Algebra I-A B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

Benchmark

focal point for instruction within lesson or task

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment

in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment

across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a formative assessment within instruction.

Strategies to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Mathematics Standards. For more information about tiered instruction, please see the Effective Tiered Instruction for Mathematics: ALL Means ALL document.

Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Mathematical Thinking and Reasoning Standards

MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs was written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:

Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR. 3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve them efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:

Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, “Does this solution make sense? How do you know?”
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students’ ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that show embedding the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	**Student Moves**	**Teacher Moves**
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. • Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* • Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. • Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance.	• Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. • Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. • Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. • Teacher provides appropriate time for student processing, productive struggle and reflection. • Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. • Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Students represent problems concretely using objects, models and manipulatives. • Students represent problems pictorially using drawings, models, tables and graphs. • Students represent problems abstractly using numerical or algebraic expressions and equations. • Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)*	• Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* • Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. • Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* • Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* • Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology.
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Students complete tasks with flexibility, efficiency and accuracy. • Students use feedback from peers and teachers to reflect on and revise methods used. • Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)*	• Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* • Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. • Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. • Teacher offers multiple opportunities to practice generalizable methods.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Students use content specific language to communicate and justify mathematical ideas and chosen methods. • Students use discussions and reflections to recognize errors and revise their thinking. • Students use discussions to analyze the mathematical thinking of others. • Students identify errors within their own work and then determine possible reasons and potential corrections. • When working in small groups, students recognize errors of their peers and offers suggestions.	• Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* • Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. • Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. • Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Students identify relevant details in a problem to create plans and decompose problems into manageable parts. • Students find similarities and common structures, or patterns, between problems to solve related and more complex problems using prior knowledge.	• Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)* • Teacher provides students opportunities to connect prior and current understanding to new concepts. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* • Teacher allows students to develop an appropriate sequence of steps in solving problems. • Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process.
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. • Students monitor calculations, procedures and intermediate results while solving problems. • Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* • Students reflect on the accuracy of their estimations and their solutions.	• Teacher provides opportunities for students to estimate or predict solutions prior to solving. • Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* • Teacher prompts students to self-monitor by continually asking, “Does this solution or intermediate result make sense? How do you know?” • Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Students connect mathematical concepts to everyday experiences. • Students use mathematical models and methods to understand, represent and solve real-world problems. • Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Students re-design models and methods to improve accuracy or efficiency.	• Teacher provides real-world context to help students build understanding of abstract mathematical ideas. • Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. • Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Teacher provides opportunities for students to apply concepts to other content areas.

Algebra I-A Areas of Emphasis

In Algebra I-A, instructional time will emphasize five areas:

(1) extending understanding of functions to linear functions and using them to model and analyze real-world relationships;
(2) solving linear equations and inequalities in one variable and systems of linear equations and inequalities in two variables;
(3) building linear functions, identifying their key features and representing them in various ways and
(4) representing and interpreting categorical and numerical data with one and two variables.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following:

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of major mathematical topics to all stakeholders.
- Benchmarks within the emphasis areas should not be taught in the order they appear in. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

			Operations with Polynomials and Radicals and Laws of Exponents	Linear, Quadratic and Exponential Functions	Solving Equations and Systems of Linear Equations and Inequalities	Building Functions, Identifying Key Features and Various Representations	Representing and Interpreting Categorical and Numerical Data
		MA.912.AR.1.1	x	x	x	x	x
		MA.912.AR.1.2			x		
		MA.912.AR.2.1			x		
		MA.912.AR.2.2		x			x

	MA.912.AR.2.3		x			
	MA.912.AR.2.4		x		x	
	MA.912.AR.2.5		x		x	x
	MA.912.AR.2.6			x		
	MA.912.AR.2.7			x		
	MA.912.AR.2.8			x		
	MA.912.AR.4.1		x		x	
	MA.912.AR.4.3		x		x	
	MA.912.AR.9.1			x		
	MA.912.AR.9.4			x		
	MA.912.AR.9.6			x		
Functions	MA.912.F.1.1		x		x	x
	MA.912.F.1.2	x			x	
	MA.912.F.1.3				x	
	MA.912.F.1.5		x		x	
	MA.912.F.1.8		x		x	x
	MA.912.F.2.1		x		x	
Financial Literacy	MA.912.FL.3.2		x			
	MA.912.FL.3.4		x		x	
Data Analysis & Probability	MA.912.DP.1.3					x
	MA.912.DP.2.4		x	x	x	x
	MA.912.DP.2.6					x

Algebraic Reasoning

MA.912.AR.1 *Interpret and rewrite algebraic expressions and equations in equivalent forms.*

MA.912.AR.1.1

Benchmark

MA.912.AR.1.1	**Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.**

Algebra I Example: Derrick is using the formula $P = 1000(1+.1)^t$ to make a prediction about the camel population in Australia. He identifies the growth factor as $(1+.1)$, or 1.1, and states that the camel population will grow at an annual rate of 10% per year.

Example: The expression 1.15t can be rewritten as$\left\{ (1.15\}^{\frac{1}{12}}\right)^{12t}$ which is approximately equivalent to 1.012^{12t}. This latter expression reveals the approximate equivalent monthly interest rate of 1.2% if the annual rate is 15%.

<u>Benchmark Clarifications:</u>
Clarification 1: Parts of an expression include factors, terms, constants, coefficients and variables.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.6
- MA.912.AR.4.1
- MA.912.FL.3.2

Terms from the K-12 Glossary

- Coefficient
- Expression
- Equation

Vertical Alignment

Previous Benchmarks
- MA.8.AR.2.1, MA.8.AR.2.2

Next Benchmarks
- MA.912.AR.5.5, MA.912.AR.5.9
- MA.912.AR.8.2
- MA.912.T.3.2

Purpose and Instructional Strategies

In grade 8, students generated and identified equivalent linear expressions, and solved multi-step problems involving linear expressions within real-world contexts. In Algebra 1-A, students generate and interpret equivalent linear and absolute value expressions and equations. In Algebra 1-B, students will continue with quadratic and exponential expressions and equations. In later courses, students will identify and interpret other functional (exponential, rational, logarithmic, trigonometric, etc.) expressions and equations.

- Instruction includes making the connection to linear and absolute value functions.
  - Students should be able to identify factors, terms, constants, coefficients and variables in expressions and equations.
    - Go beyond these popular parts of an expression and equation: the rate of change in linear functions, the starting point of a linear function, etc.
  - Look for opportunities to interpret these components in context – make these discussions part of daily instruction.

Common Misconceptions or Errors

- Students may not be able to identify parts of an expression and equation or interpret those parts within context. Ensure these are embedded throughout instruction and discussions.
  - For example, building in questions to identify these parts and discussing their connection to the context in which they represent in a routine way will help students to make these connections.

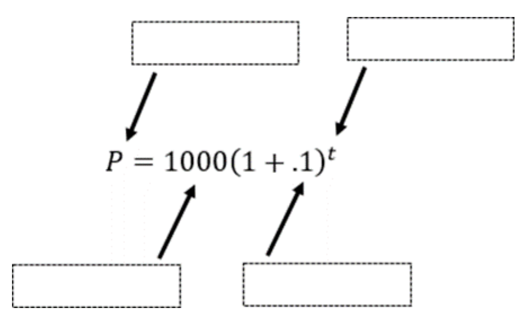
Strategies to Support Tiered Instruction

- Teacher facilitates discussions which include questions and clarifications to identify the connections of expressions and equations to the context of problems.
- Instruction provides opportunities to increase understanding of vocabulary terms.
  - For example, instruction may include a vocabulary review using a chart shown.

Term	$6x$
Coefficient	$6x$
Variable	x
Constant	6

- Teacher provides students with an expression or equation and allows them to match the parts to key vocabulary.
  - For example, teacher can provide the word bank to identify the different parts of the equation shown.

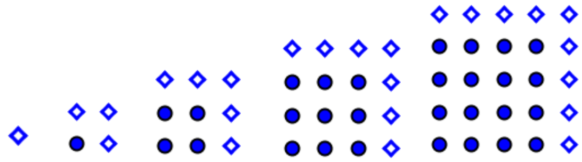
Word Bank
Initial amount/value
Final amount/value
Rate of growth
Rate of decay
Time



Instructional Tasks

Instructional Task 1 (MTR.5.1)

The algebraic expression $(n - 1)^2 + (2n - 1)$ can be used to calculate the number of symbols in each diagram below. Explain what n likely represents, how the parts of this expression relate to the diagrams, and why the expression results in the number of symbols in each diagram.



Instructional Task 2 (MTR.3.1, MTR.7.1)

Last weekend, Cindy purchased two tops, a pair of pants, and a skirt at her favorite store. The equation $T = 1.075x$ can be used to calculate her total cost where x represents the pretax subtotal cost of her purchase.

Part A. In the equation $T = 1.075x$, what does the number 1 represent? Explain using the context of Cindy's situation.

Part B. In the equation $T = 1.075x$, what does the number 0.075 represent? Explain using the context of Cindy's situation.

Instructional Items

Instructional Item 1

Identify the factors in the expression $2(3x - 1) + 2(2x + 2)$.

Instructional Item 2

A bacteria's growth can be modeled by the equation $y = 10(1 + 0.3)^{(t)}$ Identify the value that represents the starting amount of bacteria.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.2

Benchmark

MA.912.AR.1.2 Rearrange equations or formulas to isolate a quantity of interest.

Algebra I Example: The Ideal Gas Law $PV = nRT$ can be rearranged as $T = \frac{PV}{nR}$ to isolate temperature as the quantity of interest.

Example: Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for P.

Mathematics for Data and Financial Literacy Honors Example: Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for t.

Benchmark Clarifications:

Clarification 1: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.

Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2
- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.6
- MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8

- MA.912.AR.4.1
- MA.912.AR.5.3, MA.912.AR.5.6
- MA.912.FL.3.2

Terms from the K-12 Glossary

- Equation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2, MA.8.AR.2.3
- MA.8.GR.1

Next Benchmarks

- MA.912.AR.5.5, MA.912.AR.5.9
- MA.912.AR.8.2
- MA.912.T.3.2

Purpose and Instructional Strategies

In grade 8, students isolated variables in one-variable linear equations and one-variable quadratic equations in the form $x^2 = p$ and $x^3 = q$. In Algebra I, students isolate a variable or quantity of interest in equations and formulas. In Algebra I-A, equations and variables will focus on linear and absolute value. In Algebra I-B students will extend their focus of equations and variables to quadratics. In later courses, students will highlight a variable or quantity of interest for other types of equations and formulas, including exponential, logarithmic and trigonometric.

