  and the area of the base is given by the polynomial  $3x^2 - 2x$ . Part A. Find the height of the rectangular prism.

Part B: Find the length and the width of the rectangular prism.

Instructional Task 2 (MTR.3.1, MTR.4.1)

Part A. Determine the quotient of  $4m^4 + 2m^2 - 3m + 2$  and (2m + 6). Part B. Compare your strategy with a partner.

## Instructional Items

Instructional Item 1 Divide  $x^4 - 2x^3 + x^2 - 8x - 12$  by (x + 1).

Instructional Item 2

If (x - 4) is a factor of the polynomial  $2x^2 - ax - 16$ , what is the value of a?

Instructional Item 3

Determine the quotient of  $x^4 + 4x^3 - x - 4$  and  $x^3 - 1$ .

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.1.9

## Benchmark

MA.912.AR.1.9 Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions.

## Benchmark Clarifications:

Clarification 1: Instruction includes the connection to fractions and common denominators.

## **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.2
- MA.912.AR.8.1

#### Terms from the K-12 Glossary

• Rational expression



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• Least common multiple

#### Vertical Alignment

## **Previous Benchmarks**

•

MA.912.AR.1.4

• MA.8.AR.1.2, MA.8.AR.1.3

Next Benchmarks MA.912.AR.8.2(Alg 2/H), MA.912.AR.8.3(Alg2/H;PreCalc)

# Purpose and Instructional Strategies

In middle grades students perform operations with rational numbers, including fractions. In Algebra 1, students rewrote polynomial expressions in different forms by factoring, distributing, and/or combining like terms. In Math for College Algebra, they extend the work they did with operations with rational numbers and rewriting polynomial expressions to operations with rational expressions.

- Instruction includes the connection to fractions and common denominators. A rational expression, same as a fraction, can be written as a ratio. Instead of a ratio of two numbers, rational expressions are a ratio of two polynomial expressions.
- The properties of fractions can be applied to rational expressions.
  - o Simplifying rational expressions
  - Factor the numerator and denominator and extract common factors.
  - Multiply rational expressions It may be helpful to first, factor the numerator and the denominator then, multiply the numerators and multiply the denominators.
  - Divide rational expressions
     Students can multiply the first expression by the reciprocal of the second expression and follow the multiplication steps.
    - The properties of exponents can be applied to rational expressions, especially the quotient property of exponents,  $\frac{a^m}{a^n} = a^{m-n}$ , will be beneficial when simplifying rational expressions by division. Students need to be allowed to make the connection between  $\frac{x^5}{x^2}$  and  $\frac{(x+3)^5}{(x+3)^2}$ .
  - Adding and subtracting rational expressions Students can find a common denominator and then add or subtract the numerators.
- If polynomial expressions are not written in factored form, using the equivalent expression in factored form is helpful in determining the least common denominator.
- Instruction includes writing the restrictions or excluded values that make the domain 0. It is important to remind students that we list restrictions based on the original expression not the simplified expression. This is important for students as it will lead into extraneous solutions when solving rational expressions later.

## **Common Misconceptions or Errors**

- When simplifying rational expressions students cancel out terms instead of factors. Remind students to always factor before simplifying.
- When simplifying out the entire numerator students forget to write 1 as the numerator.
- When multiplying rational expressions students cross multiply instead of multiplying across.


- When adding and subtracting rational expressions students tend to forget to find a common denominator.
- When adding and subtracting rational expressions students mistakenly follow the steps of multiplication (i.e., add across the denominators and/or add across the numerators).
- Students may incorrectly divide out variable terms that are not factors. For example,  $\frac{x-2}{x-4} \neq \frac{-2}{-4}.$

#### Instructional Tasks

Instructional Task 1(MTR.3.1, MTR.5.1, MTR.7.1)

The U.S. Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficient for city driving and one for highway driving. For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high combined fuel economy. This number is not the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for x mpg in the city and y mpg on the highway, is

computed as 
$$\frac{1}{\frac{1}{2}(\frac{1}{x}+\frac{1}{y})}$$
.

- Part A. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.
- Part B. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function  $\frac{a(x)}{b(x)}$ .

Part C. Rewrite your answer from Part B in the form of  $q(x) + \frac{r(x)}{b(x)}$  where and q(x),

r(x) and b(x) are polynomials and the degree of r(x) is less than the degree of b(x).

Part D. Use your answer in Part C to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately x + 5.

#### Instructional Task 2 (MTR.5.1, MTR.7.1)

Tamisha walked 4 miles to the store to buy a new bike at an average speed of x miles per hours. She returned home riding her bike and her average speed was 2 miles faster than walking.

Part A. Write an expression for the time Tamisha takes to get to the store.

Part B. Write an expression for the time Tamisha takes to get home from the store.

Part C. Write an expression for the total time she spent on both trips.

Part D. If Tamisha spent 3 hours on both trips, what is Tamisha's average walking speed.

# Instructional Items

Instructional Item 1 (MTR.3.1)



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Perform the operation(s) on the rational expression. Write your answer as single rational expression.

100	bion.	
0	$x^2 - 5x - 24$	$5x^2$
a.	$6x + 2x^2$	8-x
b.	$\frac{9-x^2}{x^2+5x+6} \div$	$\frac{2x-6}{5x+10}$
c.	$\frac{5}{3x} + \frac{2}{7x} -$	$\frac{1}{2x}$

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



#### Last Updated: July 2024

# MA.912.AR.2 Write, solve and graph linear equations, functions and inequalities in one and two variables.

#### MA.912.AR.2.4

Benchmark	
MA.912.AR.2.4	Given a table, equation or written description of a linear function,
	graph that function, and determine and interpret its key features.
Benchmark Clarifica	tions:

*Clarification 1:* Key features are limited to domain, range, intercepts and rate of change.

*Clarification 2:* Instruction includes the use of standard form, slope-intercept form and point-slope form. *Clarification 3:* Instruction includes cases where one variable has a coefficient of zero.

*Clarification 4:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

*Clarification 5:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder notations.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.5 MA.912.F.1.6
- MA.912.F.2.1, MA.912.F.2.2, MA.912.F.2.3, MA.912.F.2.4, MA.912.F.2.5
- MA.912.AR.9.10

#### Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range (of a relation or function)
- · Rate of change
- Set-builder notation
- Slope
- x-intercept
- y-intercept

#### Vertical Alignment

revious Benchmarks Next Benchmarks	
• MA.8.AR.3.4	• MA.912.NSO.4.2(Alg2H)
MA.912.AR.2.2	• MA.912.AR.9.8(DM

# Purpose and Instructional Strategies

In grade 8, students graphed two-variable linear equations given a written description, a table, or an equation in slope-intercept form. In Algebra I, students learned to graph linear equations in standard form and furthered their practice with identifying and interpreting key features, which leads to important work with function transformations. In Math for College Algebra, students extend the concepts of graphing and interpretation of key features to other functions focusing on real-world contexts.

• While instruction for this benchmark builds and extends to other functions, students should first make the connection with their previous work with linear functions in



establishing the characteristics of graphing from various forms and representations. Providing students with opportunity to match the table, equation and graph will help to recall important features when identifying and comparing.

- Instruction includes representing domain, range and rate of change using words, inequality notation, set-builder notation, and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation may be new to students in this course.
- Words

If the domain is all real numbers, it can be written as "all real numbers" or "any value of *x*, such that *x* is a real number."

- Inequality notation
  - If the domain is all values of *x* greater than 2, it can be represented as x > 2. • Set-builder notation
  - If the range is all values of y less than or equal to zero, it can be represented as  $\{y | y \le 0\}$  and is read as "all values of y such that y is less than or equal to zero."
  - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, U, for union.
    - If the domain is all values of x less than or equal to 3, it can be represented as (-∞, 3]. If the domain is all values of x greater than 3, it can be represented as (3,∞). If the range is all values greater than or equal to -1 but less than 5, it can be represented as [-1, 5].
- In Math for College Algebra, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math	(Name)	Example	
Symbol	Meaning		
{	(Curly Bracket)	${x/x < 0}$	
	The set of	The set of all x values such that x is less	
		than 0	
	(Vertical bar)	$\{x \mid x < 0\}$	
	Such that, given	The set of all x values such that x is less	
		than 0	
U	Union, Or	(−1,3) ∪ (4,7)	
		x must be in the non-inclusive interval	
		from $-1$ to 3 or in the non-inclusive	
		interval from 7 to 7	
E	Is an element	$M = \{0, 1, 2, 3\}$	
	of	1 ∈ M	
		1 is an element of the set M	
R	The set of all	$x \in \mathbb{R}$	
	real numbers	x is an element of the set of all real	
		numbers	



- Within this benchmark, linear two-variable equations include horizontal and vertical lines. Instruction includes writing horizontal and vertical lines in the form y = 3 and x = -4 and as 0x + 1y = 3 and 1x + 0y = -4, respectively. Students should understand that vertical lines are not linear functions, but rather linear two-variable equations.
- Discussions about this topic are a good opportunity to foreshadow the use of horizontal and vertical lines as common constraints in systems of equations or inequalities.
- Instruction includes making the connection to constraints (MA.912.AR.2.5) for a given context.
- Instruction includes the use of *x*-*y* notation and function notation.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the *x* or *y*-axis when necessary.

#### **Common Misconceptions or Errors**

- Students may express initial confusion with the meaning of f(x) for functions written in function notation. To help address this, consider writing the same function in both forms simultaneously (*MTR.2.1*), i.e.,  $f(x) = \frac{2}{3}x + 6$  and  $y = \frac{2}{3}x + 6$ , to show that both f(x) and y represent the same function output(s) of the function.
- Students may misunderstand representing interval notation from smallest to largest when representing the domain or range. To address this misconception, have students sketch the graph to demonstrate their understanding of representing the domain from left to right and range from bottom to top of their graph.

# Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.7.1)

There are a total of 549 seniors graduating this year. The seniors walk across the stage at a rate of 42 seniors every 30 minutes. The ceremony also includes speaking and music that lasts a total of 25 minutes.

Part A. Write a function that models this situation. Compare your function with a partner. Part B. Graph the function you created in Part A. What is the feasible domain and range for this situation?

Part C. For how many hours will the graduation ceremony take place?

#### Instructional Task 2 (MTR.3.1, MTR.7.1)

The table shows the amount of money, *D*, in a savings account after *m* months.

Number of Months $(m)$	2	5	9	11
Dollar Amount ( <i>D</i> )	740	1100	1580	1820

Part A. Write a function that models this situation.

Part B. What is the *y*-intercept of the function you created? How does it mean in this situation?

Part C. Graph the function you created in Part A. What is the feasible domain and range for this situation?

Part D. How many months does it take to have \$3,260 in the saving account?

## Instructional Task 3 (MTR.3.1, MTR.4.1, MTR.7.1)



Martha spent \$109 renting chairs and tables for her party. Each table costs \$9.80 and the chairs \$2.00 each. The equation 2c + 9.8t = 129 represents the relationship between the chairs, *c*, the tables, *t*, and the dollar amount Martha spent renting these items.

Part A. Graph the equation that represents the situation described above. What does the point (5, 40) mean in this situation?

Part B. What is the feasible domain and range for this situation?

Part C. Identify the *x*- and *y*-intercepts. What do they mean in this situation? Explain your reasoning.

# Instructional Items

#### Instructional Item 1

Part A. Graph the function g(x) = -3.6x + 7.

Part B. Identify the domain, range and any intercepts of the function.

#### Instructional Item 2

Identify the slope and the *y*-intercept of the linear function.

x	-3	1	7	9	18
f(x)	-2.62	-1.66	-0.22	0.26	2.42

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.2.5

#### Benchmark

# MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

Algebra I Example: Lizzy's mother uses the function C(p) = 450 + 7.75p, where C(p) represents the total cost of a rental space and p is the number of people attending, to help budget Lizzy's 16th birthday party. Lizzy's mom wants to spend no more than \$850 for the party. Graph the function in terms of the context.

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain, range, intercepts and rate of change.

*Clarification 2:* Instruction includes the use of standard form, slope-intercept form and point-slope form. *Clarification 3:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

*Clarification 4:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

*Clarification 5:* Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.1.2
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.6, MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6



• MA.912.F.2

### Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range
- Rate of change
- Set-builder notation
- Slope
- *x*-intercept
- *y*-intercept

#### Vertical Alignment Previous Benchmarks

Next Benchmarks

- MA.8.AR.3.4, MA.8.AR.3.5
- MA.912.F.1.5

## Purpose and Instructional Strategies

In grade 8, students determined and interpreted the slope and *y*-intercept of a two-variable linear equation in slope-intercept form from a real-world context. In Algebra I, students solved real-world problems modeled with linear functions. In Mathematics for College Algebra, students will graph and solve problems involving linear functions.

• Instruction includes representing domain, range, constraints and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.

Words

If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."

o Inequality notation

If the domain is all values of x greater than 2, it can be represented as x > 2. • Set-builder notation

If the range is all values of y less than or equal to zero, it can be represented as  $\{y|y \le 0\}$  and is read as "all values of y such that y is less than or equal to zero."

- Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, U, for union.
  - If the domain is all values of x less than or equal to 3, it can be represented as (-∞, 3]. If the domain is all values of x greater than 3, it can be represented as (3, ∞). If the range is all values greater than or equal to -1 but less than 5, it can be represented as [-1, 5).
- In Math for College Algebra, students are introduced to new mathematical symbols.



,u	lage. All example of this is shown below.					
	Math	(Name)	Example			
	Symbol	Meaning				
	{	(Curly Bracket)	$\{x \mid x < 0\}$			
		The set of	The set of all x values such that x is less			
			than 0			
		(Vertical bar)	$\{x \mid x < 0\}$			
		Such that, given	The set of all x values such that x is less			
			than 0			
	U	Union, Or	(−1,3) ∪ (4,7)			
			x must be in the non-inclusive interval			
			from $-1$ to 3 or in the non-inclusive			
			interval from 7 to 7			
	E	Is an element	$M = \{0, 1, 2, 3\}$			
		of	1 ∈ M			
			1 is an element of the set M			
	R	The set of all	$x \in \mathbb{R}$			
		real numbers	x is an element of the set of all real			
			numbers			

Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as  $(-\infty, \infty)$ . This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Instruction includes making the connection to graphing (MA.912.AR.2.4) linear functions from any given representation.
- Instruction includes the use of *x*-*y* notation and function notation for all forms of linear functions (slope-intercept, point-slope and standard).
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that linear graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by linear functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Instruction includes the understanding that a real-world context can be represented by a linear two-variable equation even though it only has meaning for discrete values.



- For example, if a gym membership cost \$10.00 plus \$6.00 for each class, this can be represented as y = 10 + 6c. When represented on the coordinate plane, the relationship is graphed using the points (0,10), (1,16), (2,22), and so on.
- For mastery of this benchmark, students should be given flexibility to represent realworld contexts with discrete values as a line or as a set of points.
- Instruction directs students to graph or interpret a representation of a context that necessitates a constraint. Discuss the meaning of multiple points on the line and announce their meanings in the associated context (MTR.4.1). Allow students to discover that some points do not make sense in context and therefore should not be included in a formal solution (MTR.6.1). Ask students to determine which parts of the line create sensible solutions and guide them to make constraints to represent these sections.
- Instruction includes the use of technology to interpret key features and develop the understanding of constraints.

#### **Common Misconceptions or Errors**

- Students may assign their constraints to the incorrect variable.
- Students may miss the need for compound inequalities in their constraints. Students may
  not include zero as part of the domain or range.
  - For example, if a constraint for the domain is between 0 and 10, a student may forget to include 0 in some contexts, since they may assume that one cannot have negative people, for instance.
- Students may confuse an open interval with an ordered pair.

# Instructional Tasks

# Instructional Task 1 (MTR.2.1, MTR.3.1)

Sharisse is enrolling her daughter in daycare that costs \$75.00 per day and there is an initial fee to reserve the spot of \$300.

- Part A. Taking into account weekends, write a linear function that could describe this situation.
- Part B. Sharisse is planning for her monthly budget. If she only has her daughter attend daycare 3 days a week. How much would Sharisse have to pay?
- Part C. Share with a partner how you determine the amount for Part B.

# Instructional Task 2 (MTR.7.1)

A coffee truck has a high-volume urn that makes 3 gallons of coffee. The coffee truck sets up in a local park at 7:00 am and sells 12 ounces cups for \$2.50.

- Part A. Create a graph describing the relationship of how much coffee is left in the urn and how many cups of coffee are sold. Is there a linear relationship between the amount of coffee in the urn and how many cups of coffee are sold?
- Part B. What would be the domain and range for the graph created in Part A?

#### Instructional Items

#### Instructional Item 1

Use the function g(x) = 6.25 - 15x to answer the following.

Part A. Determine its intercepts.

Part B. Write a situation that could be described by this function.



#### Instructional Item 2

Tara has a lunch account in the school cafeteria. The balance, g(x), in the account at the beginning of the month can be represented by the function g(x) = -2.50x + 32, where x is the number of meals purchased.

Part A. Graph g(x).

- Part B. How much money was in Tara's account at the beginning of the month?
- Part C. What is the rate of change for the graph of this function?
- Part D. What does the slope represent in the context of the problem?
- Part E. Find g(x) when x = 11. What does this pair of values mean in the context of the problem?

#### Instructional Item 3

The homecoming committee bought 500 plastic souvenir footballs to sell at the homecoming game to raise money for a local charity. The profit (in dollars), p(x), from the sale of x footballs can be represented by p(x) = 5x - 128.

- Part A. Since the homecoming committee bought only 500 footballs, they can sell no more than this. Determine the constraint for profit the homecoming committee can earn.
- Part B. Is it possible for the profit to be exactly \$1500? Show your work and justify your answer. Part C. Is it possible for the profit to be at least \$2400? Show your work and justify your answer.

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

**Commented [FT1]:** Added instructional items 2 & 3. This is modified from Cpalms.



# MA.912.AR.3 Write, solve and graph quadratic equations, functions and inequalities in one and two variables.

#### MA.912.AR.3.7

# Benchmark

# MA.912.AR.3.7 Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

#### Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.

*Clarification 2:* Instruction includes the use of standard form, factored form and vertex form, and sketching a graph using the zeros and vertex.

*Clarification 3:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

*Clarification 4:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.8
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.6
- MA.912.F.2

#### Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Quadratic function
- Range
- Set builder notation
- x-intercept
- y-intercept

#### Vertical Alignment Previous Benchmarks

Next Benchmarks

• MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6

# Purpose and Instructional Strategies

In Algebra I, students were introduced to quadratic functions and learned to write, graph and interpret their key features. In this course, students build on the foundation to focus on quadratic functions within real-world contexts.

• Instruction includes conversations about interpreting *y*-intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and axis of



symmetry.

- When discussing end behavior, students should see a relationship between the sign of *a*, the value that determines if a quadratic function graph opens upward or downward, and the end behavior the function exhibits. Instruction presents students with the equation of the function first, before showing its graph and asking them to predict its end behavior (MTR 5.1). Using technology can help students see graph behaviors.
  - Depending on the form the function is presented in, students may be able to predict other features as well (MTR 5.1).
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than other depending on the context. In the real-world context a, b, and c may not be easy to work with so the use of technology may help students.
  - Standard Form

Can be described by the equation  $y = ax^2 + bx + c$ , where *a*, *b* and *c* are any rational number. This form can be useful when identifying the *y*-intercept, c, find the axis of symmetry using  $x = \frac{-b}{2a}$ , find the vertex of the quadratic, and

- determine whether the graph will open upwards or downwards, a.
- Factored form

Can be described by the equation  $y = (x - r_1)(x - r_2)$ , where  $r_1$  and  $r_2$  are real numbers and the roots, or x-intercepts. This form can be useful when identifying the x-intercepts, or roots.

- Vertex form Can be described by the equation  $y = a(x - h)^2 + k$ , where the point (h, k) is the vertex. This form can be useful when identifying the vertex and determining whether the graph will open upwards or downwards, a.
- Instruction includes the use of *x*-*y* notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. (Reference MA.912.AR.2.4)
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the *x* or *y*-axis when necessary.

Instruction includes opportunities for students to compare functions and show similarities between key features. It is also important that students see that the whole graph is symmetric across the vertex and if one point is found on one half of the graph, we can use its twin on the other half.

# **Common Misconceptions or Errors**

- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- When using interval notation, students may confuse when to use the parentheses and the



brackets appropriately. In these cases, ask reflective questions that will encourage the students to connect their knowledge about when to use the greater than or equal to/less than or equal to or just greater than/less than as well as the closed circle and open circles on the number line.

• When using interval notation, students may confuse when to use the *x*-values (domain/increasing, decreasing, constant, etc.) and when to use the *y*-values (range). In these cases, encourage the students to connect the graphical representation to their interval notation.

#### Instructional Tasks

Instructional Task 1 (MTR 2.1, MTR 4.1)

A given function is represented by the table below

the tuble	
x	f(x)
0	-1
0.5	0.25
1	1
1.5	1.25
2	1
2.5	0.25

Part A. Graph the function.

Part B. Determine the *y*-intercept and describe how you found it.

Part C. Over what interval is the function increasing? Over what interval is it decreasing?

Part D. What is the equation for the axis of symmetry? Describe how you found the axis of symmetry.

Part E. What is the domain of the function? What is the range of the function? Part F. Describe which representation, table or graph, is easier to use to find the key features of this function.

#### Instructional Task 2 (MTR.3.1, MTR.4.1, MTR.5.1)

Use the following function,  $f(x) = (x + 1)^2 + 4$  and  $g(x) = -\frac{1}{2}(x + 1)^2 + 4$  to answer the following.

Part A. Graph each function.

Part B. Compare the graphs and table of values and describe the transformation of

$$g(x) = -\frac{1}{2}(x+1)^2 + 4.$$

Part C. From the key features listed, determine which of them are the same and which of them are different.

key features: vertex, intercepts, domain, range, axis of symmetry, intervals where the function is increasing, decreasing, positive or negative, and end behavior.

Part D. Discuss the similarities and differences of the key features with a partner.

# Instructional Task 3 (MTR.5.1, MTR.6.1, MTR.7.1)

The height of a ball t seconds after it is thrown into the air from the top of a building can be modeled by  $h(t) = -16t^2 + 48t + 64$ , where h(t) is the height in feet.

Part A. Use technology to graph the function.

Part B. What is the height of the building? How do you know?

Part C. How many seconds does the ball rise before it starts to drop downward? Justify



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your answer.

Part D. After how many seconds does the ball hit the ground? Part E. Determine the constraints of this scenario.

# Instructional Items

Instructional Item 1 Graph the function f(x) = -2(x+6)(x+3). Identify the domain, range, vertex, and zeros of the function.

#### Instructional Item 2

Graph the function  $f(x) = \frac{1}{2}x^2 - 4x + 5$ . Identify the domain, range, vertex, zeros of the function, and the end behavior.

# Instructional Item 3

Graph the function represented in the table below. Describe the domain, range, vertex, and end behavior of the function.

x	f(x)
0	7.5
.5	11.25
1	12.5
1.5	11.25
2	7.5
2.5	1.25

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

#### MA.912.AR.3.8

Benchmark	
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine
	constraints in terms of the context.
Algebra I Example:	The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$ , where t is the number of years since 1972. In what year does the car's value start to increase?

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.

*Clarification 2:* Instruction includes the use of standard form, factored form and vertex form.

*Clarification 3:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

*Clarification 4:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

#### **Connecting Benchmarks/Horizontal Alignment**

• MA.912.AR.2.5



- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.10
- MA.912.F.1.2, MA.912.F.1.6
- MA.912.F.2

#### Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function notation
- Quadratic function
- Range
- *x*-intercept
- y-intercept

#### Vertical Alignment

#### **Previous Benchmarks**

 MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6

# Next Benchmarks • MA.912.F.1.7

#### Purpose and Instructional Strategies

In Algebra I, students were introduced to quadratic functions and learned to write, graph and interpret their key features. In Math for College Algebra, students use various types of numbers given from real-world situations that utilize quadratic functions to problem solve.

- Instructional expectation of this benchmark should reinforce core concepts and extend student knowledge through problem solving and real-world applications.
- Instruction features a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or should provide the graph or function that models the context.
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than other depending on the context. This benchmark is best taught with MA.912.AR.3.7, which provides important key features that could help determine which method to use to solve a problem considering the best set up for solving and graphing a quadratic function.
- Instruction includes the use of *x*-*y* notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation, and interval notation. (Reference MA.912.AR.2.5)
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.



- For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.
- For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- For example, if the range of a function is all real numbers, it can be represented in interval notation as  $(-\infty, \infty)$ . This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Instruction should include various methods such as quadratic function such as using the quadratic formula, factoring the quadratic and completing the square, which all lend themselves in solving quadratic equations. Using the discriminant can also support how to determine how many solutions there are to the given quadratic equation.

#### **Common Misconceptions or Errors**

- Students may find themselves stuck initially, unsure of where to start. In conversations with these students, prompt them to reflect on what they know about the context and how they can use that information to determine the requested information (*MTR.1.1*).
  - For example, students may have an equation in standard form and need to interpret the vertex in context. Prompting students to consider how they've calculated vertices in the past should lead them to choose to either convert the equation into vertex form or use the line of symmetry to help determine the vertex.
- Students misunderstanding of recognizing the vertex form is most useful for determining max or min may hinder some students from appropriately connecting concepts in problems.
  - For example, students may not understand the meaning to the vertex and will solve the original quadratic function by setting it equal to 0.
- Students may interpret f(x) = 0 as equivalent to f(0).
- Students may struggle interpreting key features and graphing from various forms of quadratic functions.
  - For example, students may be asked to find the vertex and may believe that completing the square is the only form to use. Prompting students to consider different forms will help with understanding the meaning of key features.

# Instructional Tasks

# Instructional Task 1 (MTR.2.1, MTR.5.1)

ABC Pool Company is constructing 17 feet by 11 feet rectangular pool. Along each side of the pool, they plan to construct a concrete sidewalk that has a constant width. The land parcel being used has a total area of 315 sq. ft. to construct the pool and sidewalks. The function  $f(x) = 4x^2 + 56x - 128$  represents the situation described. Transform the function to determine and interpret its zeros.

Instructional Task 2 (MTR.4.1)



During a math test, Charlie is given the equation  $4x^2 - 9x + 18 = 0$  and asked to find its discriminant.

- Part A. What is the discriminant of the equation? What does this discriminant tell Charlie about the nature of the solutions to the equation?
- Part B. Once he finds the discriminant, Charlie is asked to solve the equation. What are the solutions to the equation?
- Part C. After solving this equation, Charlie is given a new quadratic equation with real coefficients and is asked to solve it. He determines that there are two solutions, one real root and one imaginary root. Are Charlie's solutions, correct? Please explain your answer.

#### Instructional Task 3 (MTR.5.1)

- Part A. Determine the values of x that satisfy the equation  $-x^2 + 3x 1 = 0$ . If exact roots cannot be found, state the consecutive integers between which the roots are located.
- Part B. Use the equation of the axis of symmetry to sketch the graph of the function  $-x^2 + 3x 1 = y$ .
- Part C. Create a table to help identify the *x*-intercepts of the of the function  $-x^2 + 3x 1 = y$ .

Part D. Identify possible solutions to the of the function  $-x^2 + 3x - 1 = y$ .

#### Instructional Items

Instructional Item 1

The Chrysler Building, which opened in 1930, is one of New York's oldest buildings with a roof height of 925 feet (282 meters). Suppose you could conduct an experiment by dropping a small object from the roof of the Chrysler Building. Using the formula  $h(t) = -16t^2 + h_0$ , where t is the time in seconds and the initial height  $h_0$  in feet, how long would it take for the object to reach the ground, assuming there is no air resistance?

Instructional Item 2

The function, h, defined by  $h(t) = -5t^2 + 10t + 7.5$ , models the height of a diver above the water (in meters), seconds after the diver leaves the board.



Part A. How high above the water is the diving board?

Part B. When does the diver hit the water?

Part C. What is the maximum height the diver reaches during the dive?

Part D. After how long will the diver reach the same height as when he started?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



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MA.912.AR.4 Write, solve and graph absolute value equations, functions and inequalities in one and two variables.

# MA.912.AR.4.2

#### Benchmark

MA.912.AR.4.2 Given a mathematical or real-world context, write and solve onevariable absolute value inequalities. Represent solutions algebraically or graphically.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.8.1
- MA.912.AR.9.10

#### **Terms from the K-12 Glossary**

- Absolute value
- Compound inequality

# Vertical Alignment

# **Previous Benchmarks**

- MA.8.AR.2.2
  MA 912 AR 2 6
- MA.912.AR.2.6
- MA.912.AR.4.1
- MA.912.AR.4.3

# **Purpose and Instructional Strategies**

In 8th grade, students solved linear inequalities in one variable. In Algebra I, students solved one-variable absolute value equations. In Math for College Algebra, students will write and solve one-variable absolute value inequalities and one-variable linear inequalities involving other functions and systems of two-variable inequalities.

- Instruction includes making the connection between solving absolute value equations to solving absolute value inequalities. Where the solution to an absolute value equation is points, the solution to an absolute value inequality is going to be intervals.
  - For example, students should understand the following:  $|\mathbf{x}| = 3$  means the distance between x and 0 is 3;  $|\mathbf{x}| < 3$  means the distance between x and 0 is less than 3; and  $|\mathbf{x}| > 3$  means the distance between x and 0 is greater than 3.

Next Benchmarks

• MA.912.AR.3.3

- Instruction includes using words to help determine solutions.
  - For example, the equation |x| < 3 can be read as "Which values of x are less than 3 units away from zero?" This can be represented algebraically as -3 < x < 3 and the graphically solution looks like the number line below.
  - Instruction encourages students to discuss their thinking with their peers (*MTR.4.1*). Instructional focus should include conversations about the solution set algebraically and

graphically. Often students will be able to reason out their thinking, but will struggle with



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representing their thinking mathematically. Encourage this process and ask questions that will help with solving the task (*MTR.1.1*). Are there multiple ways for students to represent this problem mathematically (*MTR.2.1*)?

- Instruction includes student understanding that compound inequalities with "or" create a combining (or a union) of the solutions of the individual inequalities while compound inequalities with "and" create an overlap (or an intersection) of the solutions of the individual inequalities.
  - For example, if  $|x 1| \le 3$ , then  $x 1 \le 3$  and  $x 1 \ge -3$ .



- Students should check the solution(s) of the absolute value inequality to ensure there are no extraneous solutions.
  - For example, |x + 6| < 2x. Students will need to substitute values for *x* that is greater than 6 and less than -2.
- Instructions includes opportunities to create two inequalities from an absolute value inequality using a graphic organizer.
- Instruction includes opportunities to create two inequalities from an absolute value inequality using partially worked examples provided in a graph organizer.

Absolute Value Inequality	Inequality #1	Inequality #2
3x - 2  < 7	3x - 2 < 7	
$ 3x - 2  \ge 7$		$3x - 2 \le -7$

- Instruction includes students understanding that the quantity of an absolute value must be isolated prior to creating two inequalities.
  - For example, 5|1 4x| + 8 > 13, students will need to subtract 8 with 13 and then divide the result by 5. This will result with the following absolute value inequality: |1 4x| > 1.

• Instruction includes students understanding that if given and absolute value inequality less than (<) a negative number *i.e.* |x - 10| < -6 this will result in "no solution" and the name of the solution set will be "the empty set" or "null set". Since |x - 10| is an absolute value expression, it cannot be negative. Therefore, it is not possible for it to be less than -6. Therefore, |x - 10| < -6 will not have a solution. The set of the solutions will be considered as the "empty set" or "null set".

# **Common Misconceptions or Errors**

- Students may confuse the inequality symbols.
- Students may forget that multiplication or division by a negative number will change the direction of the inequality symbol.



- Students may incorrectly identify the solutions graphically-
- Students may misrepresent the inequality symbols on a number line and/or incorrectly identify the solutions graphically.
- Students may not create two inequalities from an absolute value inequality.
- Students may incorrectly identify the solutions graphically.
- Given that students' prior experience has dealt with equation work far more than inequality work, students will likely forget to consider the direction of the inequality symbol as they solve inequalities. To address this, have these students consider a context to understand the need to pay attention to the direction of the inequality symbol.
- Students may incorrectly distribute a number to the quantity of an absolute value.
- Students may represent the absolute value inequality using the incorrect compound inequality. It may be helpful to have students develop a graphic organizer to help organize steps.
- In interval notation, students may misinterpret the symbol for the union or intersection of two sets and indicate values incorrectly as being included as part of the solution set.

# Instructional Tasks

#### Instructional Task 1 (MTR.7.1)

Ms. Pruitt's candy company sells boxes of their famous chocolate-covered peanut butter bites. The candy is sold in boxes that must be within 95 grams of weighing 1,465 grams. Each piece weighs 42.5 grams and the box weighs 116 grams. Calculate and graph the possible quantities of chocolate-covered peanut butter bites that could be in a single box.

#### Instructional Task 2 (MTR.6.1, MTR.7.1)

Great Gardens Landscaping has been hired to build gardens around a new resort. They designed plans for two different rectangular gardens. In the first plan the length is twice the length of the width and in the second plan the length is 2 feet longer than the width. The perimeters of the gardens cannot differ more than 3 feet.

Part A. Write and solve an inequality to determine the range of the widths of the garden. Part B. What are the dimensions of the two gardens using the smallest allowable width?

#### Garden 2:

Part C. Using the largest allowable width, what is the area of the largest garden?

Garden 1:

#### Instructional Task 3 (MTR.6.1, MTR.7.1)

Tom is a machinist at Ancient City Choppers. To ensure the highest reliability of the motorcycle engines produced at the shop, Tom machines each cylinder to a 4.01' diameter with a tolerance of 0.002".

Part A. Write an absolute value inequality that could be used to determine the acceptable range of each cylinder's diameter.

Part B. Express that range graphically.

#### Instructional Task 4

The National Coffee Association says brewers should have an optimal brewing temperature of 200 degrees Fahrenheit, plus or minus five degrees.

Part A. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.



Part B. If the standard temperature in a Keurig is 192 degrees Fahrenheit, would you recommend that this product is meeting the National Coffee Association recommendations for an excellent cup of coffee? Explain.

#### Instructional Items

#### Instructional Item 1

There is a cloud layer centered at 4 miles above the sea that is 1,000 feet thick. At what elevations, x, given in miles, will a plane in that area be out of the clouds?

#### Instructional Item 2

Determine the solutions to the absolute value inequality below.  $|2x + 3| \le 6$ 

# Instructional Item 3

The manufacturing plant for a popular sugar brand fills each of its bags with 16 ounces of sugar to be sent to local grocery stores. After the bags are filled, another machine weighs them to ensure accuracy. If the bag weighs 0.7 ounces more or less than the desired weight, the bag is rejected.

Write an equation to find the heaviest and lightest bag the machine will approve. Include a verbal and graphical explanation of possible values that are acceptable.

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.AR.4.4

# Benchmark

# MA.912.AR.4.4 Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context.

#### Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior and symmetry. *Clarification 2:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.6, MA.912.AR.9.10
- MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2 (F.2.1 F.2.5)

#### Terms from the K-12 Glossary

- Absolute value
- Coordinate plane
- Domain
- Function notation



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- Intercept
- Piecewise function
- Range (of a relation or function)
- Set-builder notation
- x-intercept
- y-intercept

# Vertical Alignment

- **Previous Benchmarks** 
  - MA.912.AR.4.3

# Next Benchmarks

- MA.912.AR.3.3 (Alg 2; Math for ACT/SAT)
- MA.912.F.1.7 (Alg 2, Precalc)

## **Purpose and Instructional Strategies**

In Algebra 1, students graphed absolute value functions when given a table, equation or written description, and determined its key features. In Math for College Algebra, students solve and graph problems modeled with absolute value functions, determining key features and constraints within context.

- When making connections to transformations of functions, use graphing software to explore absolute value functions adding variability to the parent equation to see the effects on the graph. Allow students to make predictions (MTR.4.1).
- Instruction includes making the connection between the algebraic and graphical representations. In doing so, students can tie together key feature concepts of absolute value functions and provides opportunities to make connections to linear functions and its key features.
- Students should understand that there are two ways in which absolute value functions can be represented algebraically (MTR.5.1).

  - Using absolute value notation f(x) = |x|. Using piecewise notation  $f(x) = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$ 
    - The piecewise graph focuses on what should happen algebraically to the input value that will give its distance from the origin.
- Instruction includes utilizing students' knowledge of transformations to graph absolute value functions that are not parent functions (MTR.2.1).
  - For example, given f(x) = |x + 3|, to graph as a piecewise students can set the expression inside of the absolute value equal to zero to solve for the x-value where the function equals zero. So any value of x less than -3 and any value of x greater than -3.
    - Using Piecewise Notation f(x) = {x + 3, if x ≥ -3 -x 3, if x < 3
      </li>
      Using the transformation criteria, students should understand that the
    - graph of this function is a horizontal shift to the left. Points can be plotted using the slope of the function and other key features that it provides.
- Instruction reinforces the definition of absolute value as a number's distance from zero ٠ (0) on a number line. For absolute value functions, noticing where the argument of the absolute value bars will be zero can be helpful in ensuring that students graph correctly.



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• For example, if students are asked to graph y = |x + 2|, students should recognize that the absolute value will be a V-shaped graph and the sharp bend will occur at the value of x that makes the math statement of the function, |x + 2|, equal to zero. Notice in the table below the value that makes the math statement zero is -2 and to graph using a table, students can choose x values to the left and right of that critical value, -2.



- Instruction allows for the flexibility to graph absolute value functions using a variety of ways.
  - For example, the function y = |x + 2| can be graphed using transformations, recognizing that this graph shifts two units to the left and will open upward because there's a positive *a* value.
  - For example, the function y = |x + 2| can be graphed by plotting points from the vertex, using the "slope." In this case, once the vertex is plotted, as you travel 1 unit to the left and right you would go up one unit.
  - For example, for the function y = |x + 2|, students can create a T-chart and solve for the values that satisfy the function. It is important that students include negative inputs in their T-chart so they are not misled as to what the graph looks like. Students can use their understanding of the parent function of y = |x|, whose V-shaped graph has a center of 0. In creating and plotting points, students will use the vertex to find the center of the graph and then choose additional point *x*-values to use in the table to the left and right of the vertex.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation. For additional information, reference MA.912.AR.2.5.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- For example, if the range of a function is all real numbers, is can be represented in interval notation as (-∞,∞). This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Instruction includes the use of *x*-*y* notation and function notation for absolute value functions.



- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that absolute value graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by absolute value functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
  - For example, suppose you are managing a greenhouse where you need to maintain the temperature for optimal plant growth. The ideal temperature for the plants is 25°C. The function, f(x) = |x 25|, can be used to express the deviation, f(x) from the ideal temperature, where x represents the current temperature in degrees Celsius. If the temperature is within 5 degrees Celsius of the ideal temperature, then there is no significant impact on the plants. For optimal plant growth, the constraint on the function, the range will be represented as  $|x 25| \le 5$  in which restricts the domain (the temperature) to be  $20 \le x \le 30$ .
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Depending on a student's pathway in the future, it will be important to understand that there is a relationship between piecewise functions and absolute value functions when working with derivatives or integration of functions (*MTR.5.1*).
- When addressing real-world contexts, the absolute value is used to define the difference or change from one point to another. Connect the graph of the function to the real-world context so the graph can serve as a model to represent the solution (*MTR.6.1, MTR.7.1*).
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the *x* or *y*-axis when necessary.
- Problem types include applying distance to real-world problems, showing how much a value deviates from the norm.
  - For example, normal body temperature varies by person, age, activity, and time of day. The average normal body temperature is generally accepted as 98.6°F (37°C). Some studies have shown that the "normal" body temperature can have a wide range, from 97°F (36.1°C) to 99°F (37.2°C). https://medlineplus.gov/ency/article/001982.htm

The function, y = |x - 98| is represented in the graph below.



The *x*-axis corresponds to the temperature of the person in question. What does



the *y*-axis represent? However, to represent a normal temperature, the range of the graph must be limited to  $0 \le y \le 1$  or [0, 1]. Any value outside the range is either hypothermia (the surrounding temperature is too cold for the body to keep up) or hyperthermia (indicating a fever). Therefore, the final graph would be a "V" from the point (97, 1) down to (98, 0) and back up to (99,1).

• Instruction includes opportunities for students to compare functions and show similarities between key features. It is also important that students see that the whole graph is symmetric across the vertex and if one point is found on one half of the graph, we can use its twin on the other half.

#### **Common Misconceptions or Errors**

- Students may think that distributive property should be used if a constant is in front of the absolute value.
  - For example, for f(x) = 4|x 2|, students will incorrectly distribute the 4 to the absolute value function when evaluating.
- Students may not fully understand the connection to all the key features (emphasize the use of technology to help with student discovery) and how to represent them using the proper notation.
- Students may confuse the transformations when graphing.
  - For example, for f(x) = |x 2|, students may incorrectly graph the vertex two units to the left rather than two units to the right.

#### Instructional Tasks

# Instructional Task 1 (MTR.7.1)

A car starts traveling on the interstate at 9 a.m. at a speed of 70 miles per hour. The car's distance from its destination in miles, y, is represented by the function y = |100 - 75x|, where x is the number of hours since 9 a.m. The car will stop at 2 p.m.

- Part A. Find and interpret the y-intercept of the graph of the function.
- Part B. Find and interpret the vertex of the graph of the function.
- Part C. Find the intervals where the function is increasing and decreasing.

# Instructional Task 2 (MTR.3.1, MTR.5.1)

- Part A. Graph g(x) = -|x 4| + 2.
- Part B. Compare the graph from Part A to its parent function f(x) = |x|.
- Part C. Use the information from Part A and B to determine the domain and range of the function g(x).

# Instructional Items

#### Instructional Item 1

Electrical parts, such as resistors and capacitors, come with specified values of their operating parameters: resistance, capacitance, etc. However, due to imprecision in manufacturing, the actual values of these parameters vary somewhat from piece to piece, even when they are supposed to be the same. The best that manufacturers can do is to try to guarantee that the variations will stay within a specified range, often  $\pm 1\%$ ,  $\pm 5\%$  or  $\pm 10\%$ . Suppose we have a resistor rated at 680 ohms,  $\pm 5\%$ . Write an absolute value function to represent the range of possible values of the actual resistance.



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Instructional Item 2 Graph the function f(x) = -0.2|x + 7|.

Instructional Item 3 Graph f(x) = 2|x - 1| + 3 and determine when the function is positive, negative, increasing, or decreasing. \*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.



# MA.912.AR.5 Write, solve and graph exponential and logarithmic equations and functions in one and two variable.

# MA.912.AR.5.2

#### Benchmark

# MA.912.AR.5.2

# .2 Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

#### **Connecting Benchmarks/Horizontal Alignment**

• MA.912.NSO.1.2, MA.912.NSO.1.3, MA.912.NSO.1.6, MA.912.NSO.1.7

- MA.912.AR.7.1
- MA.912.AR.8.1

#### Terms from the K-12 Glossary

- Base (of an exponent)
- Exponent (exponential form)
- Exponential function

# Vertical Alignment

**Previous Benchmarks** 

- MA.912.F.1.8
- MA.912.FL.3.4

#### Next Benchmarks

- MA.912.DP.2.9 (Alg2/H; Prob&Stats; MCS; MCLA, MCDFLH)
- MA.912.C.2.4 (Calc)
- MA.912.C.2.8 (Calc)

#### Purpose and Instructional Strategies

In Algebra I, students worked with the properties of exponents and solved one-variable linear, quadratic and exponential equations. In Math for College Algebra, students will learn that there are similar rules for logarithms and when some exponential equations cannot be solved, one will use logarithms to help convert the bases. Students are to continue to work with solving exponential equations and now the focus also involves logarithmic equations. In later courses, students will use derivatives to solve different types of function problems.

- Instruction includes uses various methods to solve one-variable exponential equations.
  - For example, students can use the properties of exponents or logarithms to determine the solution for the equation  $2^x = 32$ .
    - Students can rewrite the equation as  $2^x = 2^5$ , noticing that since the base of both sides of equations are the same, then the exponents must be equivalent to one another. Therefore, x = 5.
    - Students can rewrite the equation as  $\log_2 32 = x$ , noticing that they can use the change of base formula to determine the value of x. Therefore,  $x = \frac{\log 32}{\log 2}$ , which is equal to 5.
- Instruction includes the use of the Property of Equality for Logarithmic Functions, which states If b, x and y are positive real numbers with  $b \neq 1$ , then  $\log_b x = \log_b y$  if and only if x = y.



- For example, given  $\log_6 x = \log_6 12$ , then x = 12.
- Instruction includes solving logarithmic equations by changing from a logarithmic expression to an exponential expression.
  - For example, when solving  $\log_3(4x 7) = 2$ , we can obtain an exact solution by changing the logarithm to exponential form creating the equivalent equation  $4x 7 = 3^2$ . Students should recognize that this is a one-variable linear equation 4x 7 = 9, and therefore, x = 4.

#### **Common Misconceptions or Errors**

- Students may not understand the relationship of logarithm and exponential functions and may treat them as separate concepts.
- Students may forget to check for extraneous solutions.
  - For example, when asked to solve  $\log_x 36 = 2$ , students can solve using the square method and get  $\pm 6$ . The base of the logarithm is always positive, so we discard the -6.

#### Instructional Tasks

Instructional Task 1 (MTR.7.1)

Between 7:00 a.m. and 9:00 a.m. cars arrive at Starbucks drive-thru at the rate of 24 cars per hour (0.40 car per minute). The following function can be used to determine the probability that a car will arrive within t minutes of 7:00 a.m.

$$C(t) = 1 - e^{-0.2}$$

- Part A. Determine the probability that a car will arrive within 5 minutes of 7 a.m. (that is, before 7:05 a.m.).
- Part B. Determine the probability that a car will arrive within 30 minutes of 7 a.m.
- Part C. Within how many minutes of 7:00 a.m. would the probability a car will arrive be 78%?
- Part D. What does the value C(t) approach as t becomes unbounded in the positive direction?
- Part E. Graph C(t) using graphing technology.

#### Instructional Task 2 (MTR 5.1, MTR.7.1)

Suppose that the population growth, P(x) in thousands, for a city in Florida can be modeled by the logarithmic function,  $P(x) = 3 \log_5 x^3 - 4$  with x representing the number of years since 2010.

Part A. Determine the population growth for a city in Florida at the year 2035.

- Part B. What constraints do we have on *x* considering the context?
- Part C. What is the domain of p(x)? What is the range?

# Instructional Items

Instructional Item 1

Determine the value of x that satisfies the equation  $3^{4x-1} = \frac{1}{27}$ .

#### Instructional Item 2

Solve for x in the equation  $2^x = 11$ .



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Instructional Item 3

Solve for x in the equation  $log_6(2x - 3) = log_6(x + 2)$ .

# Instructional Item 4

Determine the value of x that satisfies the equation  $2log_{10}6 - \frac{1}{3}log_{10}27 = log_{10}x$ .

#### Instructional Item 5

Solve for x in the equation  $log_2(x) = 3$ . \*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

#### MA.912.AR.5.4

# Benchmark

MA.912.AR.5.4 Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

## Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, exponential functions are limited to the forms  $f(x) = ab^x$ , where *b* is a whole number greater than 1 or a unit fraction, or  $f(x) = a(1 \pm r)^x$ , where 0 < r < 1. *Clarification 2:* Within the Algebra I course, tables are limited to having successive nonnegative integer inputs so that the function may be determined by finding ratios between successive outputs.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.1
- MA.912.F.1.1, MA.912.F.1.6,
- MA.912.F.2 (F.2.1-F.2.5)

#### Terms from the K-12 Glossary

- Exponential function
- Exponents
- Function
- Percent of change

#### Vertical Alignment

Previous Benchmarks

MA. 912.AR.1.1
MA.912.F.1.8

- MA. 912.AR.5.5(Alg2/H; MCLA)
- MA.912.AR.8.2(Alg2/H)

Next Benchmarks

• MA.912.AR.5.3

# Purpose and Instructional Strategies

In Algebra I, students wrote exponential functions that modeled relationships characterized by having a constant percent of change per unit interval. In Math for College Algebra, students continue this work, not limited to the forms  $f(x) = ab^x$ , where *b* is a real number greater than zero (0) and excluding one (1), or  $f(x) = a(1 \pm r)^x$ , where 0 < r < 1. Additionally, students further develop their understanding of this feature of exponential functions from Algebra I by emphasizing real-world context and applications.



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**Commented [EA2]:** AR.1.1 and F.1.8 are listed as previous benchmarks for AR.5.4 in Cpalms. Remove AR.5.7 from next benchmarks. This benchmark is in the MCA course.

Commented [FT3]: Referencing course red ()

- Instruction includes guidance on how to determine the initial value or the percent rate of change of an exponential function when it is not provided.
  - For example, if the initial value of (0,3) is given, students can now write the function as  $f(x) = 3b^x$ . Guide students to choose a point on the curve that has integer coordinates such as (2,12). Lead them to substitute the point into their function to find *b*. Students should recognize that exponential functions are restricted to positive values of *b*, leading to the function  $f(x) = 3(2)^x$ .
- Instruction includes interpreting percentages of growth and decay from exponential functions.
  - For example, the function  $f(x) = 500(1.16)^x$  represents 16% growth of an initial value.
    - Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.
  - For example, the function  $f(x) = 500(0.72)^x$  represents 28% decay of an initial value.
    - Guide students to discuss the meaning of the number 0.72 as a percent. They should understand it represents 72%. Taking 72% of an initial value decreases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential decay.
  - For example, the function  $f(x) = 500(1)^x$  represents an initial value that neither grows nor decays as x increases.
    - Guide students to discuss the meaning of the number 1 when it comes to growth/decay factors. They should understand it represents 100%. Taking 100% of an initial value causes the value to remain the same. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to no change in the initial value (explaining the horizontal line that shows when *b* = 1 on the graph). Students should understand that when b is 1, the function is constant rather than exponential.
- Use prior knowledge on transformations of functions to explore different forms of exponential functions.
  - For example, the function  $f(x) = 16\left(\frac{1}{2}\right)^x$  can be written as  $f(x) = 4\left(\frac{1}{2}\right)^{x-2}$  from the given table of values, showing the horizontal shift.

x	0	1	2	3
f(x)	16	8	4	2

- Instruction includes students using Euler's number, symbolized by the letter *e*, as the base in many applied exponential functions. The Euler's number, *e*, is irrational and referred to as the natural base. The number *e* can be approximated to 2.72. It is used when the relationship between two quantities is growing exponentially at a continuous rate.
- Instruction includes the use of technology.



# **Common Misconceptions or Errors**

- Students may not recognize that if *b* is greater than 1 that it represents growth and if *b* is less than one that it represents decay.
- Students may struggle differentiating between a constant rate of change and common ratio to determine which type of function they are writing.
- Students may confuse written form of decimals and percentages, e.g., 5% growth (1 + .05) = 105 % not equal to 150%.
- Remind students to use multiple points when given a table to help find the exponential function.
- Students may say the decay rate for f(x) = 325(0.64) is 64% instead of 36%.