- Instruction includes making connections to inverse arithmetic operations (refer to Appendix D) and solving one-variable equations.

4x – 2y = 8	Given
-4x + 4x – 2y = 8 – 4x	Subtraction Property of Equality
-2y = 8 – 4x	Simplify
-2y/(-2) = 8/(-2) - 4x/(-2)	Division Property of Equality
y = –4 + 2x	Simplify

- Instruction includes justifying each step while rearranging an equation or formula.
    - For example, when rearranging $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for P, it may be helpful for students to highlight the quantity of interest with a highlighter, so students remain focused on that quantity for isolation purposes. It may also be helpful for students to identify factors or other parts of the equations.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P and $\left(1 + \frac{r}{n}\right)^{nt}$ are factors so the inverse operation is division.

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

Common Misconceptions or Errors

- Students may not have mastered the inverse arithmetic operations.
- Students may be frustrated because they are not arriving at a numerical value as their solution. Remind students that they are rearranging variables that can be later evaluated

to a numerical value.

- Having multiple variables and no values may confuse students and make it difficult for them to see the connections between rearranging a formula and solving a one-variable equation.

Strategies to Support Tiered Instruction

- Instruction includes doing a side-by-side comparison of solving a multistep equation with rearranging equations and formulas. The teacher should allow students time to understand that the steps in solving both equations are the same.
  - For example, solve both equations and note the similarities in solving both types of equations.

Solving One-Variable Equations	Rearranging Equations/Formulas
Determine the height, h, in the formula $SA = 2B + Ph$ if the surface area (SA) is 537 units squared, the area of the base (B) is 112 units squared and the perimeter of the base (P) is 25 units. $537 = 2(112) + 25h$ $537 - 22 = 224 + 25h - 224$ $313 = 25h$ $\frac{313}{25} = \frac{25h}{25}$ $12.52 = h$	Isolate height, h, in the formula $SA = 2B + Ph$. $SA = 2B + Ph$ $SA - 2B = 2B + Ph - 2B$ $SA - 2B = Ph$ $\frac{SA - 2B}{P} = \frac{Ph}{P}$ $\frac{SA - 2B}{P} = h$

- Teacher provides a chart for students to use as a study guide or to copy in their interactive notebook.
  - For example, inverse operations chart below.

Inverse Operations Chart		
Addition +	↔	Subtraction −
Multiplication ×	↔	Division ÷
Square x^2	↔	Square Root \sqrtx

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Given the equation $ax^2 + bx + c = 0$, solve for x.

Part B. Share your strategy with a partner. What do you notice about the new equation(s)?

Instructional Task 2 (MTR.4.1, MTR.5.1)

Part A. Given the equation $Ax + By = C$, solve for B.

Part B. Given the equation $7x - 6y = 24$, determine the x- and y-intercepts.
Part C. What do you notice between Part A and Part B?

Instructional Items

Instructional Item 1

Solve for x in the equation $3x + y = 5x - xy$.

Instructional Item 2

The formula $d = \frac{v_o + v_t}{2} t$ relating to the translational of motion, where d represents distance, v_o represents initial velocity, v_t represents final velocity, and t represents time. Rearrange the formula to isolate final velocity.

Instructional Item 3

The area *A* of a sector of a circle with radius *r* and angle-measure *S* (in degrees) is given by $A = \frac{\pi r^2 S}{360}$ solve for the radius *r*.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2 *Write, solve and graph linear equations, functions and inequalities in one and two variables.*

MA.912.AR.2.1

Benchmark

MA.912.AR.2.1	**Given a real-world context, write and solve one-variable multi-step linear equations.**

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.1
- MA.912.AR.9.1
- MA.912.FL.3.2

Terms from the K-12 Glossary

- Linear Equation

Vertical Alignment

Previous Benchmarks

- MA.6.AR.2.2, MA.6.AR.2.3
- MA.7.AR.2.2
  MA.8.AR.2.1

Next Benchmarks

- MA.912.NSO.4.2
- MA.912.AR.9.8
- MA.912.AR.9.9

Purpose and Instructional Strategies

In grade 8, students solved one-variable multi-step linear equations in mathematical context. In Algebra I, students write and solve one-variable multi-step linear equations within a real-world context. In future courses, students will work with linear systems in three-variables and linear programing. Additionally, linear equations and linear functions are fundamental parts of all future high school courses.

- Problem types include the writing of an equation from a given context, the solving of a given equation and writing and solving an equation within context.
- Instruction includes the use of manipulatives, drawings, models and the properties of equality.
- Instruction includes the interpretation of the solution within context.
- Instruction emphasizes the understanding that solving a linear equation in one variable mirrors the process of determining x-intercepts, or roots, of the graph of a linear function.
- In many contexts, students may generate solutions that may not make sense when placed in context. Be sure students assess the reasonableness of their solutions in terms of context to check for this (*MTR.6.1*).
    - For example, if students are solving a problem where x represents the number of paintings sold at an art gallery. If the solution is $x = 6.3$, then the number of paintings sold would be 6 since a portion of a painting cannot be sold.

Common Misconceptions or Errors

- Students may experience difficulty translating contexts into expressions. In these cases, give students sample quantities to help them reason.
- Students may not use properties of equality properly.

Strategies to Support Tiered Instruction

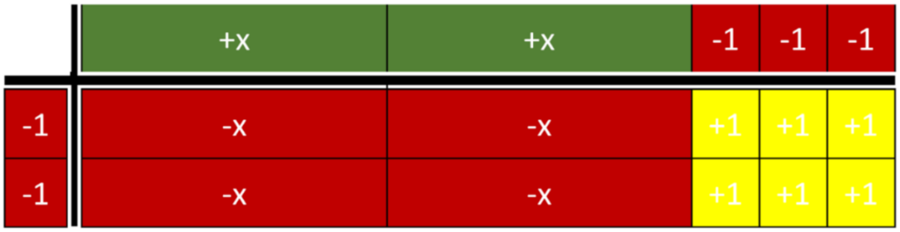
- Instruction includes opportunities to draw pictures or use bar models to represent real-world contexts.
    - For example, Kevin buys 66 markers plus 5 packs of markers. Fernando buys 48 markers plus 8 packs of markers. If Kevin and Fernando buy the same total number of markers, how many markers are in a pack?

66 Markers				P	P	P	P	P
48 Markers	P	P	P	P	P	P	P	P

- Instruction includes providing expressions and having students act out the context with props.
    - For example, Karen earns $100 a day plus $5 commission for each sale made at the store that day. In this example, give a student $100 in play money and then ask how much more they would get if they made 1 sale, 2 sales, and so on. Then ask how they could represent an unknown amount of sales.
- Instruction includes opportunities to use algebra tiles to model a multi-step equation and write the steps algebraically. For each step, ask students to identify the property of equality they would use.
- Instruction includes vocabulary development by co-creating a graphic organizer for each property of equality.

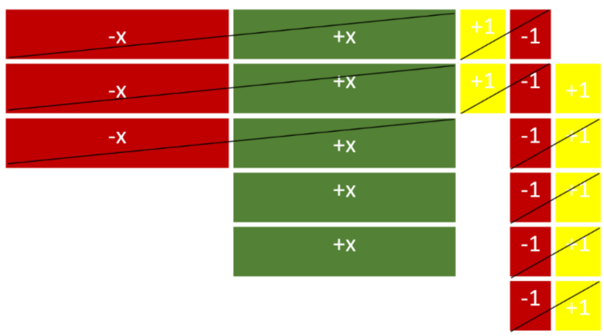
- Instruction includes the use of algebra tiles to model the distributive property or to add and subtract like terms as a problem is solved algebraically.
    - An example of modeling the distributive property using algebra tiles for $-2(2x-3)$ is shown below.

$$-2(2x-3)$$
$$-2(2x)+(-2)(-3)$$
$$-4x+6$$



    - An example of modeling combining like terms using algebra tiles for $-3x + 2 + 5x - 6 + 5$ is shown below.

$$-3x+2+5x-6+5$$
$$-3x+5x+2-6+5$$
$$2x+1$$



Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.6.1, MTR.7.1)

City A has a current population of 156,289 residents and has an annual growth of 146 residents. City B has a current population of 151,293 and has an annual growth of 363 residents.

Part A. Write an equation for when City A and City B will have the same population.
Part B. Compare your equation with a partner.
Part C. Solve your equation.
Part. D. Describe what the solutions mean within the context of the problem.

Instructional Task 2 (MTR.4.1, MTR.6.1, MTR.7.1)

Part A. Write a real-world context that could be represented with the equation
$4(x+5)=6x+12$

Part B. Define the variable in the context.
Part C. Solve the equation and discuss the solution with a partner.

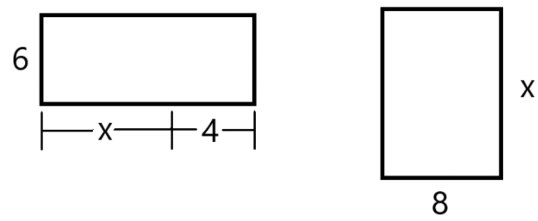
Instructional Items

Instructional Item 1

A group of friends decides to go out of town to a championship football game. The group pays $185 per ticket plus a one-time convenience fee of $15. They also each pay $27 to ride a tour bus to the game. If the group spent $2,771 in total, how many friends are in the group?

Instructional Item 2

Two rectangular fields, both measured in yards, are modeled below. What value of x, in yards, would cause the fields to have equal areas?



Instructional Item 3

A nutrition store starts a new membership program. Members of the program pay $52 to join and can purchase a canister of protein powder for $42.50. Non-members pay $49 for a canister of protein powder. After how many canisters is the total cost the same for members as it is for non-members?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.2

Benchmark

MA.912.AR.2.2	**Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.**

Benchmark Clarifications:

Clarification 1: Instruction includes the use of standard form, slope-intercept form and point-slope form, and the conversion between these forms.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.3
- MA.912.AR.4.1
- MA.912.AR.9.1
- MA.912.F.1.4
- MA.912.FL.3.2, MA.912.FL.3.4
- MA.912.DP.2.4

Terms from the K-12 Glossary

- Linear Equation
- Slope
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.7.AR.4.4
- MA.8.AR.3.2
- MA.8.AR.3.3

Next Benchmarks

- MA.912.NSO.4.2
- MA.912.AR.9.8
- MA.912.AR.9.9

Purpose and Instructional Strategies

In grade 8, students wrote linear two-variable equations in slope-intercept form from tables, graphs and written descriptions. In Algebra I, students write linear two-variable equations in all forms from real-world and mathematical contexts. In future courses, students will write systems and solve problems involving systems in three-variables and linear programming. Additionally, linear equations and linear functions are fundamental parts of all future high school courses.