# Instructional Tasks

# Instructional Task 1 (MTR.4.1, MTR.5.1, MTR.7.1)

Raylene and Caleb were working on separate biology experiments. Each student documented their cell counts over time in the chart below.

Raylene's Experiment			Caleb's E	xperiment
# of minutes	# of cells		# of minutes	# of cells
3	2400		0	160
6	1200		1	240
9	600		2	360
12	300		3	540
15	150		4	910

- Part A. Do the number of cells in Caleb's experiment increase at a constant percentage rate of change? If so, what is the percentage rate? If not, describe what is happening to the number of cells. Does this change represent growth or decay? Justify your answer.
- Part B. Write exponential functions to represent the relationship between the quantities for each student's experiment. In which experiment are the number of cells changing more rapidly? Justify your answer.
- Part C. Graph these functions and determine their key features.

# Instructional Task 2 (MTR.2.1, MTR.7.1)

The population of Zora City in 2019 was estimated to be 83,600 people with an annual rate of increase of 1.8%.

Part A. Write an equation to model future growth.

Part B. What is the growth factor for Zora City?

Part C. Use the equation to estimate the population in 2083 to the nearest hundred people.

# **Instructional Items**

Instructional Item 1

A forester has determined that the number of fir trees in a forest is decreasing by 3% per year. In 2010, there were 13,000 fir trees in the forest. Write an equation that represents the number of fir trees, *N*, in terms of *t*, the number of years since 2010.



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Instructional Item 2

Write an exponential function for the table below.

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	x	0	1	2	3	4	5
	у	1	4	16	64	256	1024

Instructional Item 3

In 2020 the world population was approximately 7.98 billion and in 2016 the world population was approximately 7.46 billion. Write an exponential function that represents the world population *y* (in billions) *t* years after 2016.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.5.6

#### Benchmark

MA.912.AR.5.6	Given a table, equation or written description of an exponential	
	function, graph that function and determine its key features.	

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.

*Clarification 2:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

*Clarification 3:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

*Clarification 4:* Within the Algebra I course, exponential functions are limited to the forms  $f(x) = ab^x$ , where *b* is a whole number greater than 1 or a unit fraction, or  $f(x) = a(1 \pm r)^x$ , where 0 < r < 1.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.2
- MA.912.AR.2.5
- MA.912.AR.3.7
- MA.912.AR.4.4
- MA.912.AR.9.6
- MA.912.F.1.1, MA.912.F.1.6
- MA.912.F.2

#### Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Percent of change
- Range (of a relation or function)
- Set-builder notation



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- x-intercept
- y-intercept

#### Vertical Alignment

Previous l	Benchmarks
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# MA.912.AR.1.1

- MA. 912.AR.5.3, MA.912.AR.5.5
- MA.912.F.1.8

- Next Benchmarks • MA.912.AR.7.2(Alg2/H; Math for ACT/SAT)
  - MA.912.AR.8.2(Alg2/H)
  - MA.912.F.1.7(Alg2/H; PreCalc; Math for ACT/SAT)

# **Purpose and Instructional Strategies**

In Algebra I, students graphed exponential functions and determined their key features, including asymptotes and end behavior. In Math for College Algebra, students continue this work, without the limitations on function forms given in Algebra I.

- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will discover asymptotes are useful guides to complete the graph of a function, especially when drawing them by hand. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
  - Instruction should not be limited to exponential functions of the form  $f(x) = ab^x$  as this was the expectation in Algebra I. Students should explore how asymptotes move as this parent function is translated.
- Emphasize reading a graph from left to right when identifying key features such as increasing and decreasing intervals.
- For students to have full understanding of exponential functions, instruction includes MA.912.AR.5.4. Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between the two types.
- Instruction includes the use of x-y notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. While the focus is on exponential functions, the information for constraints is the same. (Reference MA.912.AR.2.5).
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the *x* or *y*-axis when necessary.
- Instruction includes the use of multiple forms of exponential functions, such as

   f(x) = b<sup>x</sup>, f(x) = e<sup>x</sup> and f(x) = pe<sup>kx</sup>.

#### **Common Misconceptions or Errors**

- Students may not fully understand how to use interval notation.
- Students may not fully understand how to interpret proper function notation when determining the key features of an exponential function.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in



the problem.

- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- Students may have trouble translating from a written description to mathematical expression.
- Students may identify key features incorrectly if they don't read the graph from left to right.
- Students may get confused with positive infinity when increasing and negative infinity when decreasing.

#### Instructional Tasks

Instructional Task 1 (MTR.5.1)

Use the two functions below to answer the following questions.

$$f(x) = 1.7^x$$
$$a(x) = 0.6^x$$

- Part A. What do you expect f(0) to be? What about g(0)?
- Part B. As x gets very large, what do you expect the graph of function f to look like? What do you expect the graph of g to look like?

Part C. Graph each function using technology. How do the graphs compare with your answers from Part A and Part B?

# Instructional Task 2 (MTR.6.1, MTR.7.1)

Use the table of values that represents the population Gainesville, Florida from 1990 to 2023 to answer the following questions.

Year	Population
1990	95,602
2000	112,997
2010	124,354
2020	139,929
2021	140,398
2023	141,341

Part A. Write an exponential function to represent the population growth.

- Part B. Use the exponential function to predict the population for the year 2030.
- Part C. What is the percentage rate the population is growing?
- Part D. Graph the table of values using technology and compare to the exponential function from Part A.

https://www.biggestuscities.com/city/gainesville-florida https://worldpopulationreview.com/us-cities/gainesville-fl-population

# **Instructional Items**

# Instructional Item 1

An exponential function is given by the equation  $y = \left(\frac{12}{5}\right)^{2x}$ . What is the asymptote for the graph?

Instructional Item 2



An exponential function is given by the equation  $y = -14\left(\frac{1}{4}\right)^{1.2x} + 1$ . What is the asymptote for the graph?

Instructional Item 3

An exponential function is given.

 $y = 50(1.1)^t$ 

Part A. Does this function represent exponential growth or decay?

Part B. What would be the range of the function?

Part C. What would be the *y*-intercept of the function?

Part D. What is the constant percent rate of change of y with respect to t.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.5.7

# Benchmark

# MA.912.AR.5.7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.

*Example:* The graph of the function  $f(t) = e^{5t+2}$  can be transformed into the straight line y = 5t + 2 by taking the natural logarithm of the function's outputs.

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.

*Clarification 2:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

*Clarification 3:* Instruction includes understanding that when the logarithm of the dependent variable is taken and graphed, the exponential function will be transformed into a linear function.

*Clarification 4:* Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

# **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.1.2
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

#### Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponent


# Last Updated: July 2024

- Exponential function
- Function
- Function notation
- Linear function
- Percent of change
- Range (of a relation or function)
- Set-builder notation
- *x*-intercept
- y-intercept

# Vertical Alignment

**Previous Benchmarks** 

#### Next Benchmarks

- MA.912.AR.1.1
   MA.912.AR.5.3, MA.912.AR.5.5 MA.912.FL.3.2
- MA.912.DP.2.9(Alg2/H; Prob & Stats; MCS,MCLA,MDFLH)
  - MA.912.F.3.2(Alg2/H; MDFL/H)
- MA.912.FL.4(MDFL)

# Purpose and Instructional Strategies

In Algebra I, students worked with exponential functions in limited forms. In Math for College Algebra, students extend this work to solve and graph problems modeled with exponential functions, determining key features and constraints in terms of the context.

- Problem types include creating a function as a tool to determine requested information or providing the graph or function that models the context.
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
- Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between two.
- Instruction includes the use of *x*-*y* notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. While the focus is on exponential functions, the information for constraints is the same. (Reference MA.912.AR.2.5).
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.



- For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- For example, if the range of a function is all real numbers, is can be represented in interval notation as  $(-\infty, \infty)$ . This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that exponential graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by exponential functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.

#### **Common Misconceptions or Errors**

- Students may not fully understand how to interpret proper function notation when determining the key features of an exponential function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- Students may have a difficult time determining the constraints with fractions.
  - For example, the function  $y = m^{-n}$  where  $m^{-n} = \frac{1}{m^n}$ . In this case, the value of  $m \neq 0$ , since this would make the function undefined.

# Instructional Tasks

# Instructional Task 1 (MTR.3.1)

The population of a city is  $K = 178,563e^{0.011t}$  where t = 0 represents the population in the year 2012.

Part A. Find the population of the city in the years 2022 and 2030.

- Part B. What would be the domain?
- Part C. What is the range?

#### Instructional Task 2 (MTR.2.1)

Consider the function  $f(x) = e^{7t+3}$ .

- Part A. Generate a table of values for the function f(x).
- Part B. Use the table to graph the function.
- Part C. Find the x-intercepts for f(x) using either your table or graph from parts A and
  - В.
- Part D. Set f(x)=0 and solve. What do you notice about the values you found for x?
- Part E. Are there any asymptotes for f(x)?
- Part F. Describe the domain, range, and end behavior for f(x).



Part G. When you take the natural logarithm of the outputs for f(x), what linear function do you find?

#### Instructional Task 3 (MTR.2.1, 7.1)

Houston is due to inherit \$25,000 the day he turns 18 years old. Houston wants to know if he invests all of his inheritance into a continuously compounding interest bearing account, can he earn at least a half of a million dollars so he that he could retire early. The account has an interest rate of 6.5%. The function  $f(x) = 25,000e^{.0065t}$  represents this situation.

- Part A. Determine algebraically how many years Houston will need to work to earn half a million dollars to retire.
- Part B. Using technology, graph the functions f(x) and y = 1,000,000. What does the intersection point for these graphs represent?
- Part C. What types of constraints does one need to consider when solving this problem?

# Instructional Items

Instructional Item 1 An exponential function is given.

 $y = 17^{2m}$ 

Part A. Does this function represent exponential growth or decay?

Part B. Select four values for *m* and create a graph.

Part C. What is the domain for the function?

#### Instructional Item 2

Consider the function  $f(x) = 3^x - 1$ .

Part A. Graph f(x) on a coordinate grid.

Part B. Find the x and y intercepts of f(x).

- Part C. Describe the domain and range of f(x).
- Part D. Describe the end behavior of f(x).
- Part E. Describe any asymptotes for f(x).

# Instructional Item 3

Suppose that you have \$7,000 to invest. Which investment gives you a better return over 5 years: 8% compounded monthly or 5.75% compounded continuously? What types of constraints might you need to consider?

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.AR.5.8

#### Benchmark

MA.912.AR.5.8 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features.

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes. *Clarification 2:* Instruction includes representing the domain and range with inequality notation, interval

notation or set-builder notation.



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# Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.7
- MA.912.AR.4.4
- MA.912.AR.5.9
- MA.912.AR.9.6
- MA.912.F.1.1, MA.912.F.1.6
- MA.912.F.2 (F.2.1-F.2.5)

# Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Intercept
- Logarithmic function
- Range (of a relation or function)
- Set-builder notation
- *x*-intercept
- y-intercept

#### Vertical Alignment Previous Benchmarks

#### **Next Benchmarks**

- MA.912.AR.6.6(PreCalc; Math for ACT/SAT)
- MA.912.DP.2.9(Alg2/H; Prob & Stats; MCS,MCLA,MDFLH)

# Purpose and Instructional Strategies

In Algebra I, students graph exponential functions and determine their key features, including asymptotes and end behavior. In Math for College Algebra, students develop an understanding of logarithmic functions and the inverse relationship of exponential functions.

- Instruction includes making the connection between exponents and logarithms to compare the two in terms of functions (*MTR.5.1*).
- Instruction includes the use of *x*-*y* notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. While the focus is on logarithmic functions, the information for constraints is the same. (Reference MA.912.AR.2.5).

o Words

- If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
- $\circ$  Inequality notation



0

If the domain is all values of *x* greater than 2, it can be represented as x > 2. Set-builder notation

If the range is all values of y less than or equal to zero, it can be represented as  $\{y|y \le 0\}$  and is read as "all values of y such that y is less than or equal to zero."

- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the *x* or *y*-axis when necessary.
- Instruction includes functions with various bases, including base 10 and base *e*.

### Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithmic function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- Students may confuse finding the logarithm and natural logarithm.
- Students may not know that negative exponents create values between 0 and 1 and not less than 1.
- Students may not know what an asymptote is or how that affects the graph.
- Students may try using division to isolate when solving an exponential function rather than using equal bases or natural logs.

# Instructional Tasks

- Instructional Task 1 (MTR.3.1, MTR.4.1)
  - Given  $f(x) = \log_3(x)$ .
    - Part A. Graph the function.
    - Part B. State the domain of the function.
    - Part C. State the range of the function.
    - Part D. Determine the asymptote(s).
    - Part E. Compare the functions. Be sure to include mathematical terms such as intercepts, increasing, decreasing, positive, negative, and/or end behavior.
    - Part F. Discuss your comparison with a partner. How were your comparisons the same? How were they different?

# Instructional Task 2 (MTR.3.1, MTR.4.1)

Given  $f(x) = \log_4(x)$  and the table for g(x). Complete Parts A – F.

x	$\boldsymbol{g}(\boldsymbol{x})$
2	2.8074
4	1.4037
6	1.086



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8	0.93578
10	0.8451

Part A. Graph each function.

Part B. State the domain of each function.

Part C. State the range of each function.

Part D. Determine the asymptote(s).

Part E. Compare the functions. Be sure to include mathematical terms such as intercepts, increasing, decreasing, positive, negative, and/or end behavior.

Part F. Discuss your comparison with a partner. How were your comparisons the same? How were they different?

#### Instructional Items

*Instructional Item 1* Below is a table with values for *f* 

(	x) = location	$\log_7(x)$
	x	f(x)
	2	2.8074
	4	1.4037
	6	1.086
	8	0.93578
	10	0.8451

Part A. Graph the function.

Part B. State the domain of the function.

Part C. State the range of the function.

Part D. Determine the asymptote(s).

# Instructional Item 2

Consider the function  $f(x) = 3 \ln(4x)$ 

Part A. Using technology, graph f(x).

Part B. Describe the domain and range as well as any asymptotes if applicable.

Part C. Describe the end behavior.

Part D. Find the x and y intercepts of f(x).

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.5.9

# Benchmark

# MA.912.AR.5.9 Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
 *Clarification 2:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
 **Connecting Benchmarks/Horizontal Alignment**

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#### Last Updated: July 2024

- MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

# Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Intercept
- Logarithmic function
- Range (of a relation or function)
- Set-builder notation
- *x*-intercept
- y-intercept

#### Vertical Alignment Previous Benchmarks

#### Next Benchmarks

- MA.912.AR.7.2(Alg2/H; Math for ACT/SAT), MA.912.AR.7.3(Alg2/H; Math for ACT/SAT), MA.912.AR.7.4(PreCalc; Math for ACT/SAT)
- MA.912.AR.8.2(Alg2/H), MA.912.AR.8.3(Alg2/H; PreCalc)

# **Purpose and Instructional Strategies**

In Algebra I, students solved problems modeled with linear, exponential and quadratic functions.

- In Math for College Algebra, students solve problems modeled with logarithmic functions.
  Instruction features a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or
  - When making connections to transformations of functions, use graphing software to
  - When making connections to transformations of functions, use graphing software to explore logarithmic functions adding variability to the parent equation to see the effects on the graph. Allow students to make predictions (*MTR.4.1*).
  - Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
  - Instruction includes the use of x-y notation and function notation.



- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. While the focus is on logarithmic functions, the information for constraints is the same. (Reference MA.912.AR.2.5).
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as  $(-\infty, \infty)$ . This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Instruction includes differentiating when to use base 10 or natural logs based on the context given.

# **Common Misconceptions or Errors**

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithm function.
- Students may confuse finding the logarithm and natural logarithm.
- Students may not understand that exponentiating is the inverse action for solving a logarithmic function.
- Students may not know which base to exponentiate with to solve a logarithmic function.
- Students may not recognize that the inverse relationship that occurs with the natural log function and  $e^x$ .
- Students may not realize that they can use the inverse relationship between natural log and  $e^x$  to visualize the graph for natural log function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

# Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1 and MTR.5.1)



A video uploaded to social media initially had 75 views one minute after it was posted. The total number of views to date has been increasing exponentially and can be modeled by the function  $y = 75e^{0.2t}$ , where t represents time measured in days since the video was posted.

Part A. Convert the exponential to logarithmic form.

Part B. How many days will it take until 5000 people have viewed this video?

Part C. State the domain for the problem.

Part D. Sketch a graph that represents days 5 - 10.

Instructional Task 2 (MTR.3.1 and MTR.5.1)

Use the table and graph of  $f(x) = 2 \ln (3x^2)$  to answer the following parts.

x	$f(x) = 2\ln(3x^2)$		$\backslash$	10	/	
-2	4.9698133			5		
-1	2.1972246			$\backslash /$		
0	undefined	-10	-5	þ	5	
1	2.1972246			-5		
2	4.9698133			10		

Part A. Using technology, create a table of values and graph the function  $g(x) = 2 \ln(3x^2 - 5) - 6$ .

Part B. Compare f(x) and g(x) to describe the transformations.

Part B. Describe the domain and range of g(x) as well as any asymptotes if applicable.

Part C. Describe the end behavior of g(x).

Part D. Find the x and y intercepts of g(x).

Graph from desmos.com

# Instructional Task 3

Suppose that the population growth for a city in Florida can be modeled by the logarithmic function,  $P(x) = 3 \log_5 x^3 - 4$ 

Part A. What constraints do we have on x considering the context?

Part B. What is the domain of p(x)? What is the range?

Part C. What are the x and y intercepts and what do they represent in the context?

# Instructional Items

Instructional Item 1

Wanda invests \$10,000 in a company with earnings of 4% per year.

Part A. Write an equation in logarithmic form that describes the situation.

Part B. How long will it take to accumulate \$20,000?

Part C. How long will it take to accumulate \$25,000?

#### Instructional Item 2

Consider the function  $f(x) = \ln (5 - x)$ . Describe the domain and range for this function.

Instructional Item 3



Consider the function  $g(x) = \log_3(x^2 + 1)$ . Describe the domain, range, and end behavior of g(x).

Part D. Sketch a graph that represents the amount accumulated from the initial investment to reach the goal of \$25,000.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

MA.912.AR.7 Solve and graph radical equations and functions in one and two variables.



# MA.912.AR.7.1

#### Benchmark

# MA.912.AR.7.1 Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions.

# **Connecting Benchmarks/Horizontal Alignment**

- MA.912.NSO.1.3
- MA.912.AR.5.2
- MA.912.AR.8.1

# Terms from the K-12 Glossary

- Domain
- Intercept
- Inverse functions

# Vertical Alignment

**Previous Benchmarks** 

• MA.912.AR.2.1

- MA.912.AR.3.1
- MA.912.AR.4.1

# Purpose and Instructional Strategies

In Algebra I, students solved one variable equations which included linear, quadratic and absolute value, and also determined if the solution(s) was viable. In Math for College Algebra, students transfer their understanding of solving other types of equations to solving one variable equations involving radicals, where the variable occurs in a square root, cube root, or any higher root.

Next Benchmarks

- Instruction for solving radical equations will follow many of the same characteristics of linear, quadratic, and absolute value functions. Students will still need to isolate one variable in order to solve for the unknown value. Students will need to understand the inverse relationship between radicals and roots as well as how to convert between radical form and rational exponent form to help reveal best options to solve. (*MTR.2.1*). In general, radical equations with square roots are solved by squaring both sides of the equation and radical equations with nth roots are solved by raising both sides of the equation to the nth power.
- Instruction should include students solving an equation written in radical form or with rational exponents. For problem types with rational exponents, students should be familiar with the use of reciprocal and unit fraction to simplify the equation.
- Students will also need to understand laws of exponents (MA.912.NSO.1.3) and how best to utilize them to simplify and solve radical equations. Students should feel comfortable finding and representing the solutions to a radical equation algebraically, graphically, and numerically (with a table) (MTR.2.1).
  - a. It may be helpful to show students on a graph the intersection of the left and right expression from a given equation, which can visually reveal the solution (the



graphs intersection) at a point. This can help explain why some solutions are not viable and are extraneous.

- Essential to this lesson is student understanding of extraneous solution, which are solutions **produced** in the solving process, when students raise each side of an equation to the same exponent. If the nth root is even, all the solutions of the equation raised to the even power may not be a solution of the original equation. In this case students should always check their solutions in the original equation to see which solutions hold true.
  - a. For example, given  $\sqrt{x} = -4$ , the equation has no real solution, however, you obtain x = 16 after squaring each side.
- Students should also be able to explain how many solutions a radical equation provides as well as features of that radical equation that help determine how many solutions an equation has. (MTR.6.1).
- Instruction may include students solving an equation that has two radicals and will require involving isolating, squaring both sides and then repeating the process. It is helpful to have students recall their work with multiplying polynomials to fluently solve these types of problems.

#### **Common Misconceptions or Errors**

- Students may not understand how to determine if a solution is extraneous.
- Teachers should present the students with a few examples when extraneous solutions arise and ask the students to look for patterns for when those extraneous solutions arise. Teachers should remind students that solving an equation is finding the values of the variable that make the statement true.
- Students may not completely understand the relationships between roots and radicals so they may use the wrong root or exponent to isolate the variable when solving the equation.
- Student may square an individual term instead of squaring the entire side of the equation when trying to solve a radical equation.
  - For example, in the equation  $\sqrt{x} + \sqrt{x} + 2 = 2$ , students may square the left term on the left side of the equation and not realize the need to square the entire expression on the left side.
- Students may not understand how to use a graph or table to justify what they have found algebraically.

#### Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Create a one variable radical equation that would have one solution.

Part B. Explain what you would have to change about the equation to have two solutions. Part C. Explain what you would have to change about the equation to have no solution.

Instructional Task 2 (MTR.2.1, MTR.4.1)

An equation is given.

$$\sqrt{4x - 1} = 3$$

Part A. What is/are the solution(s)?

Part B. Describe how would you find the solution(s) on a graph.

Part C. Describe how would you find the solution(s) in a table.



#### Instructional Task 3 (MTR.4.1, MTR.5.1)

Part A. Explain what it means to get an extraneous solution to a radical equation.Part B. What features will happen in a radical equation to make an extraneous solution arise?

Part C. Give an example of a radical equation with an extraneous solution.

#### Instructional Task 4 (MTR.3.1)

One estimate for how far one can see on a clear day is given by the formula  $v = 1.225\sqrt{a}$ , where v represents the visibility in miles and a represents the altitude in feet.

- Part A. A person is parasailing and can see 64 kilometers to the horizon, use the given formula to determine how far above the ground the person parasailing.
- Part B. If you were given a value for the altitude rather than the visibility, how would that change our approach to finding a solution? What would the value for *a* mean in the context?

# Instructional Items

#### Instructional Item 1

Find the solution(s) to the following equation  $\sqrt[3]{x-3} = -5$ . Explain how you found the solution(s).

# Instructional Item 2

Find all the solution(s) to the following equation  $\sqrt[4]{x} = \sqrt[5]{x}$ .

# Instructional Item 3

Which of these equations has an extraneous solution?

a.  $\sqrt{2-x} = x$ b.  $\sqrt{5-x} = 2$ c.  $\sqrt{x} - 12 = 0$ d.  $\frac{\sqrt{4x}}{6} = x$ 

Instructional Item 4

The formula for a cone of a cone written in terms of the radius is  $r = \sqrt{\frac{3V}{\pi h}}$ 

Part A. A mound of mulch is piled in the shape of a cone with the height equal to three times the radius. Calculate the volume of such a mound of mulch whose radius equals 2.5 yards.

Part B. Is your solution that you found viable? How do you know?

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



MA.912.AR.8 Solve and graph rational equations and functions in one and two variables.

#### MA.912.AR.8.1

### Benchmark

MA.912.AR.8.1 Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

# Benchmark Clarifications:

*Clarification 1:* Within the Algebra II course, numerators and denominators are limited to linear and quadratic expressions.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.4.2
- MA.912.AR.5.2
- MA.912.AR.7.1

# Terms from the K-12 Glossary

- Linear expression
- Quadratic expression
- Rational expression
- Rational number

# Vertical Alignment

# **Previous Benchmarks**

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#### Next Benchmarks

- MA.912.AR.8.3 (Alg 2, PreCalc)
- MA.912.AR.3.1
- MA.912.AR.4.1

MA.912.AR.2.1

- MA.912.C.1.3 (Calc)
- MA.912.AR.9.7 (Alg 2)

#### **Purpose and Instructional Strategies**

In Algebra I, students solved one variable equations including linear, quadratic and absolute value, and determined if the solution(s) was viable. Students have developed a list of equation solving techniques while in Algebra and those methods have become an important building block. In Math for College Algebra, students will extend their use of solving proportions by applying this process to more complex equations, such as the case with the rational equation in one and two variables.

- While the intent of the benchmark is to focus on writing and solving rational equations, instruction should include a quick review of inverse variation,  $y = \frac{a}{x}$ , providing students with a distinction between direct variation, y = ax, helping students to understand the patterns of input and output values of such functions.
- Instruction should include a matching exploration where students can match each equation with its related graph and use it to approximate the solution(s). This will help students to connect and identify graphical characteristics of the equations.
- Students should be given opportunities to use technology to compare the equations to their related graphs and approximate solutions to help make sense of the solutions of the equations as the x-coordinates of the points of intersection and that all solutions are



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#### integers.

• For example, given the rational equation,  $\frac{1}{x+1} = \frac{3}{x}$ , students can use the graph of each side of the rational equation to identify the intersections and discuss solutions.

5		
0	5	10

- Students should understand and recognize patterns for factoring quadratic expressions from their prior studies in Algebra and be able to expand and factor expressions fluently to aid in simplifying and solving rational equations.
- Problem types include opportunities for students to solve by cross-multiplication, LCM, • solving equations with extraneous solutions and contextual problems.
  - "Cross multiplication" is best when each side of the equation is a single rational 0 expression and expressed as a proportion.
  - The "least common denominator" (product of all of prime factors in the 0 denominator, with each factor raised to the greatest power of its occurrence in any denominator) is best when a rational equation is not expressed as a proportion. (MTR.5.1).
    - For example, when solving rational equations that involve addition or subtraction, on one side of the equation, students can find the least common denominators of the rational expressions to assist in solving.

common denominators of the rational expressions to assist in solving. • Given the equation  $\frac{1}{x-2} + \frac{1}{x+3} = \frac{1}{x}$ , students can determine a common denominator of (x-2)(x+3) to rewrite the equation as  $(x-2)(x+3)\frac{1}{x-2} + (x-2)(x+3)\frac{1}{x+3} = \frac{1}{x}$ , resulting in  $\frac{x+3}{(x-2)(x+3)} + \frac{x-2}{(x-2)(x+3)} = \frac{1}{x}$ . Students should recognize that since the fractions on the left side of the equation have the same denominator, the equation can be rewritten as  $\frac{2x+1}{(x-2)(x+3)} = \frac{1}{x}$ . From here students where  $x = \frac{1}{x+3} = \frac{1}{x}$ , resulting in  $\frac{2x+3}{(x-2)(x+3)} = \frac{1}{x}$ . From here students can either solve using proportional reasoning (MA.7.AR.3) or can find common denominators between fractions on either side of the equal side and then set the numerators equivalent to one another.

- Instruction includes how to determine what the solutions to the equation are and 0 whether they are viable (MTR.6.1). Students should have experience with rational equations that produce different types of solutions: more than one, exactly one, and extraneous.
- Instruction gives students the opportunity to discuss constraints and what effect those constraints have on the solution(s) to the equations (MTR.4.1). Teachers should discuss with students why it is important to avoid values of variables that make the denominator zero.



• For example, given  $\frac{x}{x-2} = \frac{1}{x}$ , students should recognize the restrictions of domain values for x in one of the denominators, which will would result in no solution. This is best represented using technology to show both the graph and table which will reveal all solutions.

#### **Common Misconceptions or Errors**

- Students may struggle to determine if a common denominator is needed to solve a rational equation.
- If students determine a common denominator is needed, they may struggle to decide what factors needed that should be multiplied to the numerator and the denominator while keeping the rational equation equivalent.
- Students may forget to test the solutions in the original equation to determine if it is extraneous or not.
- Students may forget or incorrectly use the distributive property after solving proportions by cross multiplying when dealing with binomials.

# Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Explain how you could find the solution(s) to the rational equation given. Share your strategy with a partner.

$$\frac{3}{x} - \frac{x}{x+1} = \frac{2x}{x(x+1)}$$

### Instructional Task 2 (MTR.5.1)

Create a rational equation with the following:

- a. One solution
- b. Two solutions
- c. Infinitely many solutions
- d. An extraneous solutions

#### Instructional Task 3 (MTR.7.1)

At a local high school, sophomores are designing and printing school lanyards for the upcoming freshman class. An online printing company charges \$40 set up fee and \$2.30 for each printed lanyard.

- Part A. Create an equation to represent the average cost C(x), in dollars, per lanyard if *l* lanyards are printed using this company.
- Part B. What is the average cost per lanyard to print 55 lanyards? 105 lanyards?
- Part C. How many lanyards should be printed to have an average cost of \$2.50 or less per lanyard? Explain how you know.

# **Instructional Items**

#### Instructional Item 1

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of 30 games against his brother.



- Part A. How many games would Chase have to win in a row in order to have a 75% winning record?
- Part B. How many games would Chase have to win in a row in order to have a 90% winning record?
- Part C. Is Chase able to reach a 100% winning record? Explain why or why not.
- Part D. Suppose that after reaching a winning record of 90% in Part B, Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

#### Instructional Item 2

An equation is given below. Determine the solution(s), including any extraneous solutions.

2x	4	$3x^2$
$\overline{x+3}^+$	$\frac{1}{x^2 + 3x}$	$= \frac{1}{x}$

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



MA.912.AR.9 Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.

# MA.912.AR.9.4

Benchmark MA.912.AR.9.4 Graph the solution set of a system of two-variable linear inequalities. Benchmark Clarifications: Clarification 1: Instruction includes cases where one variable has a coefficient of zero. Clarification 2: Within the Algebra I course, the system is limited to two inequalities. **Connecting Benchmarks/Horizontal Alignment** MA.912.AR.2.4, MA.912.AR.2 Terms from the K-12 Glossary Inequality Vertical Alignment **Previous Benchmarks Next Benchmarks** MA.912.AR.2.6, MA.912.AR.2.7, • MA.912.AR.9.8(DM) MA.912.AR.2.8 **Purpose and Instructional Strategies** In Algebra I, students solved systems of linear inequalities limited to two inequalities. In Mathematics for College Algebra, students solve problems involving systems of linear inequalities with two or more linear inequalities and connect to situations where ranges of values are given rather than specific values. • For students to have full understanding of systems, instruction includes MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction. Students should recognize that straights lines will still be formed when graphing these types of equations but whose solutions contains only true points, hence the dashed lines produced instead of solid. For a full list of strategies, refer to Algebra 1, MA.912.AR.9.4, for specific guidance to support this work. Instruction includes the use of various forms of linear inequalities. o Standard Form Can be described by the inequality Ax + By > C, where A, B, and C are any rational number and any inequality symbol can be used. o Slope-intercept form Can be described by the inequality  $y \ge mx + b$ , where *m* is the slope and *b* is the

- y-intercept and any inequality symbol can be used.
  Point-slope form
- Can be described by the inequality  $y y_1 > m(x x_1)$ , where  $(x_1, y_1)$  are a point on the boundary line and *m* is the slope of the line and any inequality symbol can be used.
- Include examples where one variable has a coefficient of zero.
  - For example, a system could include the inequalities x > 5 and 5x + 3y > 2 or y > 2 and y < -x.



- Explore with students the different approaches connection to graphing solution sets of one-variable inequalities coordinate plane. (MTR. 6.1) Students should recognize whether the boundary line should be dotted (exclusive) or solid (inclusive) by taking note of the original symbol.
  - One option is for students to replace the inequality symbol with an equal sign and graph the corresponding linear equation. Then choose a test point from one of the half-planes, by substituting the coordinates of the test point into the inequality and shade appropriately.
  - Another option is when students are giving inequalities in the form y > mx + bor y < mx + b, graphing can be done without the use of test points. Students should recognize that the inequality symbol only directs where the line is shaded (above or below) for inequalities when in slope-intercept form. Students shading inequalities in other forms will need to use a test point to determine the correct half-plane to shade.
- The solution to a system of inequalities is the area where all the shading overlaps. If the areas do not overlap, it has no solution.
- Instruction includes graphing multiple inequalities and identifying points that bound the region made by the solution set.
- Instruction allows students to make connections between the algebraic and graphical representations of inequalities in two variables (*MTR.2.1*).
- Problem types include linear programming, an important tool used to help business maximize or minimize quantities. In these problem types, constraints are easily modeled and can aid companies in making needed decisions.

# **Common Misconceptions or Errors**

- Students may confuse which points are in the solution set of a system that includes inequalities (including points on the lines in a system of inequalities).
- Students may shade the wrong half-plane or graph the incorrect boundary line (solid vs. dashed).
- Students may believe that a system of inequalities will always have a solution. Point on to students that if there are no points in the rectangular coordinate system (show a graphed exampled) that simultaneously satisfy each inequality, the system will have no solution.

# **Instructional Tasks**

Instructional Task 1(MTR.4.1, MTR.6.1)

Part A. Graph the solution set to the system of inequalities.

$$y > -x + 4$$
$$y < \frac{2}{5}x + 2$$

Part B. What is one point that is a solution to the system above? Explain how you know it is a solution to the system.

Instructional Task 2 (MTR.6.1)

Part A. Graph the solution set to the system of inequalities.

 $\begin{array}{c} x + y \ge 10 \\ x \le 8 \\ y \ge 6 \end{array}$ 



Part B. List the coordinates of the boundary created by the system of inequalities in Part A. Part C. What are two points, aside from the boundary points you found in Part B that are solutions to the system above?

# Instructional Items

Instructional Item 1

Graph the solution set to the system of inequalities below.

 $x \ge 3$  $\frac{3}{5}x + y < -3$ 

Instructional Item 2

Graph the solution set to the system of inequalities below.

$$-y - 3x \le -1$$
$$2y \ge 8x - 6$$

Instructional Item 3

Graph the solution set to the system of inequalities below and list the points that form the corners of the solution set.

$$x \ge 0$$
  

$$y \ge 0$$
  

$$2x - 3y \le 6$$
  

$$y \le -x + 6$$

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.AR.9.6

# Benchmark

MA.912.AR.9.6 Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or nonviable options.

Benchmark Clarifications:

*Clarification 1:* Instruction focuses on analyzing a given function that models a real-world situation and writing constraints that are represented as linear equations or linear inequalities.

# **Connecting Benchmarks/Horizontal Alignment**

• MA.912.AR.2.4, MA.912.AR.2.5

# Terms from the K-12 Glossary

- Inequality
- Linear equation

# Vertical Alignment

**Previous Benchmarks** 

• MA.912.AR.2.6, MA.912.AR.2.7, MA.912.AR.2.8

Next Benchmarks



#### Purpose and Instructional Strategies

In Algebra I, students represented constraints as systems of linear equations or inequalities and interpreted solutions as viable or non-viable options. In Mathematics for College Algebra, students solve problems involving linear programming with two or more linear equations and/or inequalities, in addition to the fundamental concepts of systems learning in Algebra, subject to the constraints imposed by the real-world context given (*MTR.7.1*).

- Instruction includes the use of graphing utilities to help students visualize the solution to the system of equations/inequalities (*MTR.7.1*).
- For students to have full understanding of systems, instruction includes MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction (*MTR.5.1*).
- Allow for both inequalities and equations as constraints. Include cases where students must determine a valid model of a function.
  - Students often use inequalities to represent and express constraints throughout Algebra I and a list of constraints will form a system of linear inequalities. Equations can be thought of as constraints as well. In linear programming, a mathematical modeling process is used in which a linear function is maximized or minimized when given various constraints. Solving a system of equations requires students to find a point that is constrained to lie on specific lines simultaneously.
  - While most problem types can use various forms of linear equations and inequalities, using the standard form of an inequality is best for revealing the objective function, which is an algebraic expression in two or more variables describing a quantity to be maximized or minimized.
- It may be helpful to walk students through a step-by-step process of graphing the system of inequalities representing the constraints, and then find the value of the objective function at each corner (vertex), of the graphed region. The use of Desmos to point out the maximum and minimum of where the objective function occurs, at one or more points, will help to show students a visualization of the values obtained from evaluating the objective function. If the linear program problem has a solution, the solution occurs on the boundary (lines or vertices). Opportunities for discussion of possible values of the objective function and restriction to the triangular region will help students to understand viable solutions to real-world problems
- Students should understand the domain and range values shared in interval notation as it relates to constraints and inequalities.
- Instruction includes graphing multiple inequalities and identifying points that bound the region made by the solution set. Instruction also includes writing objective functions to maximize or minimize a quantity. An objective function is an algebraic expression in multiple variables that describes a quantity that can be minimized or maximized (MTR.2.1, MTR.6.1, MTR.7.1).





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 Provide students with graphs of individual regions, which represents multiple constraints, and then show the overlapping region in which the three constraints are combined to obtain the triangular overlapping region (feasible region) and the vertices of polygonal boundary (extreme points).



# **Common Misconceptions or Errors**

- Students may have difficulty translating word problems into systems of equations and inequalities.
- Students may shade the wrong half-plane for an inequality.
- Students may graph an incorrect boundary line (dashed versus solid) due to incorrect translation of the word problem or be able to identify the solutions to the objective function.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation. Teachers can encourage the students to discuss the meaning of the restriction in the context of the situation (MTR.4.1).

### Instructional Tasks

# Instructional Task 1 (MTR.7.1)

Bottled drinks and hurricane survival kits are to be transported to survivors of an area who experienced a hurricane. Each container of bottled drinks will serve 15 people and each hurricane survival kit will help 10 people.

- Part A. If x represents the number of containers of bottled drinks to be transported and y represents the number of hurricane survival kits, write the objective function that models the number of people that can be helped.
- Part B. Each truck can carry no more than 70,000 kilograms. The truck is transporting bottled drinks and hurricane survival kits. The bottled drinks weigh 15 kilograms per container and each hurricane survival kit weights 8 kilograms. If *x* represents the number of bottled drink containers to be transported and *y* the number of hurricane survival kits to be transported, write an inequality that represents this constraint.
- Part C. Each truck can transport a total volume of supplies that does not exceed 7000 cubic meters. If x represent the number of bottled drink containers to be transported and y the number of hurricane survival kits to be transported, write an inequality that represents this constraint.
- Part D. Determine how many containers of bottled drinks and how many hurricane survival kits should be transported on each truck to maximize the number of hurricane survivors that can be assisted.



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# Instructional Items

#### Instructional Item 1

Use the given objective function and system of linear inequalities representing constraints that are given to determine the maximum value of the objective function and the values of x and y for which the maximum occurs.

f(x,y) = 3x + 2y
$x \ge 0$
$y \ge 0$
$3x + y \le 9$
$x + y \ge 6$

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.AR.9.10

#### Benchmark

# MA.912.AR.9.10 Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context.

*Example:* A mechanic wants to place an ad in his local newspaper. The cost, in dollars, of an ad x inches long is given by the following piecewise function. Find the cost of an ad that would be 16 inches long.

$$C(x) = \begin{cases} 12x & x < 5 \\ 60 + 8(x - 5) & x \ge 5 \end{cases}$$

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain, range, intercepts, asymptotes and end behavior. *Clarification 2:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9

#### Terms from the K-12 Glossary

- Piecewise function
- Domain
- End Behavior
- Function
- Intercept
- Piecewise function
- Range (of a relation or function)
- Set-builder notation

# Vertical Alignment



Previous Benchmarks	Next Benchmarks
• MA.912.AR.4.3	• MA.912.AR.7.3 (Alg 2)

- MA.912.AR.8.3 (Alg 2)
- MA.912.AR.6.6 (Pre-Calc)

#### **Purpose and Instructional Strategies**

In Algebra I students solved problems modeled with various functions including linear, quadratic, exponential and absolute value. In Math for College Algebra students extend their knowledge to include piecewise functions, , interpret key features of the graph, and determine constraints in terms of the context. In other courses, students will work with piecewise functions that contain other function types.

- Instruction focuses on students applying skills to evaluate and graph piecewise functions and should recognize that a graph that may not represent a linear function can be a function.
- Instruction focuses on functions within this course.
- Students should be able to identify key features of graphs that are specific to the function types included in the piecewise defined function that they are given, and be able to recognize the piece of the function that applies to a specific domain.
- This benchmark expects students to interpret key features and determine constraints in terms of the context of a real-world problem (MTR.7.1).
  - $\circ$  For the example used in the benchmark, the piecewise function models the cost, in dollars, of an ad x inches long. The key features and constraints are as follows.
    - The domain of a function is the complete set of possible values of the input of a function or relation. The input values in this case are the size in inches of the ad length. The minimum ad length someone could possibly have is 0 inches. The maximum ad length (based on the length of traditional newspapers in the United States) would be the length of a full page, which is approximately 21 inches. The set of inputs in this case would be continuous within the minimum and maximum since the length is not limited to only whole numbers. Therefore, a reasonable domain for this context would be any length from 0 to 21 inches.
    - The range of a function is the complete set of possible values of the output of a function or relation. In this case, the output is the cost of the ad. The minimum cost of an ad would be 0 if the ad size length is 0. This function uses the subset function 12x when x < 5. The upper bound of the cost for this subset would be 12(5), or 60, but does not include 60. Once the size of the ad reaches 5 inches in length, the subset function 60 + 8(x 5) is used. The lowest cost for this subset would be 60 + 8(5 5) = 60 and the highest cost for that subset would be 60 + 8(21 5) = 188. Overall, the lowest cost would be 0 and the highest cost would be 188, so the range is all values between 0 and 188, inclusive.
    - The x-intercept occurs when the output value is 0. The y-intercept occurs when the input value is 0. In this case, the x-intercept occurs when the cost of the ad is equal to 0. The cost of the ad is 0 when the length of the ad is 0 inches. The C(x)-intercept is the starting cost of the ad when the ad length is 0. The starting cost of the ad when the ad length is 0. The starting cost of the ad when the ad length is 0 is 0 dollars.



Therefore, the *x*-intercept and the C(x)-intercept is the origin (0,0).

- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation. While the focus is on piecewise functions, the information for constraints is the same. (Reference MA.912.AR.2.5).
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is -10 < y < 24, it can be represented in interval notation as (-10, 24). This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is  $0 \le x \le 11.5$ , it can be represented in interval notation as [0, 11.5]. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is  $0 \le x < 50$ , it can be represented in interval notation as [0, 50). This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as  $(-\infty, \infty)$ . This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given. Instruction should start with a display of a piecewise function defined by two equations and opportunities for students describe each portion of the function, then allowing students to write the two descriptions with the reflective of the appropriate domain.
- When students are graphing piecewise functions, instruction should begin with students describing the domain, range and end behavior and visual models should be displayed or provided to students so that they can indicate values included. It is helpful to graph the function in two parts, using a table of coordinates to create each piece.

	(2 1) x >	- 1
x		
$x \leq 1$	f(x) = x + 5	x, f(x)
1	f(1) = 1 + 5 = 6	(1,6)
0	f(0) = 0 + 5 = 5	(0,5)
-1		
-2		
-3		

• For example, given  $f(x) = \begin{cases} x+5 & \text{if } x \le 1 \\ 2 & \text{if } x > 1 \end{cases}$  a table could look like:

- We can use the graph of the piecewise function to find the range of f. The range is  $\{y|y \le 6\}$  and the range of the other is  $\{y|y = 2\}$ . Therefore the range of the function, f, is  $\{y|y \le 6\} \cup \{y|y = 2\}$ , or  $(-\infty, 6] \cup y = 2$
- Students should understand that an absolute value function is defined as a piecewise function, which defines different rules over different intervals of its domain. Students should be given situations and asked to create piecewise functions to represent the given situations. They should also be given piecewise functions and asked to graph and find



solutions. It is helpful for students to lightly graph each line for all values of x and then erase the parts that are not included in the domain for the equation.

- Students should be able to determine constraints specific to the types of functions included in the piecewise defined function and to the real-world context presented for the piecewise function.
- Provide opportunities for technology use to support students with graphing piecewise functions. It may be helpful to build from parent functions and other key features to assist in understand the graph output.

#### **Common Misconceptions or Errors**

- Students may have a hard time determining which function in the piecewise function to use when finding the function value at a specified *x*-value.
- Student may not know how to read the piecewise function notation to graph the function correctly.
- Students may not be able to interpret the key features of the specific graphs within the piecewise defined function.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation. Teachers can encourage the students to discuss the meaning of the restriction in the context of the situation (*MTR.4.1*).

# Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.7.1)

Jessica wants to buy candy from the bulk food store. She can buy sour watermelon gummies for \$3.00 a pound. If she purchases more than three pounds of gummies the price drops by \$.25 a pound. Write a piecewise function and a graph of the function to represent Jessica's situation.

#### Instructional Task 2 (MTR.2.1, MTR.7.1)

Eric wants to plan a pizza party for the winter sports teams. He is considering pizza prices and found a place that sells small pizzas for \$12, medium pizzas for \$16, and large pizzas for \$20.

- He will choose to order small pizzas if only the wrestling team RSVPs (wrestling team has at most 10 wrestlers).
- He will choose medium pizzas if the wrestling team and soccer teams RSVP (soccer teams have at least 30 players).
- He will order large pizzas if wrestling team, soccer teams, and basketball teams all RSVP (basketball teams have at most 18 players).

Write a piecewise function and a graph of the function to represent Eric's situation. Explain how you determined the constraints for each function.

#### Instructional Task 3 (MTR.5.1)

Create three piecewise defined functions that contain the points (1,3), (3,6), and (6,12).

Instructional Items Instructional Item 1



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Part A. Graph the function 
$$f(x) = \begin{cases} 4, if -4 \le x < -2\\ 2, if -2 \le x < 0\\ 1, if \ 0 \le x < 1\\ -3, if \ 2 \le x < 4 \end{cases}$$

Part A. What is the domain of the function? Part B. What are the intercepts, if any, of this function?

Instructional Item 2

Graph the function  $f(x) = \begin{cases} x^2 + 1, & \text{if } x \ge 0\\ x + 1, & \text{if } x < -2 \end{cases}$ 

Instructional Item 3

Write a piecewise function from the graph given.



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



#### Functions

#### MA.912.F.1 Understand, compare and analyze properties of functions.

#### MA.912.F.1.1

#### Benchmark

# MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

# Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, functions represented as tables are limited to linear, quadratic and exponential.

*Clarification 2:* Within the Algebra I course, functions represented as equations or graphs are limited to vertical or horizontal translations or reflections over the *x*-axis of the following parent functions: f(x) = f(x)

$$x, f(x) = x^2, f(x) = x^3, f(x) = \sqrt{x}, f(x) = \sqrt[3]{x}, f(x) = |x|, f(x) = 2^x \text{ and } f(x) = (\frac{1}{2})^n$$

# **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9

#### **Terms from the K-12 Glossary**

- Exponential function
- Function
- Linear function
- Quadratic function
- Reflection
- Translation

#### Vertical Alignment Previous Benchmarks

#### Next Benchmarks

- MA.8.F.1
- MA.912.AR.2.2, MA.912.AR.2.3

#### **Purpose and Instructional Strategies**

In grade 8, students identified the domain and range of a relation and determined whether it is a function or not. In Algebra I, students classify function types limited to simple linear, quadratic, cubic, square root, cube root, absolute value and exponential functions. In Mathematics for College Algebra, students will classify function types within real-world context.

- The purpose of this benchmark is to lay the groundwork for students to be able to choose appropriate functions to model real-world data.
- Instruction includes the connection of the graph to its parent function. As various function types are introduced, take time to allow students to produce a rough graph of the parent function from a table of values they develop. Lead student discussion to build



connections with why these function types produce their corresponding graphs (*MTR.4.1*).

- Instruction develops the understanding that if given a table of values, unless stated, one cannot absolutely determine the function type, but state which function the table of values could represent. Discuss why function type was selected.
  - For example, if given the function y = |x| and only positive values were given in a table, one could say that table of values could represent a linear or absolute value function.
- Instruction includes opportunities to use graphing technology to graph parent functions of different equations and their key features.

#### **Common Misconceptions or Errors**

• Some students may miscalculate first and second differences that deal with negative values, especially if they perform them mentally. In these cases, have students quickly write out the subtraction expression (i.e., -14 - (-2)) so they can see that they are subtracting a negative value and should convert it to adding a positive value.

# Instructional Tasks

*Instructional Task 1 (MTR.3.1)* Determine the function types of the following:

Part A.  $f(x) = 3x^2 - 3x - 6$ Part B.  $f(x) = m^y$ Part C. f(x) = 2k + 7Part D.  $f(x) = \sqrt{x - 7}$ 



Part F.

x	f(x)
0	1600
1	800
2	400
3	200
4	100
5	50

Part G.



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x	f(x)
2	4
3	9
4	16
5	25
6	36
7	49

# Instructional Task 2 (MTR.3.1, MTR.7.1)

In 2012, there were about 223 million cell phone subscribers in the United States. During the next 5 years, the number of cell phone subscribers increased about 2.5% each year.

Part A. Classify the family function that best models this scenario.

- Part B. Write the function rule that represents this situation with x = 0 representing the year 2012
- Part C. Use the function rule to determine the number of cell phone subscribers in the United States for the year 2032.

# **Instructional Items**

Instructional Item 1

Given the table below, determine the function type that could represent it.

y -1.5 0 2.5 6 10.5	x	6	8	10	12	14	
	у	-1.5	0	2.5	6	10.5	

#### Instructional Item 2

Describe a quadratic function then give an example.

#### Instructional Item 3

A table of functions is shown using different representations. Classify the function family for each function.

Function	Function		
f(x) = 4 - 5x	$f(x) = \frac{1}{4}(x-2)^2 + 5$		
graph from desmos.com	graph from desmos.com		



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x	У		x	У
-6	2.5		-2	-0.9375
-5	2.75		-1	2.75
-4	3		0	0
-3	2.75		2	15
-2	2.5		4	255

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.F.1.2

# Benchmark

# MA.912.F.1.2 Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

*Example: The function*  $f(x) = \frac{x}{7} - 8$  *models Alicia's position in miles relative to a water stand x minutes into a marathon. Evaluate and interpret for a quarter of an hour into the race.* 

# Benchmark Clarifications:

*Clarification 1:* Problems include simple functions in two-variables, such as f(x, y) = 3x - 2y. *Clarification 2:* Within the Algebra I course, functions are limited to one-variable such as f(x) = 3x.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9

#### Terms from the K-12 Glossary

- Domain
- Function
- Function notation

#### **Vertical Alignment**

#### **Previous Benchmarks**

Next Benchmarks

- MA.8.F.1
- MA.912.AR.4.3

#### **Purpose and Instructional Strategies**

In Algebra I, students work with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Math for College Algebra, students continue to use function notation with other function types and perform operations that combine functions, including compositions of functions.