- Instruction includes making connections to various forms of linear equations to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the equation $Ax + By = C$, where A, B and C are any rational number. This form can be useful when identifying the x- and y-intercepts.
  - Slope-Intercept Form
    Can be described by the equation $y = mx + b$, where m is the slope and b is the y-intercept. This form can be useful when identifying the slope and y-intercept.
  - Point-Slope Form
    Can be described by the equation $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line. This form can be useful when a point on the line is given and the y-intercept is not easily determinable.
- Look for opportunities to point out the connection between linear contexts and constant rates of change.
- Problem types should include cases for vertical and horizontal lines.

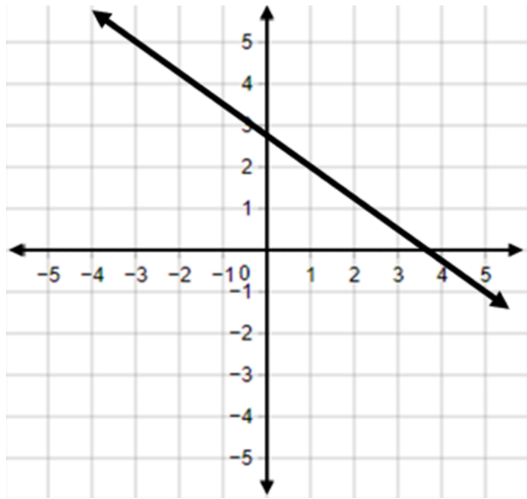
Common Misconceptions or Errors

- Students may have difficulty identifying both variables from a context. Much of their work previously has involved univariate contexts. Place emphasis on asking students what is changing in each context. Help guide their thoughts to recognize bivariate contexts as having two “things” that change in tandem.
- Students may attempt to estimate intercepts in order to continue using a linear form they prefer for some contexts. Use these opportunities to address the need for precision in mathematics.

Strategies to Support Tiered Instruction

- Instruction includes strategies from MA.912.AR.1.2 on rearranging equations to help students when they convert from one form of two-variable linear equation to another.
- Teacher models opportunities to address the need for precision in mathematics when determining intercepts.

- For example, a student could prefer to use slope-intercept form when writing two-variable linear equations. If the given information, as shown below, is two points that do not include the y-intercept, then the student may only estimate the y-intercept rather than determining it exactly. The student should realize that they could use point-slope form to write the equation without having to determine the y-



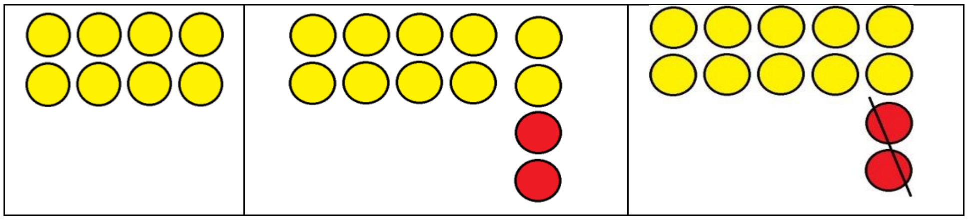
intercept

- Instruction includes explicit questions such as "What is staying the same or constant?", "What is changing or varying?" or "Is there anything else varying?"
  - For example, students are given the situation where a dog groomer charges $25 for a shampoo and hair cut plus $10 for each hour the dog stays at the groomer and are asked to write a linear two-variable equation that represents the total cost. The teacher can provide questions to help determine the constant value and two variables.

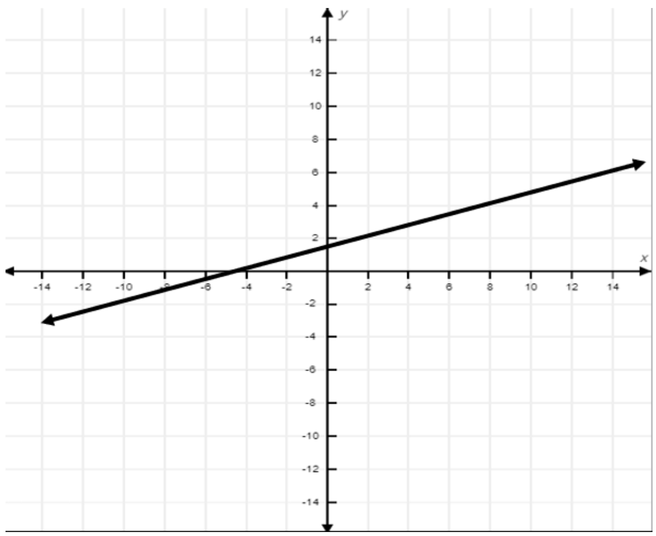
Constant	**Variable #1**	**Variable #2**
Charge for Shampoo ($25)	Number of Hours ($10 each)	Total Cost

- Instruction includes discussions about what a particular coordinate point on a line means in the context of the problem. Teachers may ask, "What does the identified point represent in the context of the problem?" and "How does the y change as x increases/decreases?"
- Teacher models finding the slope by color coding the points.
  - For example, to find the slope of a line passing through points (1,2) and (4,0), you would use $m = \frac{2-0}{1-4}$ or $m = \frac{0-2}{4-1}$.
- For students who need extra support in adding or subtracting integers, instruction includes using two-colored counters or algebra tiles to model the operation.
  - For example, given the expression $8 - (-2)$, students can use two-colored counters to find the difference as shown.

Start with 8.	There are not two negatives to subtract so you must add two zero pairs.	Subtract the two negatives to show that $8 - (-2) = 10$.



- Instruction includes a graphic organizer to identify the key features. Based on the key features identified, ask students which form of an equation would be the best.
  - For example, given the graph below, students can use an organizer to fill in some of the information.



Once the information is filled in, students can write an equation of the line and determine the rest of the key features.

Slope	x-intercept	y-intercept	Point(s) on the graph that are not intercepts	Equation of Line

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1, MTR.5.1)

Jamie bought a car in 2005 for $28,500. By 2008, the car was worth $23,700.

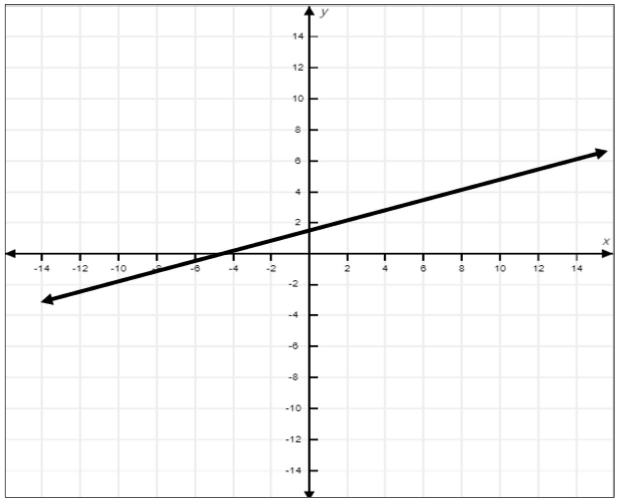
Part A. What function type could model the given situation?

Part B. What is the rate of change in the vehicle's worth per year?

Part C. Create a model that describes this situation.

Instructional Task 2 (MTR.3.1, MTR.4.1)

Use the graph below to answer the following questions.



Part A. In order to write the equation that represents this line, what information do you need?

Part B. Write a linear two-variable equation that represents the graph. Justify the linear form you chose (standard form, point-slope form, slope-intercept form).

Part C. Write a real-world situation that could represent this graph.

Instructional Items

Instructional Item 1

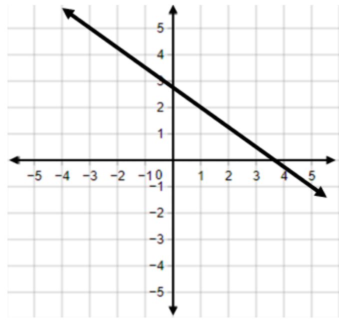
Sharon is ordering tickets for an upcoming basketball game for herself and her friends. The ticket website shows the following table of ticket options.

Number of tickets	Cost (including processing fee)
1	$42.50
2	$68.50
3	$94.50
4	$120.50
5	$146.50
10	$276.50
15	$406.50

Write a linear two-variable equation to represent the total cost C of t tickets.

Instructional Item 2

Write a linear two-variable equation that represents the graph below.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.3

Benchmark

MA.912.AR.2.3 Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.

Benchmark Clarifications:
Clarification 1: Instruction focuses on recognizing that perpendicular lines have slopes that when multiplied result in -1 and that parallel lines have slopes that are the same.
Clarification 2: Instruction includes representing a line with a pair of points on the coordinate plane or with an equation.
Clarification 3: Problems include cases where one variable has a coefficient of zero.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.1

Terms from the K-12 Glossary

- Linear Equation
- Rotation
- Slope
- Translation

Vertical Alignment

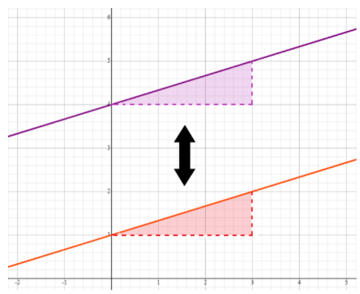
Previous Benchmarks
- MA.8.AR.3.3
- MA.8.AR.4.2
- MA.8.GR.2.1

Next Benchmarks
- MA.912.GR.1.1
- MA.912.GR.3.2, MA.912.GR.3.3

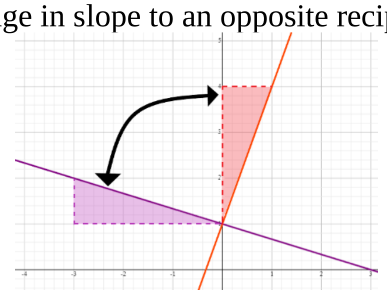
Purpose and Instructional Strategies

In grade 8, students determined whether a graphed system of linear equations resulted in one solution, no solution (parallel lines) or infinitely many solutions. In Algebra I, students write linear two-variable equations that are parallel or perpendicular to one another. In Geometry, students will use slope criteria of parallel and perpendicular lines to justify postulates, relationships or theorems.