• Instruction leads students to understand that f(x) reads as "f of x" and represents an output of a function in the same way that the variable y represents the output in x-y



notation.

- Instruction includes a series of functions with variety of inputs so that students can see the pattern that emerges (*MTR.5.1*).
  - $\circ$  For example,

$$f(x) = 2x^{2} + 5x - 7$$
  

$$f(k) = 2k^{2} + 5k - 7$$
  

$$f(-2) = 2(-2)^{2} + 5(-2) - 7$$
  

$$f(-2) = -9$$

- Students should discover that the number in parenthesis corresponds to the input or *x*-value on the graph and the output on the other side of the equal sign corresponds to the *y*-value.
  - For example, x = -2 is the input and y = -9 or f(-2) = -9 is the output.  $f(-2) = 2(-2)^2 + 5(-2) - 7$ f(-2) = -9
- Although not conventional, instruction includes using function notation flexibly.
  - For example, function notation can be seen as h(x) = 4x + 7 or 4x + 7 = h(x).
- Instruction leads students to consider the practicality that function notation presents to mathematicians. In several contexts, multiple functions can exist that we want to consider simultaneously. If each of these functions is written in *x*-*y* notation, it can lead to confusion in discussions.
  - For example, the equations y = -2x + 4 and y = 3x + 7. Representing these functions in function notation allows mathematicians to distinguish them from each other more easily (i.e., f(x) = -2x + 4 and g(x) = 3x + 7).
- Function notation also allows for the use of different symbols for the variables, which can add meaning to the function. Using function notation allows mathematicians to express the output and input of a function simultaneously.
  - For example,  $h(t) = -16t^2 + 49t + 4$  could be used to represent the height, *h*, of a ball in feet over time, *t*, in seconds. Therefore, h(3) = 7 would represent a ball 7 feet in the air after 3 seconds of elapsed time. This is equivalent to the ordered pair (3,7) but with the added benefit of knowing it is associated with the function h(t).
- When evaluating functions in two variables, students should remember to substitute the values for bath variables before solving.
- A function of two variables has inputs that are ordered pairs (*x*, *y*) and the outputs are a single real number. The domain of the function is a set of the ordered pairs and the range is the set of all possible outputs.
  - For example, if you rent a car, the cost depends on two items, the days you keep the car and how far you drive. As a function this would be represented as the cost of the car rental, R, with the inputs of the function being the days driven, d, and the miles driven, m. The function in two variables would be represented as R(d,m) in function notation.



#### **Common Misconceptions or Errors**

- Throughout students' prior experience, two variables written next to one another indicate they are being multiplied. Therefore, students may incorrectly multiply  $f \cdot x$ . Continue to reinforce the meaning of function notation using multiple representations such as with equations, tables, graphs, and in areal-world context.
- Students may need additional support in the order of operations.
  - For example, for exponential functions, many students multiply *a* by the growth factor and then raise the product to the value of the exponent.
  - For example, students may think that the multiplication is always performed before division.
- Students may forget to substitute for both variables when evaluating functions in two variables.

#### Instructional Tasks

#### Instructional Task 1 (MTR.3.1, MTR.7.1)

The original value of a painting is \$9,000 and the value increases by 7% each year. The value of the painting can be described by the function  $V(t) = 9000(1 + 0.07)^t$ , where *t* is the time in years since 1984 and V(t) is the value of the painting.

Part A. Create a table of values that corresponds to this function.

Part B. Graph the function.

Part C. Use the function to predict the value of the painting for the year 2025.

#### Instructional Task 2

Given the function f(x, y) = 3x - 5y + 6, evaluate for f(-7,5).

# Instructional Items

Instructional Item 1 Evaluate f(24), when  $f(x) = \frac{3}{2}x + 9$ .

Instructional Item 2 Given  $f(x) = 2x^4 - 0.24x^2 + 6.17x - 7$ , find f(2).

Instructional Item 3 Evaluate f(a + b, c) when  $f(x, y) = 6x^2 - 7y + 1$ .

Instructional Item 4

Given  $f(x) = -2x^2 + 4x - 5$ , find the value(s) of x when f(x) = -11.

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.F.1.3

Benchmark	
MA.912.F.1.3	Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.
Damahananlı Clamifi	tion

Benchmark Clarifications:



*Clarification 1:* Instruction includes making the connection to determining the slope of a particular line segment.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.10

# Terms from the K-12 Glossary

- Domain
- Exponential function
- Linear function
- Range (of a relation or function)
- Rate of change
- Slope

# Vertical Alignment

# **Previous Benchmarks**

- MA.8.AR.3.2
- MA.8.F.1.3
- MA.912.AR.2.2

- Next Benchmarks
  - MA.912.F.1.4(PreCalc)
  - MA.912.C.2.1(Calc),
  - MA.912.C.2.2(Calc)
  - MA.912.C.3.8(Calc),
     MA.912.C.3.9(Calc), MA.912.C.3.10
     MA.912.C.4.3(Calc)

# **Purpose and Instructional Strategies**

In Algebra I, students calculated the constant rate of change (slope) and average rate of change in real-world situations modeled by linear, quadratic, exponential and absolute value functions. In Mathematics for College Algebra, students calculate the average rate of change from other functions. In later courses, this concept leads to the difference quotient and differential calculus.

- The purpose of this benchmark is to extend students' understanding of rate of change to allow them to apply it in non-linear contexts.
- Define and discuss the average rate of change and differences between linear and nonlinear functions.
- Instruction emphasizes a graphical context so students can see the meaning of the average rate of change. Students can use graphing technology to help visualize this.
  - Starting with the linear function f(x) = 3x 2, shown below, ask students to calculate the rate of change between two points using the slope formula. Lead students to verify their calculations visually.
  - Once students have successfully used the formula, transition to the graph of  $f(x) = x^2$ .
  - Highlight the same two points and ask students to discuss what the rate of change might be between them (*MTR.4.1*). Lead students realize that while there is not a constant rate of change, they can calculate an average rate of change for an



interval. Show students that this is equivalent to calculating the slope of the line segment that connects the two points of interest.



- Once students have an understanding, ask them to find the average rate of change for other intervals, such as  $-2 \le x \le -1$  or  $0 \le x \le 2$ . As each of these calculations produce different values, reinforce the concept that nonlinear functions do not have constant rates of change (*MTR.5.1*).
- Look for opportunities to continue students' work with function notation. Ask students to find the average rate of change between f(1) and f(4) for f(x).

#### **Common Misconceptions or Errors**

- Some graphs presented to students will only display a certain interval of data. Some students may mistakenly interpret the rate of change for that entire interval rather than a given sub-interval.
- Students may confuse the average rate of change and constant rate of change.
- Students may be confused if the average rate of change is 0, even though the function is not constant.
  - For example, the average rate of change of the function  $f(x) = x^2$  from x = -1 to x = 1 is 0.

#### Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1) Jorge invests an amount of \$5,000 in a money market account at an annual interest rate of

5%, compounded monthly. The function  $f(x) = 5000 \left(1 + \frac{0.05}{12}\right)^{12x}$ , represents the value of the investment after x years.

- Part A. Find the average rate of change in the value of Jorge's investment between year 5 and year 7, between year 10 and year 12, and between year 15 and year 17.
- Part B. What do you notice about the change in value of the investment over each interval?

Instructional Task 2 (MTR.3.1, MTR.4.1) The graph of  $f(x) = 36 - x^2$  is shown.



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Part A. Determine the average rate of change on the interval [-6, -4].

- Part B. Determine the average rate of change on the interval [4,6].
- Part C. Determine the average rate of change on the interval [-6,6].
- Part D. Compare and discuss with a partner the average rate of change for parts A-C.

# Instructional Task 2 (MTR.3.1, MTR.4.1 MTR 7.1)

Jorge invests an amount of \$5,000 in a money market account at an annual interest rate of 5%, compounded monthly.

- Part A. Write the function to represent the amount of Jorge's money market account.
- Part B. Find the average rate of change in the value of Jorge's investment between year 5 and year 7, between year 10 and year 12, and between year 15 and year 17.
- Part C. What do you notice about the change in value of the investment over each interval?

# Instructional Items

#### Instructional Item 1

Use the table below to find the average gas prices in Florida between 2017 and 2019. U.S. Energy Information Administration - https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=pet&s=emm\_epmru\_pte\_sfl\_dpg

eia.gov/dnav/pel	t/hist/LeafHandler	.ashx?n=pet&s=emm	epmru	pte sf	l dpg&f=a
Year	Price		-	-	
2011	\$3.479				
2012	\$3.546				
2013	\$3.463				
2014	\$3.302				
2015	\$2.312				
2016	\$2.070				
2017	\$2.330				
2018	\$2.601				
2019	\$2.439				
2020	\$2.070				

Instructional Item 2

The graph below represents the number of producing gas wells in Kansas from 1989 to 2017. What is the average rate of change between 2000 and 2015?


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\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.F.1.6

# Benchmark



Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior and asymptotes.

*Clarification 2:* Within the Algebra I course, functions other than linear, quadratic or exponential must be represented graphically.

*Clarification 3:* Within the Algebra I course, instruction includes verifying that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

# **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.9.10

# Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range
- Rate of change
- Slope
- x-intercept
- y-intercept



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#### Algebra I B1G-M

Vertical Alignment	
Previous Benchmarks	Next Benchmarks
<ul> <li>MA.8.AR.3.5</li> <li>MA.912.F.1.5, MA.912.F.1.8</li> </ul>	<ul> <li>MA.912.F.1.7 (Algebra 2,(Alg2/H; PreCalc; Math for ACT/SAT)</li> <li>MA.912.DP.2.8 (Algebra 2, MDFL)(Alg2/H; MFDL)</li> <li>MA.912.DP.2.9 (Alg 2, MCLA, MCS,</li> </ul>
	MDFL, Prob&Stats)MA.912.DP.2.9(Alg2/H; Prob & Stats; MCS; MCLA; MFDLH)

#### **Purpose and Instructional Strategies**

In grade 8, students interpreted the slope and y-intercept of a linear equation in two variables. In Algebra I, students compared key features of two or more linear or nonlinear functions in various representations, with focus on quadratic and exponential function representations, limited to graphically only. In Math for College Algebra, students continue their work by comparing key features of linear and extends to additional types of nonlinear functions from using various representations of functions - graphically, in tables or written descriptions.

• Within this benchmark, students should recognize that a linear function has a constant rate of change and can be represented by a line and that a nonlinear function does not have a constant rate of change, so it is not a line. Students are familiar with a straight line being drawn over any interval of its domains for Linear functions, but a new important concept in this course is that a function may not be continuous at a point, resulting in discontinuity.

• Instruction includes student exploration of various models to ultimately determine that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

• Given a table, students will look at the change of y over the change of the x, to check for the slope value. If it's a linear function it will have a constant slope, but if it's not it will be a nonlinear function.



- When looking at a graph's input and output in a table, students can easily identify, using a visual model, whether there is a constant rate of change.
- For example, provide students with the following context and discuss strategies for display:

You are being contracted by a large company to provide technical services to a major engineering project. The contract will involve you advising a group of engineers for the three weeks. The company offers you a choice of two methods



of payment for your services. The first is to receive \$500 per day of work. The second is to receive payment on a scale: two cents for one total day of work, four cents for two total days, eight cents for three total days, etc. Which method of payment would you choose?

Have students choose a method of payment and begin a class discussion regarding the reasoning students used to make their choices (*MTR.4.1*). Ask if students can create a function to represent each payment method (*MTR.7.1*).

- Comparison of functions is best explored with the use of a graphing utility to compare and identify features of the 7 most common graphs used in the Math for College Algebra course. It is recommended that students complete a graphic organizer to capture the following features of each function:
  - Domain (Use Set builder, interval notation)
  - o Range
  - X-intercepts (zeros of a function)
  - o Y-intercept
  - o Increasing, decreasing or constant interval
  - Describing common behaviors of graphs
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
- It is important to have students draw graphs of functions on a graphing calculator or computer utility to examine the curves of the graph. Not every method of comparison needs to be done algebraically, but doing so graphically provides an easier view and comparison of the many characteristics.
  - For example, as students compare the *y*-values (output) of an exponential and quadratic function, they notice that an exponential curve will be higher than a quadratic curve.
- It is helpful to illustrate, by standing in the front of the class, and holding your arms up or down to simulate the end behaviors of graphs. For polynomial graphs, students should recognize patterns of end behaviors.
- Instructions should include explanation of how graphs of rational functions differ from polynomial functions and point out that rational function graphs are typically discontinuous at one or more values of x and may include asymptotes.
  - Vertical Asymptotes are determined by finding the values of x that make the denominator equal to 0.

Horizontal Asymptotes are determined by considering what happens to the function as it approaches positive and negative infinity.

#### **Common Misconceptions or Errors**

- When describing domain or range, students may assign their constraints to the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may also miss the need for compound inequalities when describing domain or range. In these cases, use a graph of the function to point out areas of their constraint that would not make sense in context.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases,



ask reflective questions to help students examine the meaning of the domain and range in the problem. Relate the range of the graph to the dependent variable.

# **Instructional Tasks**

*Instructional Task 1(MTR.3.1)* Part A. Sketch a graph for each of the functions shown on the same coordinate plane.

 $f(x) = x^2$ g(x) = |6x + 5|

$$p(x) = 4^{0.45x}$$

Part B. For what values of x will exponential function be greater than the absolute and quadratic functions?

# **Instructional Items**

Instructional Item 1

A mother and daughter decided to register for a social media group. The number of messages received by the mother on day n is given by the equation M = 4n. The graph shows the number of messages received by the daughter on day n (where n is the number of days since joining the group).



Part A. Explain the difference in the rates of change in the number of messages the mother receives and the number of messages the daughter receives.

Part B. Whose number of daily messages is increasing more rapidly? Provide justification for your answer.

#### Instructional Item 2

Which equation represents a nonlinear function?

- a. y = 4.3
- b.  $y = \pi x$
- c.  $y = \frac{3}{x}$

d. 
$$y = 2(x - 1)$$

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



# MA.912.F.2 Identify and describe the effects of transformations on functions. Create new functions given transformations.

# MA.912.F.2.1

#### Benchmark

## MA.912.F.2.1 Identify the effect on the graph or table of a given function after replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k.

# Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, functions are limited to linear, quadratic and absolute value. *Clarification 2:* Instruction focuses on including positive and negative values for k.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

#### Terms from the K-12 Glossary

- Transformation
- Translation

#### Vertical Alignment Previous Benchmarks

#### Next Benchmarks

- MA.8.GR.2
- MA.912.GR.2

# Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will examine the impact of transformations on linear, quadratic, exponential, logarithmic and absolute value functions. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
  - Have students use graphing technology to explore different parent functions.
    - In each graph, toggle on/off the graphs for f(x) + k, kf(x), f(kx) and f(x + k) to examine their impacts on the function. Use the slider to change the value of k (be sure to examine the impacts when k is positive and negative).
    - As students explore, prompt discussion (*MTR.4.1*) among them about the patterns they see as they adjust the slider (*MTR.5.1*).



- For f(x) + k, students should discover that k is being added to the output of the function (equivalent to the y-value) and will therefore result in a *vertical translation* of the function by k units.
  - Ask students to describe what values of *k* cause the graph to shift up. Which values cause it to shift down?
- For *kf*(*x*), students should discover that *k* is being multiplied by the output of the function (equivalent to the *y*-value) and will therefore result in a *vertical dilation* (stretch/compression) of the function by a factor of *k*.
  - Ask students to describe what values of *k* cause the graph to stretch up vertically. Which values cause it to compress? Which values for *k* cause the graph to reflect over the *x*-axis? What is the significance of k = -1?
- For f(x + k), students should discover that k is being added to the input of the function and will therefore result in a *horizontal translation* of the function by -k units.
  - Ask students to describe what values of *k* cause the graph to shift left. Which values cause it to shift right?
- For f(kx), students should discover that k is being multiplied by the input of the function and will therefore result in a *horizontal dilation* (stretch/compression) of the function by a factor of k.
  - Ask students to describe what values of k cause the graph to stretch horizontally. Which values cause it to compress? Which values for k cause the graph to reflect over the y-axis? What is the significance of k = -1?
- After students have a good understanding of the impact of f(x) + k, kf(x), f(kx) and f(x + k) on graphs of functions, connect that knowledge to tables of values for a function.
  - For f(x) + k, use graphing technology to display a graph of a quadratic function (like the one below) and set k = 4. Guide students to form a table and discuss its connection to the vertical translation observed on the graph.

x	f(x)	f(x) + 4
1	6	10
2	3	7
3	2	6
4	3	7

• For kf(x), use graphing technology to display a graph of a quadratic function (like the one below) and set k = 0.5. Guide students to form a table and discuss its connection to the vertical compression observed on the graph.

x	f(x)	0.5[f(x)]
1	6	3
2	3	1.5
3	2	1
4	3	1.5

• For f(x + k), use graphing technology to display a graph of a quadratic function (like the one below) and set k = 2. Guide students to form a table and discuss its connection to the horizontal translation observed on the graph. This one may be tricky for students to understand initially. For the table shown, consider x = 5. For f(x), f(5) = 6, but for g(x) = f(x + 2), g(5) = f(5 + 2), which is



equivalent to 18, which is equivalent to shifting f(7) two units to the left on the graph. Bridge this conversation with a graph of the two functions to help them understand the connection.

x	f()			
	f(x)		<i>x</i> + 2	g(x) = f(x+2)
1	6		3	2
2	3		4	3
3	2	$\rightarrow$	5	6
4	3		6	11
5	6		7	18
6	11		8	27
		1     6       2     3       3     2       4     3       5     6       6     11	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

• For f(kx), use graphing technology to display a graph of a quadratic function (like the one below) and set k = 3. Guide students to form a table and discuss its connection to the horizontal compression observed on the graph.

x	f(x)	3 <i>x</i>	f(3x)
1	6	3	2
2	3	6	11
3	2	9	38
4	3	12	83
5	6	15	146
6	11	18	258
7	18	21	326
8	27	24	443
9	38	27	578

#### **Common Misconceptions or Errors**

- Similar to writing functions in vertex form, students may confuse effect of the sign of k in f(x + k). Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k. Think about the point on the graph, that makes the argument (value in parentheses), equal zero.
  - For example, for f(x + 4), x needs to be -4 to get to zero.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of kf(x) and f(kx) by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from kf(x) and f(kx) look the same. For linear and quadratic functions, it can help to have a non-zero *y*-intercept to visualize the difference.

# Instructional Tasks

Instructional Task 1 (MTR.3.1)

Determine the equation for the graph of  $f(x) = x^2$  that has been compressed vertically by a factor of  $\frac{1}{2}$ .

Instructional Task 2 (MTR.7.1)



A square grass field has a side length of 30 meters. A square pool with the side length of x meters is to be placed in the center.

Part A. Find an equation representing the area of the grass after the pool is installed.

Part B. Graph the relation from the equation in a and adjust your graph for the valid values of x.

Part C. How does the equation and graph change with changes in the area of the field?

#### Instructional Items

Instructional Item 1

How does the graph of g(x) = f(x) - 2 compare to the graph of  $f(x) = 3(1.2)^x$ ?

#### Instructional Item 2

Describe the effect of the transformation f(2x) on the function table below.

x	f(x)
-2	4
0	0
2	4
4	16
6	36

Instructional Item 3

Given the absolute value graph of f(x) and its transformation, describe the effects on the parent function and write the expression for the transformation in terms of f(x).



\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.



Last Updated: July 2024

#### MA.912.F.2.2

#### Benchmark

MA.912.F.2.2 Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the *x*- or *y*-values or multiplying the *x*- or *y*-values by a real number.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

# Terms from the K-12 Glossary

- Coordinate plane
- Dilation
- Function notation
- Reflection
- Transformation
- Translation

#### Vertical Alignment Previous Benchmarks

• MA.8.GR.2

MA.912.GR.2

#### Next Benchmarks

- MA.912.AR.7.3(Alg2/H; Math for ACT/SAT)
- MA.912.F.1.7(Alg2/H; PreCalc; Math for ACT/SAT)

# **Purpose and Instructional Strategies**

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will examine the impact of two or more transformations of a given function. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
- When combining transformation, students will need to consider the order of the transformations.
  - Vertical Transformations affect the output value of the function
    - Given 4f(x) 2, vertically shifting by 2 and then vertically stretching by 4 does not create the same graph as vertically stretching by 4 and then vertically shifting by 2, because when we shift first, both the original



function and the shift gets stretched, while only the original function gets stretched when we stretch first.

- When students see an expression such as 4f(x) 2, we must first multiply the output value of f(x) by 4, causing a vertical stretch, and then subtract 2, causing the vertical shift. Therefore, students should be reminded that multiplication comes before addition.
- o Horizontal Transformations affects the inputs of the function
  - Given h(x) = f(5x + 2), we must first think about how the inputs to this function h relate to the inputs to function f. For example, if we know f(5) = 12, what input to h would produce that output? That means, what value of x will allow h(x) = f(5x + 2) = 12. We would need 5x + 2 = 12, and when solved for x, we would subtract 2, resulting in a horizontal shift, and then divide by 5 which gives us a horizontal compression.
  - When students see an expression such as 4f(x) 2, we must first multiply the output value of f(x) by 4, causing a vertical stretch, and then subtract 2, causing the vertical shift. Therefore, students should be reminded that multiplication comes before addition.

Combining Vertical Transformations in	First vertically stretch by and
form $af(x) + k$	then vertically stretch by $k$
Combining Horizontal Transformations	First horizontally shift by h and
in form $f(bx + h)$	then horizontally stretch by $\frac{1}{b}$
Combining Horizontal Transformations	First horizontally shift by $\frac{1}{h}$ and
in form $f(b(x+h))$	then horizontally shift by $\tilde{h}$
Horizontal and Vertical transformation	The order does not affect the
in form $f(x - h) + k$ are independent	transformation

#### **Common Misconceptions or Errors**

- Similar to writing functions in vertex form, students may confuse effect of the sign of k in f(x + k). Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Students may think that a vertical and horizontal stretch from kf(x) and f(kx) look the same. For linear and quadratic functions, it can help to have a non-zero *y*-intercept to visualize the difference.

#### Instructional Tasks

Instructional Task 1 (MTR.3.1)

$$f(x) = 0.25(x-3)^2 + 5$$

Part A. What transformations occurred to the quadratic parent function  $g(x) = x^2$ ? Part B. Is there a specific order that the transformations must occur?

Part C. Suppose  $h(x) = f(\frac{1}{2}x) - 7$ . What transformations would need to occur to map f(x) to h(x)?

Part D. What transformations would map g(x) to h(x)?

Instructional Task 2 (MTR.7.1)



The cost in dollars of ridesharing company during regular hours is modeled by the function f(x) = 2.75x + 5 where x represents the number of miles driven. The cost of a ride during high demand hours, \$8 more than 145% the cost during regular hours.

Part A. Write the function, g(x) to represent the cost of the ride during high demand hours.

Part B. Compare the cost of a 7-mile ride during regular hours and high demand hours.

#### Instructional Items

Instructional Item 1

Describe the effect of the transformation  $g(x) = 4\sqrt{2-x}$  on the function table below.

x	$f(x) = \sqrt{x}$
0	0
1	1
2	1.41
3	1.73
4	2

Instructional Item 2

Suppose g(x) is constructed by multiplying all the *y*-coordinate values for a function f(x) by  $\frac{1}{2}$ , then adding 2 to each. Describe how the graph of g(x) would change in relation to f(x).

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.F.2.3

#### Benchmark

MA.912.F.2.3 Given the graph or table of f(x) and the graph or table of f(x) + k, kf(x), f(kx) and f(x + k), state the type of transformation and find the value of the real number k.

# Benchmark Clarifications:

Clarification 1: Within the Algebra I course, functions are limited to linear, quadratic and absolute value.

# **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

#### Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment



# Last Updated: July 2024

#### **Previous Benchmarks**

- MA.8.GR.2
- MA.912.GR.2

MA.912.F.1.7(Alg2/H; PreCalc; Math for ACT/SAT)

# Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students determine the type of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- Instruction includes identifying function transformations involving a combination of translations, dilations and reflections, and determining the value of the real number that defines each of the transformations.
- Transformations can be either horizontal (changes to the input: x) or vertical (changes to the output: f(x)).
- There are three different types of transformations: translations, dilations, and reflections.
  - By combining single transformations, a parent function can become a more advanced function.



- Using a graphing utility can help students understand how changing the value of the real numbers in the function's equation changes its graph.
- Encourage student's discussion about the effects of changing the value of the real numbers *A*, *B*, *C*, and *D* in the equation of the function. Ask them to generalize their findings.

	$x \rightarrow x + C$	Shift to the left by <i>C</i> units
Translations	$x \rightarrow x - C$	Shift to the right by C units
Translations	$f(x) \to f(x) + D$	Shift up by <i>D</i> units
	$f(x) \to f(x) - D$	Shift down by D units
	$x \rightarrow Bx; B > 1$	Dilates horizontally with scale
		factor $\frac{1}{B}$ (horizontal stretching)
	$x \to \frac{1}{B}x; B > 1$	Dilates horizontally with scale
D'1		factor B (horizontal shrinking)
Dilations	$f(x) \rightarrow Af(x); A > 1$	Dilates vertically with scale
		factor A (vertical stretching)
	$f(x) \to \frac{1}{A}f(x); A > 1$	Dilates vertically with scale
		factor $\frac{1}{A}$ (vertical shrinking)



# Last Updated: July 2024

Deflections	$x \rightarrow -x$	Reflects over the <i>y</i> -axis
Kenections	$f(x) \rightarrow -f(x)$	Reflects over the <i>x</i> -axis

#### **Common Misconceptions or Errors**

- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons.
- Similar to writing functions in vertex form, students may confuse effect of the sign of k in f(x + k). Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of kf(x) and f(kx) by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from kf(x) and f(kx) look the same. For linear and quadratic functions, it can help to have a non-zero *y*-intercept to visualize the difference.

## **Instructional Tasks**

# Instructional Task 1 (MTR.3.1)

A graph and table, which represents an absolute value function, are shown below. Describe and determine the value of the real number that defines the transformation from f(x) to g(x).



## Instructional Task 2 (MTR.3.1)

Describe the transformations that maps the function  $f(x) = 2^x$  to each of the following.

$$f(x) = 2^{x} - 2 \qquad g(x) = 4^{x} + 3$$
  

$$h(x) = 2^{x-3} \qquad b(x) = 3(2^{x+1}) + 2$$

# **Instructional Items**

Instructional Item 1

Considering the graph of f(x) and g(x) below, describe the transformation and determine the value of the real number, k, that defines the transformation from f(x) to g(x).



Last Updated: July 2024



#### Instructional Item 2

Considering the table below, describe the transformation and determine the value of the real number, k, that defines the transformation from f(x) to g(x).

x	f(x)	g(x)
1	-9	-8.4
2	-13.2	-12.6
3	-17.4	-16.8
4	-21.6	-21

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

# MA.912.F.2.4

# Benchmark

# MA.912.F. 2.4 Given the graph or table of values of two or more transformations of a function, state the type of transformation and find the values of the real number that defines the transformation.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

## Terms from the K-12 Glossary

- Transformation
- Translation

# Vertical Alignment Previous Benchmarks

#### Next Benchmarks

- MA.8.GR.2
- MA.912.GR.2

• MA.912.F.1.7(Alg2/H; PreCalc; Math for ACT/SAT)

#### Purpose and Instructional Strategies



In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students determine the type of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- Instruction includes identifying function transformations involving a combination of translations, dilations and reflections, and determining the value of the real number that defines each of the transformations.
- Transformations can be either horizontal (changes to the input: x) or vertical (changes to the output: f(x)).
- There are three different types of transformations: translations, dilations and reflections.
  By combining single transformations, a parent function can become a more advanced
- By combining single transformations, a parent function can become a more advanced function.



- Using a graphing utility can help students understand how changing the value of the real numbers in the function's equation change its graph (*MTR.5.1*).
- Encourage student's discussion about the effects of changing the value of the real numbers *A*, *B*, *C*, and *D* in the equation of the function (*MTR.4.1*). Ask them to generalize their findings.

Translations	$x \rightarrow x + C$	Shift to the left by C units
	$x \to x - C$	Shift to the right by C units
(addition/subtraction)	$f(x) \to f(x) + D$	Shift up by D units
	$f(x) \to f(x) - D$	Shift down by D units
	$x \to Bx; B > 1$	Dilates horizontally with scale
		factor $\frac{1}{B}$ (horizontal stretching)
	$x \to \frac{1}{B}x; B > 1$	Dilates horizontally with scale
Dilations		factor B (horizontal shrinking)
(multiplication/division)	$f(x) \rightarrow Af(x); A > 1$	Dilates vertically with scale
		factor A (vertical stretching)
	$f(x) \to \frac{1}{A}f(x); A > 1$	Dilates vertically with scale
		factor $\frac{1}{A}$ (vertical shrinking)
Reflections (negation)	$x \rightarrow -x$	Reflects over the y-axis
	$f(x) \to -f(x)$	Reflects over the x-axis

Instruction includes determining the order for performing two or more transformations to a single function. A different order of transformations may result in a different outcome.
 It is helpful to relate the order of the transformations to the order of operations.



- Horizontal shifts
- Vertical stretching or shrinking
- Reflecting
- Vertical shift

#### Common Misconceptions or Errors

- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons (*MTR.2.1*).
- Similar to writing functions in vertex form, students may confuse the effect of the sign of k in (x + k). Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of k(x) and f(kx) by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from k(x) and f(kx) look the same. For linear and quadratic functions, it can help to have a non-zero *y*-intercept to visualize the difference.

# **Instructional Tasks**

Instructional Task 1 (MTR.3.1) Below are the graphs of f(x) and g(x). f(x)f(x)g(x)Describes the sequence of transformations that may be graph of f(x) to the

Part A. Describe the sequence of transformations that map the graph of f(x) to the graph of g(x).

Step 2

Af(Bx)

20

28

30

34

6

8

Part B. If  $f(x) = \sqrt{x}$ , write an equation for g(x).

#### Instructional Task 2 (MTR.3.1, MTR.5.1)

The tables below show a sequence of transformation for function f.

x	f(x)	
6	10	
12	14	
18	15	
24	17	



	Step 3
x	g(x) = Af(Bx) +
2	21
4	29

31

35



Part A. Describe the transformation of f(x) represented in Step 1. State the value of *B*. Part B. Describe the transformation of f(x) represented in Step 2. State the value of *A*. Part C. Describe the transformation of f(x) represented in Step 3. State the value of *D*. Part D. Write an equation for g(x) in terms of f(x).

#### Instructional Items

# Instructional Item 1

Given the table below, determine the value of A and D.

x	f(x) =  x	g(x) = A x  + D
-2	2	-4
-1	1	-1
0	0	2
1	1	-1
2	2	-4

#### Instructional Item 2

In the graph below, function g is obtained from function f through a sequence of transformations. Find an equation for g.



\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive

#### MA.912.F.2.5

# Benchmark

MA.912.F.2.5 Given a table, equation or graph that represents a function, create a corresponding table, equation or graph of the transformed function defined by adding a real number to the *x*- or *y*- values or multiplying the *x*- or *y*- values by a real number.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9



• MA.912.F.1.1

#### Terms from the K-12 Glossary

- Transformation
- Translation

## Vertical Alignment

•

# **Previous Benchmarks**

- MA.8.GR.2
  - MA.912.GR.2

Next Benchmarks • MA.912.F.1.7(Alg2/H; PreCalc; Math for ACT/SAT)

#### **Purpose and Instructional Strategies**

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will create a table, equation or graph of a transformed function defined by adding a real number to the *x* or *y* values or multiplying the *x* or *y* values by a real number.
- Instruction includes the use of a graphic software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
- Given a function f, the transformed function g(x) = f(x C) is a horizontal shift of f(x). Adding a real number, C, to all the inputs (*x*-values) of a function will result in shifting the output left or right depending on the sign of C. If C is positive, the graph will shift right. If C is negative, the graph will shift left.

	0	<u> </u>	<u> </u>		
x	(x, f(x))	f(x)	g(x) = f(x - C); C = -1	(x, g(x))	g(x)
-2	(-2,4)	4	g(-2) = f(-2+1) = f(-1) = 1	(-2,1)	1
-1	(-1,1)	1	g(-1) = f(-1+1) = f(0) = 0	(-1,0)	0
0	(0,0)	0	g(0) = f(0+1) = f(1) = 1	(0, 1)	1
1	(1,1)	1	g(1) = f(1+1) = f(2) = 4	(1, 4)	4
2	(2, 4)	4	g(2) = f(2+1) = f(3) = ?	(2,?)	



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# Algebra I B1G-M



C = -1, Shift 1 unit to the left

• Given a function f, the transformed function g(x) = f(x) + D is a vertical shift of f(x). Adding a real number, D, to all the outputs (y-values) of a function will result in shifting the output up or down depending on the sign of D. If D is positive the graph will shift up, and if D is negative the graph will shift down.

	e no negative a	- B- P				
x	(x, f(x))	f(x)	g(x) = f(x) + D; D = 1	(x, g(x))	g(x)	
-2	(-2,4)	4	g(-2) = f(-2) + 1 = 5	(-2,5)	5	
-1	(-1,1)	1	g(-1) = f(-1) + 1 = 2	(-1,2)	2	
0	(0,0)	0	g(0) = f(0) + 1 = 1	(0,1)	1	
1	(1,1)	1	g(1) = f(1) + 1 = 2	(1,2)	2	
2	(2,4)	4	g(2) = f(2) + 1 = 5	(2,5)	5	
	2	(-	f(x) + 1 (1, 2) (1, 2) (1, 2) (1, 1) (1,	↑ D	= 1, S	hift 1

• Discuss with the students that as well as translations of two-dimensional figures, adding a constant to either the input or output of a function change the position of the graph, but it



doesn't change the shape of the graph (*MTR.4.1*).

- Given a function *f*, the transformed function g(x) = Af(x) is a vertical stretch or compression of f(x). Multiplying all the outputs (y-values) of a function by a real number, A, will result in a vertical stretching or compression depending on the value of A. If A is between 0 and 1 (0 < A < 1), the graph will be vertically compressed and if A is greater than 1 (A > 1), the graph will be vertically stretched.
- If A is a negative number (A < 0), the transformed graph will be a combination of a vertical stretch or compression and a reflection over the x-axis. Discuss with students how multiplying all the y-values by -1 is the same as reflecting a two-dimensional figure over the x-axis (*MTR.4.1*).

2					
x	$(x, \overline{f}(x))$	f(x)	$g(x) = Af(x); \overline{A} = 2$	(x, g(x))	g(x)
-2	(-2,4)	4	g(-2) = 2f(-2) = 8	(-2,8)	8
-1	(-1,1)	1	g(-1) = 2f(-1) = 2	(-1,2)	2
0	(0,0)	0	g(0) = 2f(0) = 0	(0,0)	0
1	(1, 1)	1	g(1) = 2f(1) = 2	(1,2)	2
2	(2, 4)	4	g(2) = 2f(2) = 8	(2,8)	8

x	$(\boldsymbol{x},\boldsymbol{f}(\boldsymbol{x}))$	f(x)	$g(x) = Af(x); A = \frac{1}{2}$	(x,g(x))	g(x)
-2	(-2,4)	4	g(-2) = 2f(-2) = 2	(-2,2)	2
-1	(-1,1)	1	g(-1) = 2f(-1) = 0.5	(-1,0.5)	0.5
0	(0,0)	0	g(0) = 2f(0) = 0	(0,0)	0
1	(1, 1)	1	g(1) = 2f(1) = 0.5	(1,0.5)	0.5
2	(2,4)	4	g(2) = 2f(2) = 2	(2,2)	2







• Given a function *f*, the transformed function g(x) = f(Bx) is a horizontal stretch or compression of f(x). Multiplying all the inputs (*x*-values) of a function by a real number, *B*, will result in a horizontal stretching or compression depending on the value of *B*. If *B* is between 0 and 1 (0 < B < 1), the graph will be horizontally stretched by  $\frac{1}{B}$ 

and if B is greater than 1 (B > 1), the graph will be horizontally compressed by  $\frac{1}{R}$ .

• If *B* is a negative number (B < 0), the transformed graph will be a combination of a horizontal stretch or compression and a reflection over the *y*-axis. Discuss with students how multiplying all the *x*-values by -1 is the same as reflecting a two-dimensional figure over the *y*-axis (*MTR.4.1*).

x	(x, f(x))	f(x)	g(x) = f(Bx); B = 2	(x, g(x))	g(x)
-4	(-4,16)	16	g(-4) = f(-8) = ?		
-2	(-2,4)	4	g(-2) = f(-4) = 16	(-2,16)	16
-1	(-1,1)	1	g(-1) = f(-2) = 4	(-1,4)	4
0	(0,0)	0	g(0) = f(0) = 0	(0,0)	0
1	(1,1)	1	g(1) = f(2) = 4	(1,4)	4
2	(2,4)	4	g(2) = f(4) = 16	(2, 16)	16
4	(4, 16)	16	g(4) = f(8) = ?		

• Discuss with students the meaning of g(x) = f(2x). In this case, the output value, g(x), is the same as the output value of f(x) at an input that is twice the size.

x	(x, f(x))	f(x)	$g(x) = f(Bx); B = \frac{1}{2}$	(x, g(x))	g(x)
-4	(-4,16)	16	g(-4) = f(-2) = 4	(-4,4)	4
-2	(-2,4)	4	g(-2) = f(-1) = 1	(-2,1)	1
-1	(-1,1)	1	$g(-1) = f\left(-\frac{1}{2}\right) = ?$		
0	(0,0)	0	g(0) = f(0) = 0	(0,0)	0
1	(1,1)	1	$g(1) = f\left(\frac{1}{2}\right) = ?$		
2	(2,4)	4	g(2) = f(1) = 1	(2,1)	1
4	(4, 16)	16	g(4) = f(2) = 4	(4, 4)	4

• Discuss with students the meaning of  $g(x) = f(\frac{1}{2}x)$ . In this case the output value, g(x) is the same as the output value of f(x) at an input that is half the size. Example:  $g(4) = f(\frac{1}{2} \cdot 4) = f(2) = 4$  (*MTR.4.1*).



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# Algebra I B1G-M



#### **Common Misconceptions or Errors**

- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons (*MTR.2.1*).
- Some students misinterpret how the parameters of the equation of a transformed function are affected by a horizontal translation. This may indicate that students do not understand the relationship between the graph and the equation of the function.
  - For example, a student may think that g(x) = f(x + 1) is a horizontal translation to the right because of the positive addend for *x*. One potential teaching strategy would be using a graphing utility to graph the function  $f(x) = (x C)^2$  creating *C* as slider, and then allowing students to explore the translation results as the value of the slider changes.
- Some students may have difficulties understanding that multiplying the input of a function by a number greater than 1 will result in a horizontal compression of the graph instead of a stretching. It is important to point out that multiplying the *x*-value does not change the original value of the input. Because the input is being multiplied by a number greater than 1, a smaller input in the transformed function is needed to obtain the same output from the original function. One potential teaching strategy would be using a graphing utility to graph the function  $f(x) = (Bx)^2$  creating *B* as slider, and then allowing students to explore the stretching/compression results as the value of the slider changes from 0 to 2. Remind students that negative values of *B* will result in a vertical reflection of the function.

# Instructional Tasks

Instructional Task 1 (MTR.2.1)

The figure shows the graph of a function *f* whose domain is the interval  $-4 \le x \le 4$ .



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Algebra I B1G-M



Part A: Sketch the graph of each transformation described below and compare it with the graph of f. Explain what you see.

a. 
$$q(x) = f(x) + 2$$

b. 
$$h(x) = f(x+2)$$

c. 
$$k(x) = 2f(x)$$

d. 
$$r(x) = f(2x)$$

Part B: The points labeled M, N, P on the graph of f have coordinates M = (-4, -5), N = (0, -1, ) and P = (-4, 4). Complete the table below with the coordinates of the points corresponding to M, N, P on the graphs of g, h, k and r?

f(x)	<b>g</b> ( <b>x</b> )	h(x)	k(x)	r(x)
(-4, -5)				
(0, -1, )				
(-4, 4)				

#### Instructional Items

Instructional Item 1 (MTR.3.1)

Given the function f(x) = |x|, graph the function f(x) and the transformation

g(x) = f(x - 3) on the same axes. What do you notice about the *x*-intercepts of g(x)?

# Instructional Item 2 (MTR.3.1)

Given the function  $f(x) = \log x$ , graph the function f(x) and the transformation g(x) = 3f(x) on the same axes. Describe the transformed function, g(x), as it relates to the graph of f(x).

#### Instructional Item 3 (MTR.3.1)

A function f(x) is given. Create a table for the functions below.

a. g(x) = f(x) + 5b. h(x) = f(2x)

2x)					
	x	2	4	6	8
	$f(\mathbf{x})$	1	3	7	1

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



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#### MA.912.F.3 Create new functions from existing functions.

# MA.912.F.3.2

Benchmark	
MA.912.F.3.2	Given a mathematical or real-world context, combine two or more
	functions, limited to linear, quadratic, exponential and polynomial,
	using arithmetic operations. When appropriate, include domain
	restrictions for the new function.
Benchmark Clarifica	tions:
Clarification 1: Instr	uction includes representing domain restrictions with inequality notation, interval
notation or set-builde	er notation.
Clarification 2: With	in the Mathematics for Data and Financial Literacy course, problem types focus on
money and business.	
Connecting Benc	hmarks/Horizontal Alignment
• MA.912.AF	R.2.4, MA.912.AR.2.5
• MA.912.AF	R.3.7, MA.912.AR.3.8
• MA.912.AF	R.4.4
• MA.912.AF	R.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
• MA.912.F.1	1.1
Terms from the	K-12 Glossary
Domain	
<ul> <li>Function</li> </ul>	
Function no	tation
• Set-builder	notation
Vertical Alignme	ent
<b>Previous Benchma</b>	arks Next Benchmarks
• MA.7.AR.1	MA.912.C.2.3 (Calc), MA.912.C.2.4
• MA.8.AR.1	(Calc)
• MA.912.AF	R.1.4, MA.912.AR.1.7
Purpose and Inst	tructional Strategies Integers
In middle grades, s	tudents performed operations on linear expressions. In Algebra I, students
performed addition	, subtraction, multiplication and division with polynomials. In Math for
College Algebra, st	udents continue to use function notation and they combine functions using
arithmetic operation	ns.
<ul> <li>When approx</li> </ul>	ppriate, the domain restrictions will be determined for the new function.
Students wi	ll evaluate the solution when combining two functions for a provided input.
• In this benc	hmark, students will combine functions through addition, subtraction,

- multiplication, and division. This process will utilize their prior experience with polynomial arithmetic in MA.912.AR.1 from Algebra I.
  - In mathematical contexts, combinations through addition may be represented as (f + g)(x) or as h(x) = f(x) + g(x).
  - In mathematical contexts, combinations through subtraction should be represented as (f g)(x) or as h(x) = f(x) g(x).



- In mathematical contexts, combinations through multiplication should be represented as  $(f \cdot g)(x)$  or as  $h(x) = f(x) \cdot g(x)$ .
- In mathematical contexts, combinations through division should be represented as  $\left(\frac{f}{g}\right)(x)$  or as  $h(x) = \frac{f(x)}{g(x)}$  where  $g(x) \neq 0$ . Additionally, for division, it can be represented using the division symbol.
- Explain that when functions are combined using addition, subtraction, and multiplication, the domain of the resulting function is only the inputs (x-values) that are common to the domains of the original functions. The domain of f + g, f g and  $f \cdot g$  is the intersection of the domains of f and g.
- When combining functions by division, students will need to consider values in the domain of the quotient that should be restricted. Values that should be restricted are those that would cause the denominator to equal zero.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation. Reference MA.912.AR.2.5 for understanding various ways to write constraints.
- Instruction may include the use of graphs and tables to foster a deeper understanding of operations with functions.
- Students have previous knowledge of linear and quadratic functions and should make the connection that by multiplying two linear functions, it results in a quadratic function.
  - For example, to determine revenue, one could multiply the function that represents price of a product by the function that represents quantity of a product that will be sold at that price.

#### **Common Misconceptions or Errors**

- When multiplying and dividing functions students may struggle working with exponents, remind students of the multiplication and division properties of exponents, include negative exponents.
- When finding the domain of the new function, students find the union of the domains instead of the intersection. Emphasize that they need to find what the domains have in common.
- When finding the domain of  $\frac{f}{g}$  students forget to identify the restrictions on the domain. Remind students that they always need to find the values that make g(x) = 0 and exclude them from the domain of  $\frac{f}{g}$ .

# **Instructional Tasks**

# Instructional Task 1 (MTR.5.1)

Using the graphs and functions below, sketch a graph of the function h(x) given that h(x) = f(x) + g(x).

 $f(x) = \frac{1}{4}x^3 - 4$ 

$$g(x) = \frac{1}{2}x + 1.$$



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# **Instructional Items**

Instructional Item 1 Instructional Item 1 Given  $f(x) = x^2 + 2x + 4$  and g(x) = -2x + 5, find f(x) + g(x), f(x) - g(x),  $f(x) \cdot g(x)$ , and  $\frac{f(x)}{g(x)}$ . Determine the domain for each function.

Instructional Item 2

Let  $f(x) = x^3 + 4x^2 - 20x - 48$  and  $g(x) = x^2 + 2x - 24$ . a. Determine the simplified function for  $h(x) = \frac{f(x)}{g(x)}$ . b. Determine the restrictions on the domain for h(x).

\*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.



## MA.912.F.3.4

# Benchmark

MA.912.F.3.4	Represent the composition of two functions algebraically or in a table.
	Determine the domain and range of the composite function.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1, MA.912.F.1.2
- Terms from the K-12 Glossary
  - Domain
  - Function
  - Function notation

#### Vertical Alignment

**Previous Benchmarks** 

#### Next Benchmarks

# MA.912.C.2.3 (Calc), MA.912.C.2.4 (Calc) MA.912.F.3.5 (PreCalc)

# Purpose and Instructional Strategies Integers

In Algebra I, students work with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Math for College Algebra, students continue to use function notation with other function types and combine functions with arithmetic operations, including compositions of functions.

- Emphasize that combining function with arithmetic operations is not the same as composition of functions. This is in benchmark MA.912.F.3.2.
- Define function composition and demonstrate composing functions with several examples. Explain that function composition is an operation that can be performed on two functions so the output of one function becomes the input of another to form a new function. The resulting function is known as a composite function.
- Introduce the notation for composition of functions as (f ∘ g)(x) = f(g(x)) and explain that we read the left-hand side as "f composed with g at x," and the right-hand side as "f of g of x."
- Emphasize the difference between f(g(x)) and g(f(x)) and explain that function composition is not commutative. Explain that it is important to follow the order of operations when evaluating a composite function, we evaluate the inner function first.
- Students should have practice with finding the domain and range of the composite function and the potential differences in domain of the original function types to the domain of the composite functions.
- Students should have practice with creating a composite function given:
  - $\circ$  Two equations
    - An equation and a table
    - Two tables
- Students should have practice with decomposing a given composite function and identifying the two equations of the original functions.
  - For example, consider the composite function  $h(x) = (2x 3)^3 8$ .



- Identify the two functions f(x) and g(x), where h(x) = f(g(x)) $f(x) = x^3 - 8$  and g(x) = 2x - 3
- Instruction includes determining the domain and range of the composite function. Explain that the domain of the composite function is all inputs x, such that x is in the domain of g and g(x) is in the domain of f.

#### **Common Misconceptions or Errors**

- Students may confuse finding a composition of a function with applying the four basic operations to combine two functions.
- When decomposing a composite function, students may not know which function was plugged into the other to get the composition of functions.
- Students sometimes believe that variables represent just a fixed number, so they struggle understanding that a variable can also represent a function.
- Students struggle evaluating functions for numerical inputs, so it is more difficult for them evaluating for inputs that are functions.
- Students may not understand the function notation for composition thinking that f(g(x)) means to multiply function g by function f.
- Students may confuse the composition  $(f \circ g)$  with the product  $(f \cdot g)$ . Emphasize that the composition means "evaluate g at x, then evaluate f at the result g(x)."
- Some students might thing that  $(f \circ g)$  is the same as  $(g \circ f)$ . Emphasize that composition is not commutative.
- When finding the domain of a composite function, sometimes students find the union of the domains instead of the intersection. Emphasize that they need to find what the domains have in common, the intersection. Also, students sometimes forget to include the restrictions on the domain of the inner function on the domain of the composite function. Remind them to use the most restrictive domain.

#### Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Your parents purchased a new reclining chair for the living room. The chair will be delivered to your home. The cost of the chair is d dollars, the tax rate is 6.5%, and the delivery fee is \$50.

- Part A. Write a function f(d) for the purchase amount, d, and the delivery fee. State the domain.
- Part B. Write another function g(d) for the cost of the chair after taxes. State the domain.
- Part C. Write the function f(g(d)) and interpret its meaning. State the domain of f(g(d)).
- Part D. Write the function g(f(d)) and interpret its meaning. State the domain of g(f(d)).
- Part E. Which results in a lower cost to you, f(g(d)) or g(f(d))? Explain why.

#### Instructional Task 2 (MTR.7.1)

Let f be the function that assigns to a temperature in degrees Celsius its equivalent in degrees Fahrenheit. Let g be the function that assigns to a temperature in degrees Kelvin its equivalent in degrees Celsius.

Part A. Explain what x and f(g(x)) represent in terms of temperatures, or explain why there is no reasonable representation.



- Part B. Explain what x and g(f(x)) represent in terms of temperatures, or explain why there is no reasonable representation.
- Part C. Given that  $f(x) = \frac{9}{5}x + 32$  and g(x) = x 273, find an expression for f(g(x)).
- Part D. Find an expression for the function h which assigns to a temperature in degrees Fahrenheit its equivalent in degrees Kelvin.

# **Instructional Items**

Instructional Item 1 (MTR.3.1)

- Given  $f(x) = 3x^2 + x + 4$  and g(x) = x + 4.
  - a) Determine  $f \circ g$ .
  - b) Determine  $g \circ f$ .
  - c) Determine the domain of each composite function.

Instructional Item 2 (MTR.3.1) Find the composition of the functions  $f(x) = \frac{5}{x+3}$  and  $g(x) = x^2 + 6$ .