- Instruction includes allowing students to explore the transformations of two lines using graphing software or other technology. If students don't have their own computers, use your own and let students direct the exploration.
  - For example, using graphing software, ask students to use the sliders to discover two lines that are parallel. Once students achieve this, write the equations of these two lines on the board and ask them to find two different lines that are parallel. Write the equations for these lines on the board. Repeat this for multiple pairs of lines. As students explore, ask them to find patterns in the equations they develop (*MTR.5.1*). Guide their discussion to focus on the fact that parallel lines have equivalent slopes (*MTR.4.1*).
  - Repeat this exercise for perpendicular lines.
- When students establish connections and understanding of slopes of parallel and perpendicular lines, tie them back to their work with transformations in Grade 8, with instruction of the benchmark MA.8.GR.2.1. (*MTR.5.1*).
  - For example, parallel lines can be translated to coincide with each other without changing slope.



  - For example, perpendicular lines can be rotated 90° about the point of intersection to coincide with each other. Use slope to draw congruent triangles on each line and show the change in slope to an opposite reciprocal after rotation.



- Once students understand slope relationships, instruction should guide them to utilize slope and a point on the line to develop the point-slope equation for the line.
  - Students can also develop slope-intercept equations for the line. Students should have prior knowledge from their work in MA.912.AR.2.2 that x and y in a linear equation represent points on the corresponding line. Direct students to see that substituting the slope of a line (m) and the coordinates of a point on that line (x, y) into slope-intercept form ($y = mx + b$) allows them to solve to find the y-intercept.

Common Misconceptions or Errors

- Some students may forget to change the sign of the slope when working through perpendicular line problems. Others may change the sign correctly but forget to make the slope a reciprocal. In both cases, consider having the student sketch a rough graph or use graphing software to check their result.

Strategies to Support Tiered Instruction

- Instruction provides opportunities to identify parallel or perpendicular lines using a graphing tool, graphing software or sketching a rough graph.
- Instruction includes opportunities to graph a linear function on a coordinate grid. Teacher models using a piece of spaghetti or other straight item (such as a pencil) to show a line parallel or perpendicular to the one they graphed and estimate the slope of the straight item. Students can compare that estimate with the slope of the parallel or perpendicular line they determined algebraically.
  - For example, if the given line has a slope of $-\frac{1}{2}$ and the student determines incorrectly that the perpendicular slope is -2, they can visually see the mistake when using a straight object that is placed perpendicular to the graph of the given line.
- Teacher provides opportunities to practice identifying the slope of parallel and perpendicular lines using a graphic organizer. To follow up this activity, have students do a card match where they match a linear equation to the slope of a parallel line and the slope of a perpendicular line.
  - For example, the organizer below could be used to identify the slope of the parallel and perpendicular line of the given equation.

Linear Equation	Slope of the Given Line	Slope of a Parallel Line	Slope of a Perpendicular Line
$y = \frac{2}{3}x + 5$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{3}{2}$

Instructional Tasks

Instructional Task 1(MTR.4.1)

Part A. A vertical line that passes through the point $(-0.42, 7.8)$ is perpendicular to another line. What could be the equation of the second line?

Part B. Graph both lines on a coordinate plane. What do you notice about the lines?

Part C. Compare your graph with a partner.

Instructional Task 2 (MTR.4.1)

Sarah is walking on the sidewalk in a city shown in the graph. Each grid square represents 1 square block. Shawna is walking on a street parallel to Sarah represented by equation $y=\frac{3}{2}x+2$. Shawna wants to meet up with Sarah at the point (4, 5). What would be the equation of the line that Shawna would take to walk perpendicular to Sarah’s path to meet her?

Instructional Items

Instructional Item 1

Write a linear function for a line that passes through the point $(4, -3)$ and is perpendicular to $y = 0.25x + 9$.

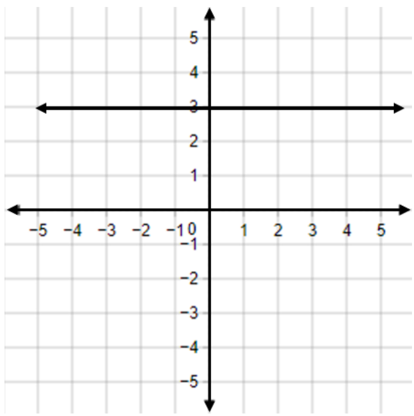
Instructional Item 2

Write an equation for the line that passes through $(3, 14)$ and is parallel to the line that passes through $(10, 2)$ and $(25, 15)$.

Instructional Item 3

Write a linear function for a line that is parallel to $f(x) = \frac{1}{3}x - 8$ and passes through the point $(-7, 0)$.

Instructional Item 4



Part A: What is the slope of a line that is parallel to the given graph?
Part B: Write a linear equation that is perpendicular to the given graph.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.4

Benchmark

MA.912.AR.2.4	**Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.**

Benchmark Clarifications:
Clarification 1: Key features are limited to domain, range, intercepts and rate of change.
Clarification 2: Instruction includes the use of standard form, slope-intercept form and point-slope form.
Clarification 3: Instruction includes cases where one variable has a coefficient of zero.
Clarification 4: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
Clarification 5: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder notations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.3, MA.912.F.1.5

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Range
- Rate of Change
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.4

Next Benchmarks

- MA.912.NSO.4.2
- MA.912.AR.9.8, MA.912.AR.9.9, MA.912.AR.9.10
- MA.912.F.1.6

Purpose and Instructional Strategies

In grade 8, students graphed two-variable linear equations given a written description, a table or an equation in slope-intercept form. In Algebra I, students graph linear functions from equations in other forms, as well as tables and written descriptions, and they determine and interpret the domain, range and other key features. In later courses, students will graph and solve problems involving linear programming, systems of equations in three variables and piecewise functions.

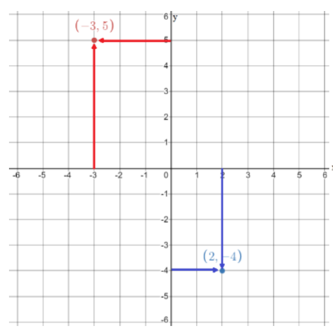
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality Notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-Builder Notation
    If the domain is all values of x less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."
- Within this benchmark, linear two-variable equations include horizontal and vertical lines. Instruction includes writing horizontal and vertical lines in the form $y = 3$ and $x = -4$ and as $0x + 1y = 3$ and $1x + 0y = -4$, respectively. Students should understand that vertical lines are not linear functions but rather linear two-variable equations.
  - Discussions about this topic are a good opportunity to foreshadow the use of horizontal and vertical lines as common constraints in systems of equations or inequalities.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.

Common Misconceptions or Errors

- Students may express initial confusion with the meaning of $f(x)$ for functions written in function notation.
- Students may confuse the x or y coordinates that correspond to domain and range.

Strategies to Support Tiered Instruction

- Teacher provides equations in both function notation and x-y notation written in slope-intercept form and models graphing both forms using a graphing tool or graphing software *(MTR.2.1)*.
    - For example, $f(x) = \frac{2}{3}x + 6$ and $y = \frac{2}{3}x + 6$, to show that both $f(x)$ and y represent the same outputs of the function.
- Teacher provides instruction using a coordinate plane geoboard to provide students support in graphing a linear function.
    - For example, given the y-intercept and slope of line, teacher first puts a peg at the y-intercept. Then, the teacher models using the slope to graph a second point by moving a second peg from the y-intercept "up and over" (or "down and over") to another point. Students can check their second point by substituting it into the given equation of the line. If the equation makes a true statement, then the student has graphed the second point correctly. If the equation makes a false statement, then the student has not graphed the second point correctly.
- For students who need extra support in plotting points, teacher provides instruction using a coordinate plane geoboard to find the x-value on the x-axis and the y-value on the y- axis. Then, the teacher models moving to find the coordinate point, where the x- and y- value meet.
    - The graph below shows how a teacher could model plotting the points $(-3, 5)$ and $(2, -4)$.



- Teacher provides an Anchor Chart to clarify vocabulary terms for x and y coordinates.

x	**y**
domain	range
input	output
x	f(x)

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.7.1)

There are a total of 549 seniors graduating this year. The seniors walk across the stage at a rate of 42 seniors every 30 minutes. The ceremony also includes speaking and music that lasts a total of 25 minutes.

Part A. Write a function that models this situation.

Part B. What is the slope of the function you created? How does that translate to this situation?

Part C. Graph the function you created in Part A. What is the feasible domain and range for this situation?

Part D. For how many hours will the graduation ceremony take place?

Instructional Items

Instructional Item 1

A linear function has a y-intercept of -3 and a slope of 0.

Part A. Graph the function.

Part B. Identify the domain and range. Write them in inequality notation.

Part C. Identify the x-intercept.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.5

Benchmark

MA.912.AR.2.5 **Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.**

Algebra I Example: Lizzy's mother uses the function $C(p) = 450 + 7.75p$, where $C(p)$ represents the total cost of a rental space and p is the number of people attending, to help budget Lizzy's 16^{th} birthday party. Lizzy's mom wants to spend no more than $850 for the party. Graph the function in terms of the context.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain, range, intercepts and rate of change.

Clarification 2: Instruction includes the use of standard form, slope-intercept form and point-slope form.

Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

Clarification 5: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1
- MA.912.F.1.5, MA.912.F.1.8
- MA.912.DP.2.4

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Range
- Rate of Change
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.4
- MA.8.AR.3.5

Next Benchmarks

- MA.912.NSO.4.2
- MA.912.AR.9.8, MA.912.AR.9.9, MA.912.AR.9.10
- MA.912.F.1.6
- MA.912.DP.1.1, MA.912.DP.1.2

Purpose and Instructional Strategies

In grade 8, students determined and interpreted the slope and y-intercept of a two-variable linear equation in slope-intercept form from a real-world context. In Algebra I, students solve real-world problems that are modeled with linear functions when given equations in all forms, as well as tables and written descriptions, and they determine and interpret the domain, range and other key features. Additionally, students will interpret key features and identify any constraints. In later courses, students will graph and solve problems involving linear programming, systems of equations in three variables and piecewise functions.

- This benchmark is a culmination of MA.912.AR.2. Instruction here should feature a variety of real-world contexts.
- Instruction includes representing domain, range and constraints using words, inequality notation and set-builder notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality Notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-Builder Notation
    If the domain is all values of x less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."
- Instruction includes the use of x-y notation and function notation.
- This benchmark presents the first opportunity for students to represent constraints in the domain and range of functions. Students should develop an understanding that linear graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by linear functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
  - For example when a variable represents an amount of time, negative values would not be possible and therefore be a constraint.

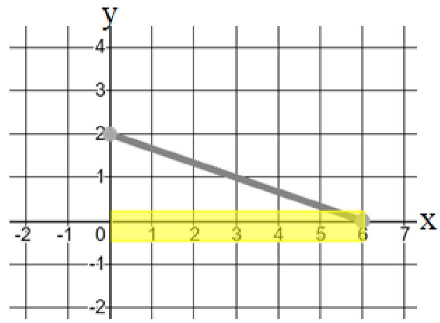
- Instruction includes the understanding that a real-world context can be represented by a linear two-variable equation even though it only has meaning for discrete values.
    - For example, if a gym membership cost $10.00 plus $6.00 for each class, this can be represented as $y = 10 + 6c$. When represented on the coordinate plane, the relationship is graphed using the points $(0,10)$, $(1,16)$, $(2,22)$, and so on.
- For mastery of this benchmark, students should be given flexibility to represent real-world contexts with discrete values as a line or as a set of points.
- Instruction directs students to graph or interpret a representation of a context that necessitates a constraint. Discuss the meaning of multiple points on the line and announce their meanings in the associated context (*MTR.4.1*). Allow students to discover that some points do not make sense in context and therefore should not be included in a formal solution (*MTR.6.1*). Ask students to determine which parts of the line create sensible solutions and guide them to make constraints to represent these sections.
- Instruction includes the use of technology to develop the understanding of constraints.
- Instruction includes the connection to scatter plots and lines of fit (MA.912.DP.2.4) and the connection to systems of equations or inequalities (MA.912.DP.9.6).

Common Misconceptions or Errors

- Students may express initial confusion with the meaning of $f(x)$ for functions written in function notation.
- Students may assign their constraints to the incorrect variable.
- Students may miss the need for compound inequalities in their constraints. Students may not include zero as part of the domain or range.
    - For example, if a constraint for the domain is between 0 and 10, a student may forget to include 0 in some contexts, since they may assume that one cannot have zero people, for instance.

Strategies to Support Tiered Instruction

- Teacher provides equations in both function notation and x-y notation written in slope-intercept form and models graphing both forms using a graphing tool or graphing software *(MTR.2.1)*.
    - For example, $f(x) = \frac{2}{3}x + 6$ and $y = \frac{2}{3}x + 6$, to show that both $f(x)$ and y represent the same outputs of the function.
- Instruction provides opportunities for identifying the domain and range on the x- and y-axis respectively using a highlighter.
    - For example, Tim bought 2 cubic feet of fertilizer and used a little every day on his lawn for 6 months, and the amount of fertilizer decreases at a constant rate as shown on the graph. The domain of the function in this context is $0 \leq x \leq 6$.

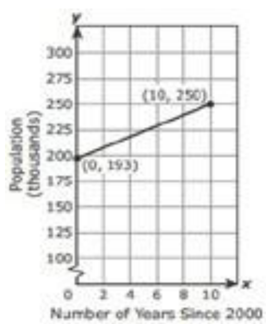


- Teacher provides context to visualize and determine if it would make sense for the function to extend to a given area.
    - For example, if Garrison bought a house in 2014 and the price increases at a constant rate, he could model this by graphing a linear function where x represents the time since 2014. The domain could include negative values if he wanted to show the estimated price of the house before 2014.
    - For example, if the temperature in Alaska is at 14 degrees Fahrenheit at 6:00 am and drops at a constant rate, this can be modeled by graphing a linear function where t represents the time since 6:00 am. The range could include negative numbers to show the temperature below 0 degrees Fahrenheit.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

The population of St. Johns County, Florida, from the year 2000 through 2010 is shown in the graph below. If the trend continues, what will be the population of St. Johns County in 2025?



Instructional Task 2 (MTR 4.1, MTR.7.1)

Devon is attending a local festival downtown. He plans to park his car in a parking garage that operates from 7:00 a.m. to 10:00 p.m. and charges $5 for the first hour and $2 for each additional hour of parking.

Part A. Create a linear graph that represents the relationship between the price and number of hours parked.

Part B. What is an appropriate domain and range for the given situation?

Part C. Discuss with a partner whether there are constraints on the domain and range.

Instructional Task 3 (MTR.7.1)

Suppose you fill your truck's tank with fuel and begin driving down the highway for a road trip. Assume that, as you drive, the number of minutes since you filled the tank and the number of gallons remaining in the tank are related by a linear function. After 40 minutes, you have 28.4 gallons left. An hour after filling up, you have 26.25 gallons left.

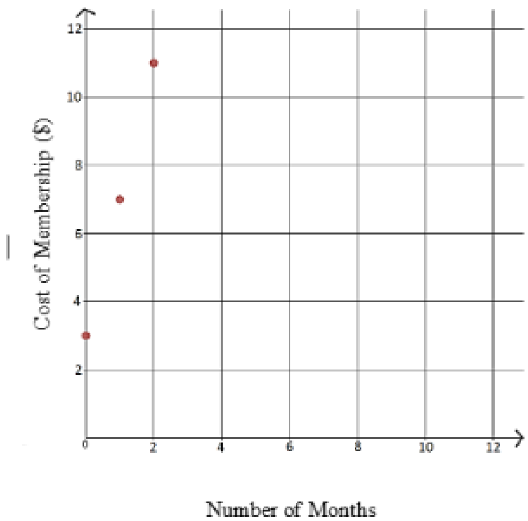
Part A. Graph this relationship.

Part B. Determine how many hours it will take for you to run out of fuel.

Instructional Items

Instructional Item 1

Mohammed wants to reach his fitness goals and decides to join a local gym to start his journey to a healthier lifestyle. The gym membership cost for students is shown in the graph below.



What is the range for the given situation?

a. $\{x|x \geq 0\}$

b. $\{y|y \geq 3\}$

c. $\{3, 7, 11\}$

d. All Real Numbers

Instructional Item 2

The band is selling cookie dough as a fundraiser for instruments at their annual concert. They start with 100 boxes, and they sell approximately 20 boxes per hour. The function

$$C(t) = 100 - 20t$$

represents the number of boxes of cookie dough remaining after hours.

Select all the statements that are true.

a. The domain is all real numbers

b. The y-intercept is 0.

c. The range is all real numbers

d. The range is $0 \leq y \leq 100$

e. The y-intercept is 100

f. The boxes of cookie dough increase 20 boxes per hour.

MA.912.AR.2.6

Benchmark

MA.912.AR.2.6 **Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.**

Algebra I Example: The compound inequality $2x \leq 5x + 1 < 4$ is equivalent to $-1 \leq 3x$ and $5x < 3$, which is equivalent to $-\frac{1}{3} \leq x < \frac{3}{5}$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1
- MA.912.AR.4.1
- MA.912.AR.9.1, MA.912.AR.9.4, MA.912.AR.9.6

Terms from the K-12 Glossary

- Linear Expression

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.2

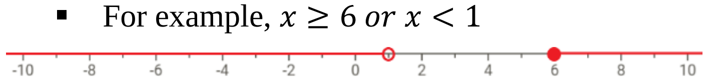
Next Benchmarks

- MA.912.AR.3.3
- MA.912.AR.9.8

Purpose and Instructional Strategies

In grade 8, students solved two-step linear inequalities in one variable. In Algebra I, students solve one-variable linear inequalities, including compound inequalities. In later courses, students will solve problems involving linear programming and will solve one-variable quadratic inequalities.

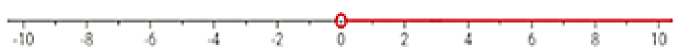
- Instruction includes representing the solution using words, inequality notation, set-builder notation and a graph.
  - Words
    For example, the solution to the compound inequality 2 < x + 1 < 5 could be represented as all real numbers greater than one and less than four.
  - Inequality Notation
    For example, if the solution is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-Builder Notation
    For example, if the domain is all values of x less than or equal to zero, it can be represented as
  - $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."
    Graph
    - For example, $x \geq 6$ *or* $x < 1$



- Instruction emphasizes that solutions for compound inequalities with an *or* statement contain *all* the solutions for *both* inequalities and that solutions for compound inequalities with an *and* statement contain only the solutions that *both* inequalities *have in common.* Allow for student discovery of this by discussing whether each point on a number line is a solution for its compound inequality (*MTR.4.1*).

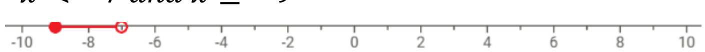
  - For example, plot various points on a number line and ask student whether certain points are a solution to the compound inequality $x < -6$ or $x \geq 8$.
- Instruction includes student understanding that compound inequalities with *or* create a combining (or a *union*) of the solutions of the individual inequalities while compound inequalities with *and* create an overlap (or an *intersection*) of the solutions of the individual inequalities.
- For mastery of this benchmark, students do not need to have familiarity with the terms "unions" and "intersections" as this will be part of a later course.
- Instruction builds understanding that compound inequalities may have different looks graphically.
  - Graphs of Compound Inequalities with ***or***
    - $x \geq 6 \text{ or } x < 1$

    - $x < -5 \text{ or } x \geq -9$ (Solution is all real numbers)

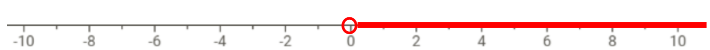
    - $x \geq 3 \text{ or } x > 0$
    - 

  - Graphs of Compound Inequalities with ***and***
    - $x \geq 6 \text{ and } x \leq 1$ (There are no solutions)

    - $x < -7 \text{ and } x \geq -9$



    - $x \geq 3 \text{ and } x > 0$



Common Misconceptions or Errors

- Given that students' prior experience has dealt with equation work far more than inequality work, students will likely forget to consider the direction of the inequality symbol as they solve inequalities.
- When creating inequalities, look for students to experience confusion with the phrase "less than."
- Students may interchange the meanings of "at most" and "at least".

Strategies to Support Tiered Instruction

- Teacher models which direction to shade when graphing inequalities. Teacher additionally models and co-creates a number line while providing a think-aloud explaining that after solving for x, you must select values for x. If the value selected creates a true statement, have students place a dot on that value. If it creates a false statement, have students place an "x" on that value. At the beginning, students may need to choose a few values before being able to accurately shade. The same thing can be done for compound inequalities by replacing values for x in both inequalities.