Instructional Item 3 (MTR.3.1) Express the function  $h(x) = \sqrt{x^2 + 5x + 6}$  as a composition of the two functions f and g so that h(x) = f(g(x)). \*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

#### MA.912.F.3.6

Benchmark		
MA.912.F.3.6	Determine wh	ether an inverse function exists by analyzing tables,
	graphs and eq	uations.
Connecting Ber	nchmarks/Horizo	ontal Alignment
• MA.912.A	AR.5.6, MA.912.A	R.5.7, MA.912.AR.5.8, MA.912.AR.5.9
• MA.912.F	7.1.6	
Terms from the	e K-12 Glossary	
Inverse fu	nction	
Vertical Alignm	nent	
<b>Previous Benchr</b>	narks	Next Benchmarks
• MA.9	12.F.1	MA.912.F.3.8 (PreCalc),
		MA.912.F.3.9(PreCalc)

Purpose and Instructional Strategies Integers

In Algebra I, students analyzed linear, quadratic, exponential and absolute value functions. In Mathematics for College Algebra, extend their knowledge of analyzing those functions as well as other functions to determine whether an inverse function exists by analyzing tables, graphs, and equations (*MTR.2.1*).

• Students will need to use what they have learned about key features and characteristics and patterns that are evident with each function type to determine whether the functions



they see represented in graphs, tables, and/or equations are functions that are invertible (*MTR.5.1*).

- In Algebra 1 students used the vertical line test to determine if the graph represents a function or not. When dealing with a graph, students can use the horizontal line test to determine if an inverse to the function exists. The horizontal line test allows a student to check if a function is one-to-one, meaning there is only one *x*-value for each *y*-value. In order to use the horizontal line test, students can draw a horizontal line and check to see if it intersects the graph of the function in all places at exactly one point. If it does, then the given function should have an inverse that is also a function.
- Instruction includes discussing what adjustments could be made to the function to make it where an inverse to the function will exist (*MTR.5.1*).
- Instruction includes using compositions of functions to determine whether two functions are inverses of each other (*MTR.2.1*).
- Instructions could include using a graphic organizer for students to use to help them understand how to transpose the input and output values in a table to determine the inverse of a function.

#### **Common Misconceptions or Errors**

- Students may confuse the horizontal line test (used to determine if the function has an inverse) with the vertical line test which determines if the graph represents a function.
- Students may not understand how to transpose the input and output values in a table to determine the inverse of a function.
- Students may not understand that it is possible for a function to have an inverse that may not be a function.

# Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Create a function that has an inverse. Explain using a table, graph, and equation how you know that an inverse exists.

#### Instructional Task 2 (MTR.5.1)

Create a function for which an inverse does not exist. Explain what about that function makes it not invertible. Explain what adjustments could be made to the function to make it invertible.

#### Instructional Task 3 (MTR.2.1, MTR.4.1)

Determine whether an inverse function exists for the function given below in the graphical and numerical representations. Explain what features you found in the graph as well as the table that helped you determine whether the function had an inverse or not.





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# Instructional Items

Instructional Item 1

Determine whether an inverse to the following function represented in the table exists. Explain.

x	$f(x)=2x^2-3x-6$
-2	-4
-1	-7
0	-6
1	-1
2	8

#### Instructional Item 2

Determine whether an inverse to the following function represented by the following graph exists. Explain.

					4				
 10	-8	-6	-4	-2	0	2	4		
					-2				

# Instructional Item 3

Determine whether an inverse to the following function represented with the equation exists. Explain.

 $f(x) = \ln\left(x\right)$ 

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.

# MA.912.F.3.7

# Benchmark

MA.912.F.3.7 Represent the inverse of a function algebraically, graphically or in a table. Use composition of functions to verify that one function is the inverse of the other.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes the understanding that a logarithmic function is the inverse of an exponential function.

#### **Connecting Benchmarks/Horizontal Alignment**

- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.6



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Terms from the K-12 Glossary

Inverse functions

Vertical Alignment Previous Benchmarks

#### Next Benchmarks

- MA.912.F.3.8(PreCalc),
- MA.912.F.3.9(PreCalc)
- MA.912.C.2.6(Calc)

#### **Purpose and Instructional Strategies Integers**

In Algebra I, students identified and interpreted key features for linear, quadratic, exponential and absolute value functions. In Mathematics for College Algebra, students also identified and interpret key features for the same functions as well as logarithmic functions and use those skills around analyzing tables, graphs and equations to represent an inverse to a function given to them (*MTR.2.1, MTR.5.1*).

- Instruction includes noticing patterns such as in transposing of the input and output values in a table to create an inverse of a function.
- Instruction includes the understanding that an inverse to a function does not always represent a function. Students should be able to know how to look at two functions in any representation and determine if they are inverse functions.
- Instruction includes the connection to MA.912.F.3.6 to help students be able to represent inverses to given functions, if they exist, algebraically, graphically or in a table. If students are not able to determine whether an inverse to a given function exists by examining the features given in the function, they may struggle to be able to represent the inverse of the given function in various representations. In this benchmark, students are also expected to use compositions of functions, which is a connection to MA.912.F.3.4, to verify that one function is the inverse of the other.
- Students need to show understanding that certain function types are inverses of each other such as quadratic, exponential and logarithmic functions. Students should also demonstrate an understanding that a linear function will have a linear inverse, if the inverse of the function exists (*MTR.5.1*).
- Instruction includes giving students a function represented in a table, graph, or equation and asking them to determine if the function is invertible (able to produce an inverse function) or non-invertible (not able to produce an inverse function). Once they have determined if the inverse function is able to be produced, then students should be expected to represent that inverse in multiple ways. In a table, students should notice that the input and output values transpose with each other. Instructions could include using a graphic organizer for students to use to help them understand how to transpose the input and output values in a table to determine the inverse of a function In a graph, students should notice that the points reflect over the line y = x.
- Students should be able to use compositions of function to verity that two functions are inverses of each other. When they compose the functions, they should demonstrate an understanding that if they end up with anything other than *x* as their solution to the compositions of functions than those functions are not inverses of each other.
- During instruction teachers may ask the students to use another representation (table or graph) to look at the two functions to justify their response as to whether the functions are inverses of each other. If they can see graphically that two functions are inverses of each other but their algebraically they do not get an *x* when composing the functions, it will



prompt the students to go back and check their work. Likewise, if they see graphically or in a table that the two functions are not inverses of each other, but their composition of functions does produce an x they will be prompted to go back and check their algebra *(MTR 6.1)*.

Instruction includes opportunities for students to explore the idea that when functions are inverses of each other, they can compose the functions both ways [f(g(x)) and g(f(x))] and still get "x" as the solution. While compositions are not commutative, in this special case, compositions can be done in either order.

#### **Common Misconceptions or Errors**

- Students may think that they should get a numerical value (most commonly zero or 1) for their solution when composing the two functions in order for the functions to be inverses, rather than getting the solution "*x*."
- Students may not understand how the graphs of inverse functions will reflect over the line y = x. Students may not realize that it has to reflect over the line y = x rather than just be a reflection over any line.
- Students may not understand how to look for patterns in the table that will justify that a function is an inverse of their given function.

#### Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.7.1)

The table below shows the number of households in the U.S. in the years.

Year	1998	1999	2000	2001	2002	2003	2004
Households (in thousands)	97,107	98,990	99,627	101,018	102,528	103,874	104,705

Part A. Find a linear function, *h*, which models the number of households in the U.S. (in thousands) as a function of the year, *t*.

Part B. Represent the inverse of the linear function you wrote in Part A in a table, graph, and expression.

#### Instructional Task 2 (MTR.5.1, MTR.4.1)

Let *f* be the function defined by  $f(x) = 10^x$  and g be the function g(x) defined by  $f(x) = \log_{10} x$ .

- a. Sketch the graph of y = f(g(x)). Explain your reasoning.
- b. Sketch the graph of y = g(f(x)). Explain your reasoning.
- c. Let f and g be any two inverse functions. For which values of x does f(g(x)) = x?
   For which values of x does g(f(x)) = x?

#### Instructional Task 3 (MTR.2.1, MTR.6.1)

Find the inverse to the function  $f(x) = x^2 + 3$ .

a. Represent the inverse in an equation/expression, a table, and a graph.

b. Verify that the inverse you found was an inverse algebraically using compositions of functions.

#### Instructional Items

Instructional Item 1

Which of these is **true** for the inverse of the function  $f(x) = (x + 1)^2$  on the domain  $x \le -1$ ?

 $\leq -1$ a.  $f^{-1}(x) = \sqrt{x} - 1$ 



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b. 
$$f^{-1}(x) = -\sqrt{x} - 1$$
  
c.  $f^{-1}(x) = \sqrt{x} - 1$   
d.  $f^{-1}(x) = -\sqrt{x} - 1$ 

#### Instructional Item 2

Find the inverse of the function  $f(x) = 3^x$ 

#### Instructional Item 3

Find the inverse of the function represented in the table. Represent the inverse in a table.  $r = \int f(x) - 2x^2 - 3x - 6$ 

x	$f(x)=2x^2-3x-6$
-2	-4
-1	-7
0	-6
1	-1
2	8

#### Instructional Item 4

Verify that the following functions are inverses of each other using compositions of functions.

$$f(x) = 2^x$$
$$g(x) = \log_2 x$$

\*The strategies, tasks and items included in the BIG-M are examples and should not be considered comprehensive.



# Appendix A. Summary of Changes

The purpose of this table is to provide a summary of changes from the latest version of the B1G-M. For questions or feedback on the B1G-M, please direct them to <u>BESTMath@fldoe.org</u>.

Benchmark	<b>B1G-M</b> Component	Change Made
MA.912.NSO.1.1	Connecting	REMOVED benchmarks MA.912.AR.3.1 and
	Benchmarks/Horizontal	MA.8.AR.1.2
	Alignment	
MA.912.NSO.1.1	Vertical Alignment	ADDED MA.4.FR.1.4 as a previous benchmark;
		REMOVED MA.8.NSO.1.1 as a next benchmark;
		REMOVED hyperlink and text
MA.912.NSO.1.1	Purpose and	6th bullet @ the 1st open bullet, the example was
	Instructional Strategies	UPDATED using names of individuals and
		removing confusing wording: "For example, if Jeff
		owes \$12.47 and Mary owes \$6.50, most students
		can quickly see and understand that Jeff has less
MA 012 NGO 1 1	C	ADDED
MA.912.NSO.1.1	Common Missensentions on	ADDED comma after points in the concluding
	Frors	paragraph of the 1st bullet.
MA 912 NSO 1 1	Strategies to Support	MODIFIED 3 <sup>rd</sup> bullet REMOVED the word
1111.912.1100.111	Tiered Instruction	"over" after \$34 and insert "rather than"
MA.912.NSO.1.1	Instructional Task	REWORDED "individual" to "individually" in 2
		places: "groups or individually: if
		individually"
MA.912.NSO.1.2	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.1.2	Strategies to Support	ADDED "on Saturday," before Thomas's and "on
	Tiered Instruction	Thursday," before Derrek's; MODIFIED the
		football picture to make sense with the actual use
		of x's and o's on playbooks and color coding to
		match the number line
MA.912.NSO.1.2	Instructional Tasks	REWORDED Instructional Task 1 with clarifying
		language: "On Thursday, Calvin borrowed \$46.68
		from his mom to purchase a new video game. He
		mowed yards on the weekend to earn money to
		pay his mom back. After paying his mom back,
		Calvin had 412/32 left on Monday"; ADDED a
MA 010 MGO 1 0		negative to 4.5
MA.912.NSO.1.3	Connecting	ADDED benchmark MA.912.NSO.1.4
	Alignment	
MA 912 NSO 1 3	Vertical Alignment	REMOVED hyperlink and text
MA 012 NSO 1 2	Purpose and	CHANGED the 2nd open bullet to read "Distance
INIA.912.INSU.1.5	Instructional Strategies	is not depicted as a negative number, thus absolute
	msu actional su alegles	value "
		value



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Domohanoula	DIC M Common on t	Change Made
Benchmark	BIG-M Component	Change Made
MA.912.NSO.1.3	Common	ADDED an open bullet at the beginning that deals
	Misconceptions or	with the subsequent bullets: Would you describe
	Errors	how far you traveled with a negative number?";
		CLARIFIED the car will drive in reverse, not
		backwards; REWORDED the example on walking
		to a door from the desk and back: "If you walk
		from your desk to the door backwards (your back
		is facing the door), did you walk a negative
		distance?"; EDITED the amusement park question
		to include sequencing and reworked the question
		stems: "You walk forward 10 feet in line, then
		the line makes a U-turn and you walk 30 feet. A
		U-turn happens in the line again and you travel an
		additional 16 feet before boarding the ride. How
		do you determine the distance you traveled whilst
		traveling in line?"; REMOVED the last open
		bullet (may appear on page 22 currently)
MA.912.NSO.1.3	Strategies to Support	REMOVED the word "students" before error
	Tiered Instruction	analysis in the $2^{nd}$ closed bullet
MA 912 NSO 1 3	Instructional Tasks	REWORDED instructional task 2 for clarification
1111.912.1100.11.5	instructional rusks	purposes: "The table below shows the change in
		rainfall amount from the last 5 years ".
		REWORDED Task 3 to be listed as a "task" not
		item then changed the question stem and parts A
		and B accordingly: Part A reads "Compare the
		distance of 4 and -4 in relation to 0 " and Part B
		reads "Explain how this relates to the definition of
		absolute value "
MA 912 NSO 1 4	Connecting	ADDED MA 912 NSO 1 3 MA 912 NSO 4 1
MA.712.1050.1.4	Banchmarks/Horizontal	ADDED MA./12.1000.1.3, MA./12.1000.4.1
	Alignment	
MA 012 NSO 1 4	Vertical Alignment	PEMOVED hyperlink and text
MA 012 NSO 1.4	Promo a se and	A hullet DEMOVED the word "and" and
MA.912.NSO.1.4	Purpose and	ADDED a service often "direction"
MA 012 NGO 1 4	Instructional Strategies	ADDED a comma alter direction
MA.912.NSO.1.4	Common	CHANGED "physical" to "physically" in bullet 2
	Misconceptions or	
	Errors	
MA.912.NSO.1.4	Instructional Items	CHANGED "higher altitude" to "greater depth"
MA.912.NSO.2.1	Connecting	REMOVED MA.912.DP.1.4 and ADDED
	Benchmarks/Horizontal	MA.912.1.3
	Alignment	
MA.912.NSO.2.1	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.2.1	Purpose and	EDITED the Area Model table with appropriate
	Instructional Strategies	zeros; Notated that *formatting* needs to be
	-	reworked so decimals line up.


		-
Benchmark	<b>B1G-M</b> Component	Change Made
MA.912.NSO.2.1	Strategies to Support	REWORDED the 2 <sup>nd</sup> solid bullet point for
	Tiered Instruction	clarification: "Teacher encourages students with a
		firm understanding of multiplying and dividing
		fractions to convert decimal values into equivalent
		fractional forms and then perform the desired
		operation and convert the solution back to decimal
		form."
MA.912.NSO.2.2	Connecting	REMOVED MA.912.AR.3 and MA.912.DP.1.4;
	Benchmarks/Horizontal	ADDED MA.912.AR.1.3
	Alignment	
MA.912.NSO.2.2	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.2.2	Purpose and	EDITED the equation in the $2^{nd}$ open bullet to a
	Instructional Strategies	verbal expression from an algebraic one:
	C C	"canask the question, what number multiplied
		by 5/8 gives the value 3/4?"
MA.912.NSO.2.2	Common	EDITED 2nd ("Students may incorrectly assume
	Misconceptions or	that multiplication") and 3rd bullet points for
	Errors	grammar:("Students may incorrectly")
MA.912.NSO.2.2	Strategies to Support	EDITED 1st bullet point for grammar and
	Tiered Instruction	clarification: ("Teacher encourages students with a
		firm understanding for multiplying and dividing
		decimals to convert fractional values to their
		equivalent decimal forms and then perform the
		desired operation and convert the solution back to
		fractional form."); EDITED 5th bullet point for
		clarification: "with a common denominator,
		dividing the numerators and then dividing the
		denominators."
MA.912.NSO.2.3	Connecting	ADDED benchmark MA.912.GR.2.3 and
	Benchmarks/Horizontal	MA.912.GR.2.4
	Alignment	
MA.912.NSO.2.3	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.2.3	Purpose and	EDITED first ("within real-world contexts."
	Instructional Strategies	AND third bullet for grammar and clarification:
		"Students should have experience using
		technology with decimals and fractions as they
		occur in the real world (MTR.7.1). This experience
		will help prepare students to work with all"
MA.912.NSO.2.3	Common	EDITED for grammar only, bullets 5 ("Students
	Misconceptions or	may incorrectly assume that" AND sixth bullet
	Errors	("Students may incorrectly assume that
		division")



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Benchmark	B1G-M Component	Change Made
MA.912.NSO.2.3	Strategies to Support Tiered Instruction	EDITED bullets 3, 4, 6, AND 10 for grammar and consistency in wording throughout the document. Bullet 3: "Teacher encourages students with a firm understanding of multiplying and dividing fractions to convert decimal values into equivalent fractional forms and then perform the desired operation and convert the solution back to decimal form." Bullet 4: Teacher encourages students with a firm understanding of multiplying and dividing decimals to convert fractional values into equivalent desired operation and convert the solution" Bullet 6: "provides graph paper to use while" Bullet 10: "with a common denominator, dividing the numerators, and then dividing the denominators."
MA.912.NSO.3.1	Connecting Benchmarks/Horizontal Alignment	REMOVED benchmarks MA.912.NSO.2.2, MA.912.NSO.2.3, and MA.912.AR.3.4; ADDEDED benchmarks MA.912.NSO.3.2 and MA.912.NSO.3.4
MA.912.NSO.3.1	Terms from the K-12 Glossary	REMOVED "Rectangular Array"
MA.912.NSO.3.1	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.3.1	Purpose and Instructional Strategies	<b>Factorization LCM and visual CHANGED</b> ; <u>1st</u> <u>bullet and then 2nd open bullet wording</u> : "Use Prime Factorization to find the LCM. [image] The LCM of 6 and 15 is 30 and is represented as 2 x 3 x 5. Discuss with students why 3 is listed only once. To use the strategy of prime factorization to find the LCM of two numbers, students must know what to do when a prime factor repeats. [image] The LCM of 180 is represented as 2 x 2 x 3 x 3 x 5. [image] The LCM of 210 is represented as 2 x 3 x 5 x 7. <u>1st bullet and then 3rd open bullet</u> , ADDED "Using the Ladder Method (Successive Division) [image]. <u>2nd bullet and then 2nd OPEN</u> <u>bullet under Factorization and 1st image</u> : "Using the Ladder Method for GCF. [image] The GCF of 18 and 24 is 2 x 3 = 6. A student may start with any factor that can evenly divide into the given numbers. A student may have started with 3 or 6. [image with 3 steps of Ladder Method] The GCF of 96 and 138 is 2 x 3 = 6. A student may start with any factor that can evenly divide into the given numbers. Each 'rung' of the ladder is created



Benchmark	<b>B1G-M</b> Component	Change Made
		factor." End of last bullet: REPLACE the word "factors" with "multiple".
MA.912.NSO.3.1	Instructional Tasks	Task 1 ADDED clarifying wording to part A: "If they both start a new book on Wednesday, how many days will it take for them to start another new book on the same day?". Task 2 ADDED the task: "Instructional Task 2 What is the greatest common factor of 636 and 132? Explain your method for solving."
MA.912.NSO.3.1	Instructional Items	CHANGED the values to meet the benchmark and intent on both items: "Instructional Item 1 What is the greatest common factor of 85 and 187? Instructional Item 2 What is the least common multiple of 18 and 12?"
MA.912.NSO.3.2	Connecting Benchmarks/Horizontal Alignment	REMOVED benchmark MA.912.NSO.3.1, located in the wrong place & format; ADDED benchmarks MA.912.NSO.3.1 and MA.912.NSO.3.4
MA.912.NSO.3.2	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.3.2	Purpose and Instructional Strategies	EDITED grammar and formatting for bullets 2 ("Students should not use the multiplication") AND 5 ("and the factors to sum in the parenthesis. Given the common factor multiplied by the sum of two whole numbers, determine")
MA.912.NSO.3.2	Common Misconceptions or Errors	CLARIFIED wording to meet the intent of the benchmark in bullet 2: "Students may incorrectly think that they always must factor out the greatest common factor." CHANGED to 3 total bullets.
MA.912.NSO.3.2	Strategies to Support Tiered Instruction	1st open bullet (2nd sentence) and 2nd open bullet (2nd sentence), changed "co-constructs" to "co-creates".
MA.912.NSO.3.2	Instructional Tasks	EDITED to clarify it is one of the Greatest Common Factor: "Rewriteis one of the common factors of 211 and 66"
MA.912.NSO.3.3	Connecting Benchmarks/Horizontal Alignment	ADDED benchmark MA.912.NSO.3.4
MA.912.NSO.3.3	Terms from the K-12 Glossary	ADDED "Natural Number"
MA.912.NSO.3.3	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.3.3	Purpose and Instructional Strategies	EDITED the last bullet point for grammar: "The expectation of this benchmark is the application of



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Benchmark	<b>B1G-M</b> Component	Change Made
	<b>FF</b>	exponents, not determining the value in
		expressions with multiple operations."
MA.912.NSO.3.3	Common	In the lastbullet - REMOVED "number".
	Misconceptions or	
	Errors	
MA.912.NSO.3.3	Strategies to Support	2nd bullet, 1st sentence, place a "period" after
	Tiered Instruction	"factor" and change the next sentence to: "Teacher
		encourages" 3rd bullet, REMOVED "exponent"
		and replace with "algebra" tiles.
MA.912.NSO.3.3	Instructional Tasks	Task 2 was CHANGED so the exponent is a 4
	T 1T.	instead of an odd value of 3.
MA.912.NSO.3.3	Instructional Items	Item 2 was CHANGED to (-3) raised to the power
MA 012 NGO 2 4	Compositions	014. DEMOVED statement: ADDED has shown by
MA.912.NSO.3.4	Connecting Bonohmarks/Horizontal	MA 012 NSO 2 1 MA 012 NSO 2 2
	Alignment	MA.912.NSO.3.1, MA.912.NSO.3.2, MA 912 NSO 3 3: REMOVED "stray:" hullet
MA 912 NSO 3.4	Vertical Alignment	REMOVED hyperlink and text
MA 912 NSO 3 4	Common	EDITED 5th bullet for clarification with vellow
MA.712.1050.3.4	Misconceptions or	highlights after "to beln students identify factors
	Errors	aujckly. If students believe a number is prime.
		students should try to find a number that can
		produce the ones digit and try numbers that end
		with that digit."
MA.912.NSO.3.4	Strategies to Support	EDITED the number of prime numbers listed in
	Tiered Instruction	the first bullet point for clarification: "the first
		seven prime numbers as 2, 3, 5, 7, 11, 13, 17
		and"; EDITED the values given in the 3rd solid
		bullet point to read as follows: " rules for 2, 3, and
		5, to assist" EDITED the last bullet and
		highlighted for clarification: "Teacher models how
		to find a number that can produce the ones digit
MA 012 NSO 2 5	Connecting	PEMOVED benchmark MA 012 NSO 2 1:
WIA.912.NOU.3.3	Benchmarks/Horizontal	ADDED benchmark MA 012 AP 3.5
	Alignment	ADDED UCHCHIHAIK MA.712.AK.J.J
MA.912.NSO.3.5	Vertical Alignment	REMOVED hyperlink and text
MA.912 NSO 3 5	Purpose and	CLARIFIED wording to meet the intent of the
1.1.1.912.1100.3.3	Instructional Strategies	benchmark to include terminating and repeating
	Stategies	decimals in the first sentence; other minor
		grammatical errors. First paragraph: "positive
		rational numbers into equivalent forms to include
		terminating decimals. This skill is extended in
		grade 7 to also include negative rational numbers
		and repeating decimals." Fourth bullet: "decimal
		and percent equivalence to "Sixth bullet: "that



Bonchmark	B1C-M Component	Change Made
Dentimial K	DIG-MI Component	percents are a number and are worth specific
		amounts in different contexts "
		amounts in unrerent contexts.
MA.912.NSO.3.5	Instructional Items	EDITED Instructional Item 3, to change the order
		of the information to include: "can be written as
		the decimal and is equivalent to
		percent."
MA.912.NSO.4.1	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.4.1	Purpose and	EDITED grammar in the first paragraph ("In
	Instructional Strategies	grade 7, students perform all operations") and
		bullet 1 ("beneficial for students to learnfor
		more situations. Given a real-world context,
		develop a conceptual model to represent the
		situation. Students can describe"), bullet 3, under
		open bullet 2 (at Two-Color Counters " A
		positive and a negative together make a zero pair.
		So, if I have" and at bullet 3, under open bullet 3
		(at Number Lines' last sentence "in the positive
		direction (right), they land on -5. ), built 4
		(removed the word SUM .), AND bullet 5 ( the
		direction twice. ")
MA 012 NSO 4 2	Connecting	ADDED bonchmorks MA 012 NSO 1 1
MA.912.NSO.4.2	Benchmarks/Horizontal	MA 912 NSO 1 4 noted "This is all of NSO 1"
	Alignment	MA.912.1050.1.4, noted This is an of 1050.1
MA.912.NSO.4.2	Vertical Alignment	REMOVED hyperlink and text
MA.912.NSO.4.2	Purpose and	HIGHLIGHTED first bullet and open bullet
	Instructional Strategies	underneath the first bullet for clarification
MA.912.NSO.4.2	Common	EDITED bullet 4 to ADD "two-" in front of
	Misconceptions or	"colored counters." AND bullet 6 ("Students may
	Errors	say that a value divided by itself is zero because
		they subtract instead of divide. Students should
		remember that any number")
MA.912.NSO.4.2	Strategies to Support	EDITED bullet 5 for consistency: ADDED "two-"
	Tiered Instruction	in front of colored counters.
MA.912.AR.1.1	Purpose and	EDITED for grammar in the first paragraph
	Instructional Strategies	("within equations. In grade 6, students extend
		this knowledge") AND bullet 1 ("connections
		to the properties of operations, including"),
MA.912.AR.1.1	Purpose and	The introductory paragraph, 3rd sentence: "In
	Instructional Strategies	grade 6, students extend" 1st bullet:
		"operations," Next to last bullet: REMOVED



Benchmark	B1G-M Component	Change Made
	•	"and", ADDED a comma after coefficients and
		ADDED arithmetic in front of operations
MA.912.AR.1.1	Common	REWORDED the 3rd bullet point for clarification:
	Misconceptions or	"Students may oversimplify a problem by using
	Errors	the process of circlingwords and trying to
		eliminate information without considering the
		context. (MTR.2.1); Instead, teachers and students"
MA.912.AR.1.1	Instructional Tasks	Instructional Task 1, Part B, ADDED a "?" at the end.
MA.912.AR.1.1	Instructional Items	RENUMBERED instructional item 3 to reflect it is
		#3 and re-worded this item: " itewer than the sum
MA.912.AR.1.2	Connecting	ADDED benchmark MA.912.NSO.1.1:
	Benchmarks/Horizontal	REMOVED MA.912.DP.1.6
	Alignment	
MA.912.AR.1.2	Purpose and	Word changes in bullet 4 ("symbol. This type of
	Instructional Strategies	problem will create"), bullet 5 ("on a number
		line. This also includes showcasing"), AND
		bullet 7 ("includes the understanding of
		how")
MA.912.AR.1.2	Instructional Items	ADDED word to Instructional Item 3: "video
		games for no more than"
MA.912.AR.1.3	Connecting	ADDED benchmarks MA.912.NSO.2.1,
	Benchmarks/Horizontal	MA.912.NSO.2.2; REMOVED MA.912.NSO.4.2
MA 012 AD 1.2	Alignment Deemaaa and	EDITED to match the formation of other
MA.912.AK.1.5	Purpose and Instructional Stratagies	EDITED to match the first percentaiting of other
	Instructional Strategies	being substituted will also be integers. In grade 7
		students will evoluate "); EDITED 2nd solid
		bullet to define that constants were limited to
		integers reading as: "built from constants (limited
		to integers) "• EDITED 3rd bullet for grammar
		("This is a way a student can demonstrate")
MA.912.AR.1.3	Common	EDITED 1st bullet ADDED after incorrectly: "use
	Misconceptions or	the order of operations when generating equivalent
	Errors	expressions." REMOVED "using the order of
		operations".
MA.912.AR.1.3	Common	ADDED a fourth bullet point and 1st open bullet
	Misconceptions or	example: "Students may substitute without using
	Errors	the correct operation of multiplication. For
		example, 3 <i>c</i> , where c is 2; a student may write 32
		instead of 3(2) or 3 x 2."