- Teacher provides instruction involving real-world contexts where students must determine the direction of the inequality symbol on a number line.
- Teacher supports student understanding on why the inequality symbol changes direction when multiplying or dividing by a negative.
  - For example, students are provided with a numerical inequality and a series of operations where they put in the inequality after each operation to determine when the inequality is flipped and when it is not.

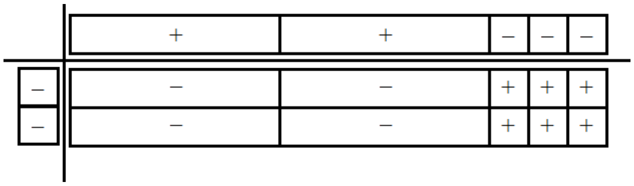
	$5 > -3$
Add -6 to both sides.	$-1 > -9$
Subtract 3 from both sides.	$-4 > -12$
Multiply both sides by 2.	$-8 > -24$
Divide both sides by -4.	$2 < 6$
Subtract 4 from both sides.	$-2 < 2$
Multiply both sides by -3.	$6 > -6$

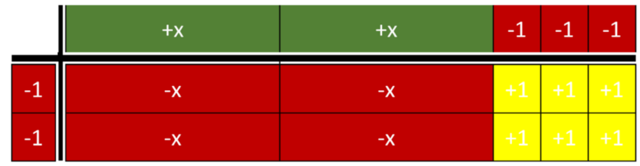
- Teacher provides a highlighter to identify the phrases "is less than," "is greater than," "is less than or equal to," and "is greater than or equal to" when writing inequalities.
- Teacher provides clarification on "less than" and "is less than", since "less than" has been used for subtraction in prior grade levels.
- It is helpful to compare the placement of the word "is" in the two statements: "eight less than four times a number is …" which is the same as "4n – 8 = …" and "eight is less than four times a number" which is represented as "8 < 4n".
- Teacher provides an anchor chart with inequality symbols and possible associated key words including "at most" and "at least".

$<$	$\leq$	$>$	$\geq$
Less than	Less than or equal to	Greater than	Greater than or equal to
Fewer than	Maximum	More than	Minimum
Does not include	At most	Does not include	At least

- Instruction includes the use of algebra tiles to model the distributive property or to add and subtract like terms as a problem is solved algebraically.
  - An example modeling the distributive property using algebra tiles for $-2(2x - 3)$ is shown below.

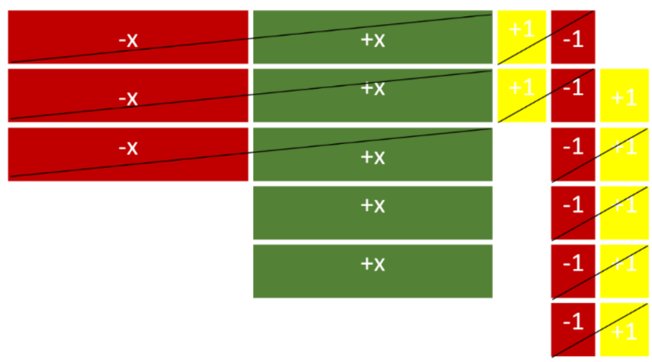
$$-2(2x - 3)$$
$$-2(2x) + (-2)(-3)$$
$$-4x + 6$$





- An example modeling combining like terms using algebra tiles for $-3x + 2 + 5x - 6 + 5$ is shown below.

$$-3x + 2 + 5x - 6 + 5$$
$$-3x + 5x + 2 - 6 + 5$$
$$2x + 1$$



- Instruction includes providing expressions and have students act out the context with props.
  - For example, Karen earns a base salary of $3500 plus $200 commission for each sale made. She wants to earn at least $4500 this month, and the company will only pay at most $5300 each month. In this example, give a student play money and then ask how much money they make after 1 sale, 2 sales, and so on this month. Then ask them what the least number of sales they want to make and the most number of sales they would want to make, and to represent this using an inequality.
- Instruction includes vocabulary development by co-creating a graphic organizer for each property of inequality.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Each month, Phone Company A charges $105 for unlimited talk and text plus an additional $11.50 for each gigabit of data used. Phone Company B charges $28.50 per gigabit of data used, and the price includes unlimited talk and text.

Part A. How many gigabits of data could a person use monthly to make the total cost from Phone Company B more expensive than Phone Company A?

Part B. Compare your solution with a partner.

Part C. What are all of the possible gigabits a person could use to make the total cost from Phone Company B more expensive than Phone Company A?

Instructional Task 2 (MTR.7.1)

Tamar is shopping for food for an upcoming graduation party for her senior class. She starts with $350 and hopes to retain between $100 and $125 after her purchases to hire a DJ. She spends a total of $163 on food and plans to buy 24-packs of soda that are priced at $8 each. What range of 24-packs of soda can Tamar afford to buy?

Instructional Items

Instructional Item 1

The expression $45h + 225$ represents the total cost in dollars of renting a large party tent from a local rental store for h hours. Jamal's parents are willing to spend between $500 and $750 for the tent.

Part A. Write an inequality to represent the possible lengths of time the tent can be rented.

Part B. Using your inequality in Part A, what is one possible length of time the tent could be rented? Give your answer in number of hours.

Instructional Item 2

Graph the solution to the inequality $-\frac{4}{5} - 9.2w < 1.24w - 3.75$.

Instructional Item 3

You are planning a road trip, and you want to ensure that the fuel level in your car stays within an acceptable range, between 8 and 15 gallons. Let g represent the number of gallons of gas in your tank. You want to make sure that your fuel level is neither too low nor too high. Write an inequality to model the situation

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.7

Benchmark

MA.912.AR.2.7	**Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real- world context.**

Benchmark Clarifications:

Clarification 1: Instruction includes the use of standard form, slope-intercept form and point-slope form and any inequality symbol can be represented.

Clarification 2: Instruction includes cases where one variable has a coefficient of zero.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.1
- MA.912.AR.9.1, MA.912.AR.9.4, MA.912.AR.9.6

Terms from the K-12 Glossary

- Coordinate Plane

- Linear Expression

Vertical Alignment

Previous Benchmarks

MA.8.AR.3.3

Next Benchmarks

- MA.912.AR.3.9
- MA.912.AR.9.8

Purpose and Instructional Strategies

In grade 8, students wrote two-variable linear equations in slope-intercept form. In Algebra I, students write two-variable linear inequalities. In later courses, students will solve problems involving linear programming and will write two-variable quadratic inequalities.

- Instruction includes making connections to various forms of linear inequalities to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the inequality $Ax + By > C$, where A, B and C are any rational number and any inequality symbol can be used. This form can be useful when identifying the x- and y-intercepts.
  - Slope-Intercept Form
    Can be described by the inequality $y < mx + b$, where m is the slope and b is the y-intercept and any inequality symbol can be used. This form can be useful when identifying the slope and y-intercept.
  - Point-Slope Form
    Can be described by the inequality $y - y_1 \leq m(x - x_1)$, where (x_1, y_1) is a point on the boundary line and m is the slope of the line and any inequality symbol can be used. This form can be useful when the y-intercept is not readily apparent.
- Look for opportunities to point out the connection between linear contexts and constant rates of change.
- Problem types should include cases for vertical and horizontal lines.
- Instruction includes the connection to one-variable linear inequalities and their solutions on a number line.
  - For example, provide the inequality $x \geq 2$ and a coordinate plane for students to determine points that satisfy the inequality. Students will determine that all points on the half-plane to the right of $x = 2$ contain solutions.

Instruction includes the connection to graphs of the solution sets of one-variable inequalities on a number line; recognizing whether the boundary line should be dotted (exclusive – meaning not included in the solution set) or solid (inclusive – meaning included in the solution set).

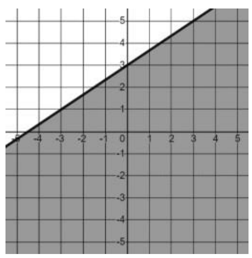
Common Misconceptions or Errors

- Students often choose the incorrect inequality symbol when interpreting graphs or contexts. Help these students develop the habit of using a test point to check their work. Any point can be used, but many students find it easiest to use the origin (0, 0) as it often makes mental calculation much quicker.
- When creating inequalities, look for students to experience confusion

with the phrase “less than.”

Strategies to Support Tiered Instruction

- Instruction includes opportunities to use a highlighter to identify the phrases “is less than,” “is greater than,” “is less than or equal to,” and “is greater than or equal to” when writing inequalities.
- Instruction includes opportunities to identify a test point to substitute into an inequality to determine which symbol should be used when writing the inequality. It is usually easiest to use the origin $(0,0)$ as it makes mental calculations easier. If the point selected creates a true statement, their inequality is true. If it creates a false statement, their inequality is false.
  - For example, given the graph below, one could use the test point $(0,0)$ to determine whether the inequality $y \leq \frac{2}{3}x + 3$ or $y \geq \frac{2}{3}x + 3$ represents the graph. When substituting in $(0,0)$ for $y \geq \frac{2}{3}x + 3$, it results in the inequality $0 \geq 3$ which is a false statement. When substituting in $(0,0)$ for $y \leq \frac{2}{3}x + 3$, it results in the equality $0 \leq 3$ which is a true statement. Therefore, the inequality $y \leq \frac{2}{3}x + 3$ represents the given graph.



- Teacher provides clarification on “less than” and “is less than”, since “less than” has been used for subtraction in prior grade levels.
  - It is helpful to compare the placement of the word “is” in the two statements: “eight less than four times a number is …” which is the same as “4n – 8 = …” and “eight is less than four times a number” which is represented as “8 < 4n”.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

A carpenter makes two types of chairs: a lawn chair and a living room chair. It takes her 3 hours to make a lawn chair and 5 hours to make a living room chair.

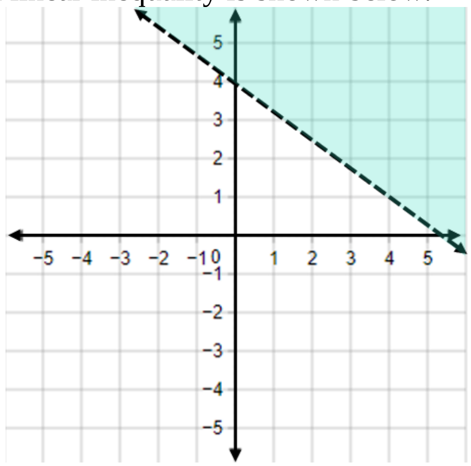
Part A. If the carpenter works a maximum of 55 hours per week, write a two-variable linear inequality to describe the number of possible chairs of each type she can make in a week.

Part B. What is one possible combination of lawn chair and living room chair that the carpenter can make in one week?

Part C. What is one possible combination of lawn chair and living room chair that the carpenter could make in one month?

Instructional Task 2 (MTR.3.1, MTR.4.1)

The graph of the solution set to a linear inequality is shown below.



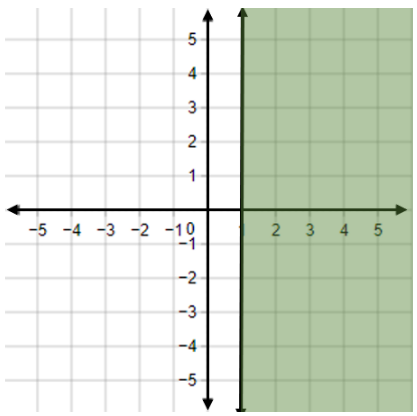
Part A. Create a linear inequality that corresponds to this graph.

Part B. Create a real-world situation that could be described by the graph and the inequality created in Part A.

Instructional Items

Instructional Item 1

The graph of the solution set to a linear inequality is shown below. Create a linear inequality that corresponds to this graph.



Instructional Item 2

The Drama Club sold sodas and sandwiches in the cafeteria to raise money for costumes. The sodas were sold for \$2 each and the sandwiches were sold for \$5 each. In order to cover the cost of expenses, the Drama Club needed to raise more than \$80. Which inequality represents this situation?

a. $2x + 5y < 80$
b. $2x + 5y > 80$
c. $(2 + 5)x > 80$
d. $(2 + 5)x < 80$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.8

Benchmark

MA.912.AR.2.8 Given a mathematical or real-world context, graph the solution set to a two- variable linear inequality.

Benchmark Clarifications:

Clarification 1: Instruction includes the use of standard form, slope-intercept form and point-slope form and any inequality symbol can be represented.

Clarification 2: Instruction includes cases where one variable has a coefficient of zero.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.1, MA.912.AR.4.3
- MA.912.AR.9.1, MA.912.AR.9.4, MA.912.AR.9.6

Terms from the K-12 Glossary

- Coordinate Plane
- Linear Expression

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.4

Next Benchmarks

- MA.912.AR.3.10
- MA.912.AR.9.8

Purpose and Instructional Strategies

In grade 8, students graphed linear two-variable equations written in slope-intercept form. In Algebra I, students graph the solution set to a two-variable linear inequality. In later courses, students will solve problems involving linear programming and will graph the solutions sets of two-variable quadratic inequalities.

- Instruction includes the use of linear inequalities in standard form, slope-intercept form and point-slope form. Include examples in which one variable has a coefficient of zero such as $x < -\frac{17}{5}$.
- Instruction includes the connection to graphing solution sets of one-variable inequalities on a number line; recognizing whether the boundary line should be dotted (exclusive) or solid (inclusive). Additionally, have students use a test point to confirm which side of the line should be shaded (*MTR.6.1*).
- Students should recognize that the inequality symbol only directs which side of the line is shaded (above or below) for inequalities when in slope-intercept form. Students shading inequalities in other forms will need to use a test point to determine the correct half-plane to shade.

Common Misconceptions or Errors

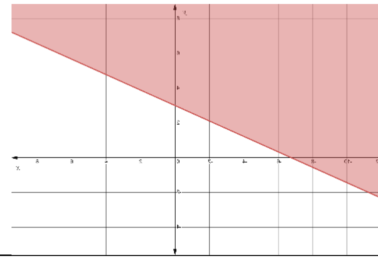
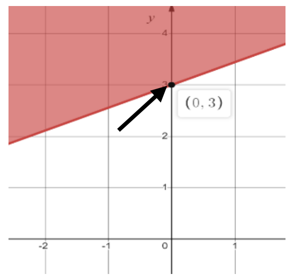
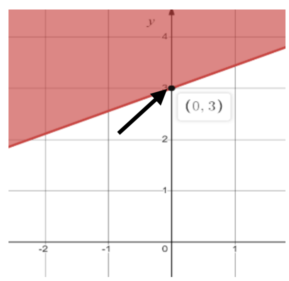
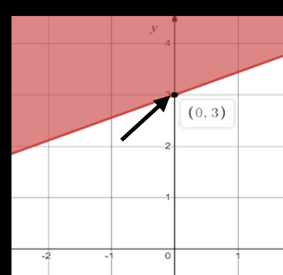
- Students often choose to shade the wrong half-plane when graphing two-variable linear inequalities.
- Students may think that the inequality symbol's orientation always determines the side of the line to shade.
  - For example, students may say that inequalities with a less than symbol should be shaded below the line while inequalities with a greater than symbol should be shaded above the line. This typically happens after graphing multiple inequalities in slope-intercept form. To address this, provide counterexamples to this such as $3x - 2y < 15$ *or* $-4x - 7 \geq y$. Use these counterexamples to emphasize the benefit of using a test point to confirm the direction of shading.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to use a highlighter to identify the phrases "is less than," "is greater than," "is less than or equal to," and "is greater than or equal to" when writing inequalities.
- Teacher provides instruction modeling how to correctly identify the solution set of a linear inequality given in slope-intercept form. After graphing, students can circle the y-intercept. If the inequality is in form $y < mx + b$ or $y \leq mx + b$, the solution set is the half-plane that contains the y-axis values below the y-intercept. If the inequality is in form $y > mx + b$ or $y \geq mx + b$, the solution set is the half-plane that contains the y-axis values above the y-intercept.
- Teacher provides counterexamples to address the misconception that the inequality symbol's orientation always determines the side of the line to shade.
  - For example, $3x - 2y < 15$ *or* $-4x - 7 \geq y$ can be used to emphasize the benefit of using a test point to confirm the direction of shading.
- Instruction includes opportunities to graph the boundary line of a system of inequalities, based on an inaccurate translation from word problem. To assist in determining the boundary line for the system, students can create a graphic organizer like the one below.

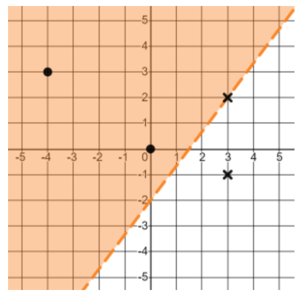
<	≤	>	≥
Less than	Less than or equal to	Greater than	Greater than or equal to
Fewer than	Maximum	More than	Minimum
Does not include	"No More Than" or "At most"	Does not include	"No less than" or "At least"
Dashed line	Solid Line	Dashed line	Solid Line

- Instruction includes making the connection between the algebraic and graphical representations of a two-variable linear inequality and its key features.
  - For example, teacher can provide a graphic organizer such as the one below.

Algebraic Representation	**Graphical Representation**
$y \geq 3 + \frac{4}{9}x$	
The y-intercept is located at the point (0,3).	
The y-intercept is located at the point (0,3).	
The y-intercept is located at the point (0,3).	
The slope of the boundary line is $\frac{4}{9}$.	From any point on the boundary line, the next point can be found by moving up/down 4 units and then moving right/left 9 units.
$\geq$	The boundary line is solid with the solution set shaded above the boundary line.

- Instruction includes opportunities to identify a test point to substitute into an inequality. It is usually easiest to use the origin (0,0) as it makes mental calculations easier. If the point selected creates a true statement, their inequality is true, and they should shade in the half- plane containing that point. If it creates a false statement, they should shade in the half- plane not containing that point. By using a test point, students avoid the mistake of thinking that the direction of the inequality determines the shading.

- For example, the points $(-4,3)$, $(0,0)$, $(3,2)$ and $(3,-1)$ were used to determine where to shade for the inequality $4x - 3y < 6$ shown below.



Instructional Tasks

Instructional Task 1 (MTR.7.1)

Penelope is planning to bake cakes and cookies to sell for an upcoming school fundraiser. Each cake requires $1\frac{3}{4}$ cups of flour and each batch of cookies requires $2\frac{1}{4}$ cups of flour Penelope bought 3 bags of flour. Each bag contains around 17 cups of flour.

Part A. Assuming she has all the other ingredients needed, create a graph to show all the possible combinations of cakes and batches of cookies Penelope could make.

Part B. Create constraints for this given situation.

Instructional Items

Instructional Item 1

Graph the solution set to the inequality $y + 3 > -2(x - 2)$.

Instructional Item 2

Fred has $9 in his pocket and wants to buy chips and candy for a movie he is going to watch. Chips cost $1.50 per bag and candy is $1 a box. Graph the solution set that represents the amount of chips and candy Fred can buy.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4 *Write, solve and graph absolute value equations, functions and inequalities in one and two variables.*

MA.912.AR.4.1

Benchmark

MA.912.AR.4.1	**Given a mathematical or real-world context, write and solve one-variable absolute value equations.**

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.1
- MA.912.F.2.1

Terms from the K-12 Glossary

- Absolute Value

Vertical Alignment

Previous Benchmarks

- MA.6.NSO.1.3
- MA.6.NSO.1.4
- MA.7.NSO.2.1

Next Benchmarks

- MA.912.AR.3.1
- MA.912.AR.4.4
- MA.912.F.2.3

Purpose and Instructional Strategies

In grade 6, students solved problems involving absolute value, including context related to distances, temperatures and finances. In grade 7, students solved multi-step order of operations with rational numbers including absolute value. In Algebra I, students write and solve absolute value equations in one variable. In later courses, students will solve and graph real-world problems that are modeled with absolute value functions.

- In Algebra I, instruction includes absolute value equations in the form $d = |ax + b| + c$ or $d = a|x + b| + c$, where $a \neq 0$ and b, c, and d are rational numbers. In Algebra I Honors, instruction can include other forms, such as $dx = |ax + b| + c$ or $|dx + e| = |ax + b| + c$, that may result in extraneous solutions.
- Instruction reinforces the definition of absolute value as a number's distance from zero (0) on a number line. Distance is expressed as a positive value; example: $|3| = 3$ and $|-3| = 3$. The numbers 3 and –3 are each three units away from zero on a number line (*MTR.2.1*).
- Instruction includes asking for solutions of absolute value equation in word form.
  - For example, the equation $|x| = 7.1$ can be read as "What values of x have absolute value equivalent to 7.1?"
- Instruction focuses on recognizing that there are either two solutions or no solutions to an absolute value equation.
  - The equation $|x| = -8$ has no solution because the absolute value of a number cannot be negative.
  - The equation $|5x - 2| = 10$ has two solutions by the definition of absolute value; one of which satisfies $5x - 2 = 10$ and the other satisfies $5x - 2 = -10$.