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D 1 1	DIG NG (	
Benchmark	BIG-M Component	
MA.912.AR.1.3	Strategies to Support	ADDED the steps to work out and color code the
	Liered Instruction	solution for the expression: "a" and "a" values
		should be in RED and "c" and "c" values should be
		in GREEN.
		$-5a^{2}+c$ ; a=-3 and c=-12
		$-5(-3)^{2}+(-12)$
		-5(9)+(-12)
		-45+(-12)
MA.912.AR.1.3	Instructional Items	REPLACED the value I with the variable p and
		omit 2nd variable "p"
MA.912.AR.1.4	Purpose and	REPLACED "whole number" with "integer" in the
	Instructional Strategies	first paragraph and ADDED "of" in front of
		"algebra tiles" in bullet 5.
MA.912.AR.1.4	Common	EDITED for grammar: The first bullet reads as:
	Misconceptions or	"Students may incorrectly think there is only one
	Errors	right answer or equivalent value." DELETED the
		2nd sentence in first bullet for
		consistency/clarification.
MA.912.AR.1.4	Strategies to Support	EDITED for grammar bullet three: DELETED the
	Tiered Instruction	"s" at the end of "distributive"; Bullet three,
		DELETED "and allowing the studentsas well."
MA.912.AR.1.4	Instructional Tasks	Updated image for task 1 AND Task 2 has been
		<b>ADDED to include:</b> Instructional Task 2
		(MTR.2.1) How can you show the two expressions,
		2(x + 5) and $2(x) + 2(5)$ , are equivalent? Explain.
MA.912.AR.2.1	Connecting	DELETED benchmark MA.912.NSO.4.1
	Benchmarks/Horizontal	
	Alignment	
MA.912.AR.2.1	Purpose and	EDITED for consistency in wording and grammar;
	Instructional Strategies	For paragraph 1: "In grade 5, students evaluated
		numerical expressions and simplified them on both
		sides of an equation to determine if they were true
		or falseThe substituted values in grade 6 are
		limited to integers. In grade 7 students learn"
MA.912.AR.2.1	Common	DELETED conflicting language in bullet one
	Misconceptions or	("between expressions where more than one
	Errors	value can satisfy the statement.") and in bullet two
		("may incorrectly think there is only one way to
		accurately evaluate it. For example, if students are
		asked if the value of $x = 8$ is true given the
		equation $2x - 3(x-4) = 4,")$
MA.912.AR.2.1	Strategies to Support	EDITED to ADD a comma after 4, REMOVED
	Tiered Instruction	duplicated value of "5" and replaced with "6".
MA.912.AR.2.1	Instructional Tasks	Task 2, part B, CHANGED "choose" to "chooses".
MA.912.AR.2.1	Enrichment Tasks	ADDED rational numbers b, which is "21/5" & f.
		which is "-25/7"



Bonchmark	B1C M Component	Change Made
MA 012 AP 2 2	Burnoss and	EDITED for grammar in bullet 4: "validation of
MA.912.AK.2.2	Instructional Strategies	their steps will introduce algebraic proofs."
MA 012 AP 2 2	Instructional Itams	EDITED for grammar in Instructional Itam 4 to
MA.912.AK.2.2	instructional items	change "table top" to "tabletop" in 3 places
MA 012 AP 2 2	Durness and	EDITED for grammar in bullet 6: "Their stand will
MA.912.AK.2.3	Instructional Strategies	introduce algebraic proofs "
MA 012 AP 24	Connecting	DEL ETED benchmark MA 012 NSO 2.2
MA.912.AK.2.4	Panahmarka/Horizontal	DELETED benchinark MA.912.NSO.2.2
	Alignment	
MA 912 AR 24	Common	EDITED for grammar: DELETED "instead" and
MA.)12.AK.2.4	Misconceptions or	REPLACED with "may need to": DELETED "try
	Errors	to" and DELETED "rules and"
MA 912 AR 24	Strategies to Support	EDITED to ADDED open space (empty square
WIT 1.912.7 HX.2.4	Tiered Instruction	box) for value rather than a variable in locations
MA 912 AR 2.4	Instructional Tasks	EDITED in tasks 1, 2, and 3 for formatting of
11111.912.111(.2.1	instructional Tusks	fractions See formatting changes of fractions to
		the right.
MA 912 AR 2.4	Instructional Items	EDITED in item 2 for the formatting of fractions
		See formatting changes of fractions to the right.
MA 912 AR 3.1	Connecting	REMOVED benchmarks MA 912 NSO 2.2 and
WIT 1.912.7 HX.5.1	Benchmarks/Horizontal	MA 912 NSO 3 5
	Alignment	111111111111111111111111111111111111111
MA.912.AR.3.1	Purpose and	EDITED for the formatting of fractions in bullets 9
	Instructional Strategies	and 10. Images updated.
MA.912.AR.3.1	Strategies to Support	EDITED for the formatting of fractions in open
	Tiered Instruction	bullet 1. See image to the right.
MA.912.AR.3.1	Instructional Tasks	EDITED for grammar in Task 2, Part C: "Does the
		ratio of 40 to 27 have the same meaning as
		27:40?"
MA.912.AR.3.1	Instructional Items	EDITED for the formatting of fractions in Item 3.
		See image to the right.
MA.912.AR.3.2	Connecting	REMOVED MA.912.NSO.2.2 and ADDED
	Benchmarks/Horizontal	MA.912.NSO.3.1
	Alignment	
MA.912.AR.3.2	Common	EDITED for grammar by changing 2nd bullet to
	Misconceptions or	"represent" instead of "represented".
	Errors	
MA.912.AR.3.2	Instructional Tasks	EDITED Task 1 for grammar: "When buying
		ground beefADDED "ground" in front of beef
		for Part A. DELETED "the" in front of ground
		beef for Part B.
MA.912.AR.3.2	Instructional Items	EDITED to include place value rounding at the
		end of Instructional Item 1: "apples? (round to the
		nearest cent)"



Benchmark	B1G-M Component	Change Made
MA.912.AR.3.3	Connecting	DELETED MA.912.NSO.2.2: ADDED
	Benchmarks/Horizontal	MA.912.NSO.3.1
	Alignment	
MA.912.AR.3.3	Vertical Alignment	DELETED MA.8.AR.3.2
MA.912.AR.3.3	Instructional Tasks	EDITED table in instructional task 1 for value
		formatting of fractions - See image to the right;
		EDITED table in task 2 to include Additional
		rows; ADDED two parts to instructional task 2,
		"Part A. Using a common ratio, complete the table
		when x is 7. Part B. Using a common ratio,
		determine an x and corresponding y value."
MA.912.AR.3.3	Instructional Items	EDITED grammar for clarification for Item 1:
		"Using this recipe, complete the table to determine
		the amount" and end with a period.
MA.912.AR.3.4	Connecting	ADDED MA.912.NSO.3.1; DELETED
	Benchmarks/Horizontal	MA.912.NSO.2.2
	Alignment	
MA.912.AR.3.4	Purpose and	EDITED for grammar at bullet 2, open bullet 1:
	Instructional Strategies	"when determining how much"
MA.912.AR.3.4	Instructional Items	EDITED fraction in Item 1
MA.912.AR.3.5	Connecting	ADDED benchmarks MA.912.NSO.1.1 and
	Benchmarks/Horizontal	MA.912.NSO.3.1; DELETEDD benchmark
	Alignment	MA.912.NSO.2.2
MA.912.AR.3.5	Vertical Alignment	ADDED benchmarks MA.8.M.1.1 and
N/4 010 4D 0.5	D 1	MA.8.FR.2.2
MA.912.AR.3.5	Purpose and	ADDED clarifying language about previous
	Instructional Strategies	learning to the introduction, insert this sentence
		after sentence 1: Students also learned how to
		convert measurements within the same
		measurement system. REPLACED image of table
		"dollars", in the last hullet point DELETED phrase
		for elevitication: "including conversions of gross
		and volumes and conversions that are commonly
		used in the science classroom "
MA 012 AP 3 5	Common	DEWORDED the 1st bullet point for grammar and
MA.912.AK.3.3	Misconceptions or	clarification: " for solving problems in a real-
	Frors	world context as needed and analyzing the
	LIIOIS	nrohlem "
MA.912.GR.1.1	Connecting	ADDED benchmark MA 912 NSO 1.3
1011.912.010111	Benchmarks/Horizontal	
	Alignment	



Benchmark	<b>B1G-M</b> Component	Change Made
MA.912.GR.1.1	Purpose and	EDITED introductory paragraph for grammar and
	Instructional Strategies	consistency: "first quadrant. In grade 6, students
	_	plot ordered pairsIn grade 7, students will apply
		their knowledge of plotting order pairs"; In
		bullet 2, CLARIFIED coordinate pairs are also
		called coordinates: "calling ordered pairs a
		coordinate pair or coordinates. CLARIFIED again
		in bullet 3: "plotting ordered pairs on coordinate
		planes with different scales." and OPEN bullet
		ADDED for the example. ADDED a 4th bullet
		point about cardinal directions: "Instruction
		includes describing movement on the coordinate
		plane from the origin to the right, to the left,
		horizontally, vertically, or as a cardinal direction
		(north, east, south, west)."
MA.912.GR.1.1	Common	CLARIFIED language regarding point location in
	Misconceptions or	the 2nd bullet point: Students may not recognize
	Errors	that a point" and 2nd bullet point, 1st open
		build: For example, the location of the point on
		ard hullet point: "Students may not understand the
		numbering " ADDED a 4th bullet point: "Students
		may incorrectly label the scale of the axes "
MA 912 GR 1 1	Strategies to Support	DELETED extra bullet underneath bullet 4 Bullet
1111.912.010111	Tiered Instruction	4. open bullet DELETED "For" and replaced with
		"Another".
MA.912.GR.1.1	Instructional Tasks	ADDED fractional coordinates to task 1: A (2,
		5/2), D (3/2, -5), and G (0, 0.6) and at Part D:
		"over which axis or axes are they reflected?
		Explain." REWORDED task 3 to clarify
		expectations and ADDED parts: Instructional Task
		3 "Create an ordered pair, given $x > 0$ and $y > 0$ .
		Then determine which quadrant or axis each
		ordered pair described will lie"; Part A. $(-x, y)$ ;
14 010 CD 1.0		Part B. $(-x, 0)$ ; Part C. $(x, -y)$ ; Part D. $(0, y)$
MA.912.GR.1.2	Connecting	REMOVED MA.912.NSO.1.1
	Benchmarks/Horizontal	
MA 012 CD 1 2	Augnment Durpose and	EDITED for grommer in the introductory
WIA.912.UK.1.2	I upose and Instructional Stratagica	paragraph's last 2 sentences: "for the subtraction of
	monucuonal suategies	rational numbers in grade 7 In grade 8 ".
		DELETED in the 2nd hullet point: "Students
		should not be penalized when performing
		operations": ADDED a sentence at the end of the
		2nd open hullet point: "Students should not be
		operations"; ADDED a sentence at the end of the 2nd open bullet point: "Students should not be



Benchmark	<b>B1G-M</b> Component	Change Made
		penalized if they convert from one form to another when performing operations."
MA.912.GR.1.2	Common Misconceptions or Errors	Moved 3rd bullet point under 2nd bullet point as the 2nd open bullet example.
MA.912.GR.1.2	Strategies to Support Tiered Instruction	In bullet 4 the last sentence, it should read: "Once the number line is drawn"
MA.912.GR.1.2	Instructional Tasks 1 & Instructional Items 1	In Instructional Task 1, the first sentence: CHANGED "grid" to "plane"
MA.912.GR.1.3	Purpose and Instructional Strategies	EDITED introductory paragraph to read: "In grade 5, students found the perimeterIn grade 6, students plot theand determine the perimeter or area. Students may alsoIn grade 7, students will extend their knowledge to find the areas of other quadrilaterals." EDITED 3rd bullet to read: "When discussing area and perimeter, allow flexibility for students to use" EDITED 2nd open bullet (which is under the 3rd bullet) to read: "where the garden should go; the length of each box on the gridNathanial's garden has corners at $(0,9)$ , $(8,9)$ , $(0,2)$ and $(8,2)$ backyard. Find the perimeter to determine the amount of wood needed to create a barrier for the garden."
MA.912.GR.1.3	Common Misconceptions or Errors	EDITED 1st bullet to read: "y-coordinate in an ordered pair." 2nd bullet: DELETED "the difference between".
MA.912.GR.1.3	Strategies to Support Tiered Instruction	DELETED "to" after "the distance between" in both open bullets
MA.912.GR.2.1	Connecting Benchmarks/Horizontal Alignment	ADDED MA.912.NSO.2.2 and MA.912.NSO.2.3
MA.912.GR.2.1	Purpose and Instructional Strategies	EDITED introductory paragraph to read: "In grade 5, students found the area of a rectangle with fractional and decimal side lengths using visual models and formulas. In grade 6, students will use modelsto derive the formula of the area of a triangle." In grade 7, students will decompose and quadrilaterals to find area." EDITED 1st bullet to read: "any right triangle with the same base and height will always form a rectangle. Therefore, the triangle will have an area" DELETED "show" in the 2nd bullet. In the last bullet, an apostrophe was ADDED to students.



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Benchmark	<b>B1G-M</b> Component	Change Made
MA.912.GR.2.3	Strategies to Support Tiered Instruction	1st bullet, DELETED comma after "and volume". CLARIFIED 3rd bullet to read: "manipulatives, such as unit cubes, andconcept of area, perimeter and volume."; <b>ADDED image</b> under bullet 3
MA.912.GR.2.3	Instructional Tasks	ADDED "rectangular" in front of "prism" and "prisms".
MA.912.GR.2.4	Connecting Benchmarks/Horizontal Alignment	REMOVED benchmark MA.912.NSO.3.5
MA.912.GR.2.4	Purpose and Instructional Strategies	CLARIFIED language in the introductory paragraph to read: "In grade 5, students found areaand right rectangular pyramidsstudents will calculate."; In the 1st bullet, DELETED "the symbolic and" in the 2nd sentence. 3rd bullet, 2nd sentence: DELETED the -s in "figures". In the 1st open bullet under bullet 4, ADDED "is" in front of 'needed'.
MA.912.GR.2.4	Common Misconceptions or Errors	EDITED 1st bullet to read: "determine the different two-dimensional figures that make up the net of the three-dimensional figure." ADDED bullet 3: "Students may confuse rectangular prisms and right rectangular pyramids."
MA.912.GR.2.4	Strategies to Support Tiered Instruction	ADDED a rectangular prism NET visual for the 1st open bullet if it is not already there; See image 1 to the right; and 2 nets (one is a square pyramid and the other is a rectangular pyramid) to the 2nd open bullet.
MA.912.GR.2.4	Instructional Items	EDITED item 1 to read: "Carl is making a box to ship a gift How much cardboard, in square inches, is required to make the box?" (DELETED last sentence in this first item.)
MA.912.DP.1.1	Previous Benchmarks	ADDED MA.8.DP.1.2
MA.912.DP.1.1	Purpose and Instructional Strategies	Introductory paragraph: DELETED the -s in "students" in the 1st sentence; 2nd sentence CHANGED "statistical" to "statistics" and ADDED "and begin" and DELETED "information and". In the first open bullet and 1st sentence, DELETED "there" and REPLACED with "it"; DELETED "gathering" and REPLACED with "collecting". DELETED bullet 4 and open bullet that follows. CHANGED titles for the table to read as: "Examples of Statistical Questions" & "Non- Examples of Statistical Questions". In the bullet directly underneath the table, in the 2nd sentence, DELETED "this"; DELETED "survey" and REPLACED with "statistical". <b>NEW bullet 5</b> ,



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Benchmark	<b>B1G-M</b> Component	Change Made
		DELETED "well written"; the remainder should read as follows: "Statistical questions address the followinginterest and measurement of interest. These types of questions should be formulated to anticipate answers that vary." <b>NEW bullet 6</b> should read: "Instruction includes students recognizing that statistical questions exist to collect data that can be organized, analyzed and interpreted. This includes determining the measure of center and measure of variation." <b>Bullet 6</b> <b>under open bullet</b> , DELETED "or on average", as well as commas in front of and after it. <b>NEW</b> <b>bullet 7 should read:</b> "to data that can be gathered by students themselves but can also be found by researching or using existing data sets." <b>Bullet 7 under open bullet</b> , DELETED "are the ten largest cities in Florida". It should read: "What was the temperature in Miami in the month of July?" can be considered a statistical question because the data had already been collected."
MA.912.DP.1.1	Common Misconceptions or	DELETED 2nd bullet beginning with "students may think"
MA.912.DP.1.1	Strategies to Support Tiered Instruction	REMOVED 2nd bullet and open bullet. NEW bullet 2, in the last sentence, ADDED "the" in front of the teacher and DELETED "they can" and continued with: "to convert it to a statistical question." NEW bullet 3, DELETED "co- construct" and REPLACED with "co-creates"; DELETED "and" after "graphic organizer" & REPLACED with "that"; also DELETED 1st two open bullets and keep the "Example of a graphic organizer is shown below" as the only open bullet under NEW bullet 3. In the table, DELETED the 2nd example under all columns. CHANGED "other" to "Non-statistical" and ADDED a NEW 3rd example under all columns: 1st column in the table: "Data-gathering but non-statistical"; 2nd column in the table: "Often a question that is on a survey" 3rd column in the table: "Does your father work from home?"
MA.912.DP.1.2	Connecting Benchmarks/Horizontal Alignment	ADDED benchmarks MA.912.NSO.2.1 and MA.912.NSO.2.2; REMOVEDD benchmark MA.912.NSO.3.5



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Benchmark	<b>B1G-M</b> Component	Change Made
MA.912.DP.1.2	Purpose and Instructional Strategies	Introductory paragraph, 2nd sentence: ADDED an "a" in front of numerical data. Bullet 1 should read: "Instruction includes using numerical data generated by statistical questions." 3rd bullet ADDED "as a whole" at the end of the sentence. 4th bullet should read: "Instruction includes how to calculate and interpret the mean (MTR.4.1)." In Bullet 5 @ the 1st open bullet A at the end: "Is it representative of the whole?" Bullet 6 ADDED "(mean and median)" after measures of center and ADDED "(range)" after measures of variability. Bullet 8 DELETED "The data set can be represented in" and REPLACED with "Instruction includes multiple representations of a given data set such as" Bullet 9 should read: "Instruction includes graphical representations both horizontally" Bullet 10 is ADDED: "Instruction includes interpreting the median as splitting the data into two halves (50% above and below the median)."
MA.912.DP.1.2	Common Misconceptions or Errors	1st bullet DELETED 2nd sentence about "average" and ADDED "Statisticians prefer to use the term "mean" rather than "average" since the term "average" is often used to refer to "typical" or "in the middle." 2nd bullet should read: "confuse mean, median, mode and range."
MA.912.DP.1.2	Strategies to Support Tiered Instruction	Bullet 3, 1st sentence: CHANGED "average" to "mean" and in the 2nd sentence change to: "the middle of data set does not indicate that the amount is the mean."
MA.912.DP.1.2	Instructional Item 1	EDITED the question stem and data set to read: "store. He is asked to report his sales of running shoes for each day of the week, in dollars."
MA.912.DP.1.3	Connecting Benchmarks/Horizontal Alignment	ADDED benchmarks MA.912.NSO.2.1, MA.912.NSO.2.2; REMOVED MA.912.NSO.3.5
MA.912.DP.1.3	Purpose and Instructional Strategies	Bullet 5 ADDED "the advantages and disadvantages of" after "Instruction focuses on"; DELETED "plots' benefits and disadvantages" and CHANGED to "plots".
MA.912.DP.1.3	Common Misconceptions or Errors	ADDED a 3rd bullet point: "Students may incorrectly believe the median always splits the box portion of the box plot in half."



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Benchmark	B1G-M Component	Change Made
MA.912.DP.1.3	Strategies to Support Tiered Instruction	DELETED "co-constructs" and REPLACED with "co-creates". Make sure image to the right is under the open bullet under the table to illustrate a box plot with the median not centered in the box.
MA.912.DP.1.3	Instructional Item 1	ADDED at the end of the sentence to read: "of the employees."
MA.912.DP.1.4	Connecting Benchmarks/Horizontal Alignment	REMOVED benchmark MA.912.NSO.3.5
MA.912.DP.1.4	Purpose and Instructional Strategies	ADDED "see glossary" for "outlier" in bullet 6.
MA.912.DP.1.4	Strategies to Support Tiered Instruction	1st bullet, CHANGED 2nd sentence: "Teacher co- creates a graphic organizer to compare and contrast the difference" EDITED 2nd bullet: DELETED ", providing instruction when needed. Teacher", DELETED comma after "with students" and ADDED "and" before "provides". EDITED 3rd bullet MOVED "in" in front of "students" to after "students". EDITED 1st open bullet: DELETED "is" after "conversations can be had about what" and REPLACED with "it".
MA.912.DP.1.4	Instructional Task 1	ADDED "Instructional strategies for this task:" and numbered 1-4: "1. Provide the histograms below to students, either individually or whole group. 2. Have students examine the histogram for 2 minutes and record their observations. 3. Have students share with a partner their thinking. 4. Then as a whole group, create a cohesive"
MA.912.DP.1.5	Connecting Benchmarks/Horizontal Alignment	ADDED benchmarks MA.912.NSO.1.1, MA.912.DP.1.3, and MA.912.DP.1.4; REMOVED benchmarks MA.912.NSO.2.3 and MA.912.NSO.3.5
MA.912.DP.1.5	Purpose and Instructional Strategies	In the introductory paragraph, sentence 2: DELETED "include" and REPLACED with "ADDED". 1st SQUARE bullet ADDED "to" in front of the first "represent".
MA.912.DP.1.5	Strategies to Support Tiered Instruction	In Bullet 4 DELETED "co-construct" and REPLACED with "co-creates".
MA.912.DP.1.6	Purpose and Instructional Strategies	ADDED wording at the beginning of the last bullet: "Instruction includes the use of technology to help with connecting measuresplot. This can"
MA.912.DP.1.6	Strategies to Support Tiered Instruction	3rd bullet ADDED wording: "initial data sets and changed data"
MA.912.DP.1.6	Instructional Task 1	Introductory paragraph, REPLACED "on" with "in" Part B ADDED "that" after "hours"



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Benchmark	B1G-M Component	Change Made
MA.912.DP.1.6	Instructional Item 1	EDITED Part A: DELETED "If he took his temperature on the Sunday of the next week and it was 103.2 degrees F," and capitalized "Which" and CHANGED "measures" to "measure". INSERTED a NEW sentence at the beginning of Part B: "He took his temperature on the following
		Sunday, and it was 103.2 [degree sign] F.