Common Misconceptions or Errors

- Students may forget that absolute value refers to a distance and is a positive number or zero.
- Students may forget many absolute value equations will produce two solutions.
- Students may forget some equations have no solutions.

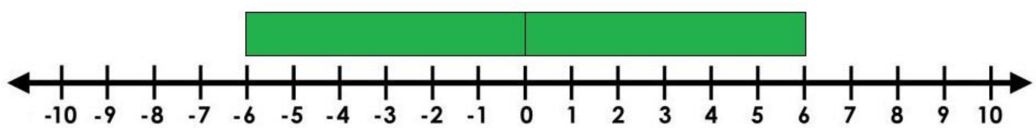
Strategies to Support Tiered Instruction

- Teacher provides a three-column table with equations already sorted into two-, one-, or no-solutions. Teacher asks what is noticed about the equations in each column and class determines the headers to be used (Two Solutions, One Solution, No Solutions). Then, students sort additional equations into columns. In Algebra I Honors, equations with extraneous solutions should be included.

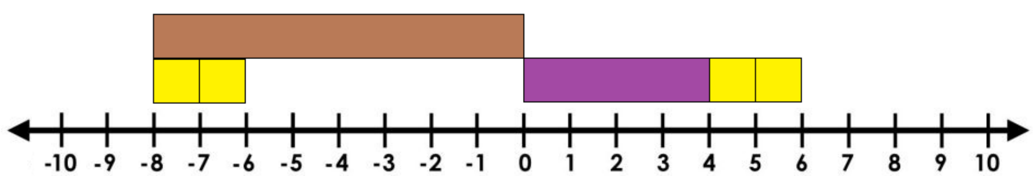
-

Two Solutions	One Solution	No Solutions
$\lvert x \rvert = 6$	$\lvert x \rvert = 0$	$\lvert x \rvert = -6$
$\lvert 2x - 5 \rvert = 8$	$\lvert 2x - 5 \rvert = 0$	$\lvert 2x - 5 \rvert = -8$
$\left\lvert \frac{2}{3}x - 7 \right\rvert = 23$	$\left\lvert \frac{2}{3}X - 7 \right\rvert = 0$	$\left\lvert \frac{2}{3}X - 7 \right\rvert = -23$

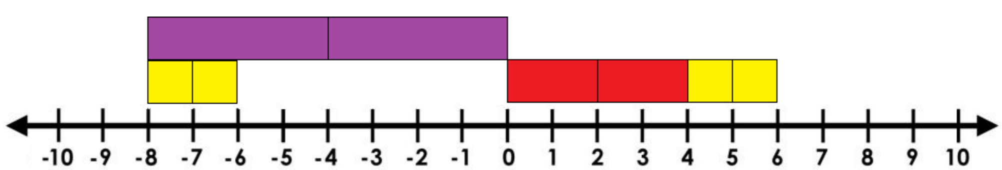
- Teacher provides absolute value equations and has students then sort them into a three-column graphic organizer with column headers of: two solutions, one solution and no solution. In Algebra I Honors, equations with extraneous solutions should be included.
- Teacher models absolute value equations on the number line to show that absolute value is the distance from zero. By the visual representation, students will see how absolute value equations produce two solutions. Manipulatives, such as Cuisenaire rods, can be used to represent the constants in each equation
    - For example, the absolute value equation $\lvert x \rvert = 6$ can be modeled as shown.



    - For example, the absolute value equation $\lvert x + 2 \rvert = 6$ can be modeled as shown.



    - For example, the absolute value equation $\lvert 2x + 2 \rvert = 6$ can be modeled as shown.



Instructional Tasks

Instructional Task 1 (MTR.6.1, MTR.7.1)

Donna and Kayleigh both go to the same high school. Donna lives 21 miles from the school. Kayleigh lives 6 miles from Donna.

Part A. Write an absolute value equation to represent the location of Kayleigh's house in relation to the high school.

Part B. How far could Kayleigh live from her school?

Instructional Task 2 (MTR.6.1, MTR.7.1)

Jay has money in his wallet, but he doesn't know the exact amount. When his friend asks him how much he has he says that he has 50 dollars give or take 15.

Part A. Write an absolute value equation to model this situation.

Part B. How much money could Jay have in his wallet?

Instructional Task 3 (MTR.6.1, MTR.7.1)

A car dealership is having a contest to win a new truck. In order to win a chance at the truck, you must first guess the number of keys in the jar within 5 of the actual number. The people who are within this range then get to try a key in the ignition of the truck. Suppose there are 697 keys in the jar. Write an absolute value equation that will reveal the highest and lowest guesses in order to win a chance at the truck. What are the highest and lowest guesses that will qualify for a chance to win?.

Instructional Items

Instructional Item 1

The difference between the temperature on the first day of the month, t_1, and the temperature on the last day of the month, 74 degrees, is 6 degrees. Write an equation involving absolute value that represents the relationship among t_1, 74 and 6.

Instructional Item 2

Determine the solutions of the equation below.

$$3|x-2|+14=14$$

Instructional Item 3

Determine the solutions of the equation below.

$$|\frac{1}{4}\ x-3|=10$$

Instructional Item 4

Determine the solutions of the equation below.

$$|\frac{3}{2}x+2|+10=0$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4.3

Benchmark

MA.912.AR.4.3 **Given a table, equation or written description of an absolute value function, graph that function and determine its key features.**

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior and symmetry.

Clarification 2: Instruction includes representing the domain and range with inequality notation, interval

notation or set-builder notation.
Clarification 3: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.2.1

Terms from the K-12 Glossary

- Absolute Value
- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.6.NSO.1.3, MA.6.NSO.1.4
- MA.7.NSO.2.1

Next Benchmarks

- MA.912.AR.4.4
- MA.912.AR.9.10
- MA.912.F.1.6
- MA.912.F.2.3

Purpose and Instructional Strategies

In middle grades, students graphed linear equations in two variables. In Algebra I, students graph absolute value functions and determine key features. In later courses, students will solve real-world problems involving absolute value functions and piecewise functions.

- In Algebra I, for mastery of this benchmark use $y = a|x - h| + k$ where a is nonzero and h and k are any real number.
  - The vertex of the graph is (h, k).
  - The domain of the graph is set of all real numbers and the range is $y \geq k$ when $a > 0$.
  - The domain of the graph is set of all real numbers and the range is $y \leq k$ when $a < 0$.
  - The axis of symmetry is $x = h$.
  - The graph opens up if $a > 0$ and opens down if $a < 0$.
  - The graph $y = |x|$ can be translated h units horizontally and k units vertically to get the graph of $y = a|x - h| + k$.
  - The graph $y = a|x|$ is wider than the graph of $y = |x|$ if $|a| < 1$ and narrower if $|a| > 1$.
- Instruction includes the understanding that a table of values must state whether the function is an absolute value function.
  - For example, if given the function $y = |x|$ and only positive values of x were given in a table, one would only have part of the graph. Discuss the importance of providing enough points in a table to create an accurate graph.

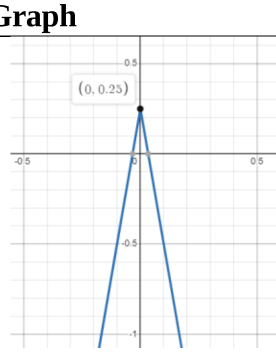
- When making connections to transformations of functions, use graphing software to explore $y = a|x - h| + k$ adding variability to the parent equation to see the effects on the graph. Allow students to make predictions (*MTR.4.1*).
- Instruction provides opportunities to make connections to linear functions and their key features.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$, a quadratic function of the form $y = a(x - h)^2 + k$, and an absolute value function of the form $y = \text{a}\,|\text{x} - \text{h}| + \text{k}$. *(MTR.5.1)*
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the domain is all values of x less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."
- When addressing real-world contexts, the absolute value is used to define the difference or change from one point to another. Connect the graph of the function to the real-world context so the graph can serve as a model to represent the solution (*MTR.6.1*, *MTR.7.1*).
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.

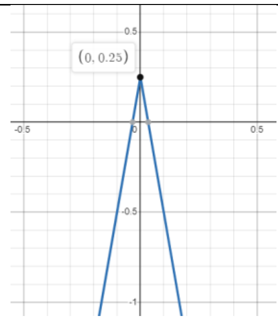
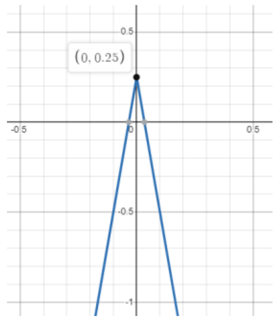
Common Misconceptions or Errors

- Students may not fully understand the connection of all of the key features (emphasize the use of technology to help with student discovery) and how to represent them using the proper notation.

Strategies to Support Tiered Instruction

- Teacher provides opportunities for students to comprehend the context or situation by Teacher models using a graphing tool or graphing software to help students discover the key features and their connections to the absolute value equation.
- Teacher provides a colored visual of a two-variable absolute value equation and its graph.

Key Feature	Graph	Equation
Vertex (h, k)	 $(0, 0.25)$	$y = -7.5|x| + 0.25$

Range $y \leq k$ since $a < 0$	 $y \leq 0.25$	$y = -7.5\|x\| + 0.25$
Axis of Symmetry $x = h$	 $x = 0$	$y = -7.5\|x - 0\| + 0.25$

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Graph the function $f(x) = -\frac{1}{2}|x - 4| + 6$ and determine its domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior and symmetry.

Instructional Task 2 (MTR.3.1)

The graph, $q(x)$, is translated 5 units left and 7 units down from $f(x) = |x - 2|$.

Part A. Graph the absolute value function, $q(x)$.
Part B. Identify the vertex of $q(x)$.
Part C. What is the interval that shows when the function is positive? Negative?
Part D. What is the domain of $q(x)$ when the function is increasing? How does this interval differ from the domain of $q(x)$.
Part E. How would the graph of $q(x)$ change if the a value was $-\frac{1}{3}$?

Instructional Items

Instructional Item 1

Given the table of values for an absolute value function, graph the function.

x	$f(x)$
-2	4
-1	4
0	6
1	8

2	10

Instructional Item 2

Identify key features of the function, $h(x) = -\frac{1}{2}|x + 3| - 4$. Key features include when the function is positive, negative, increasing, decreasing, end behavior, x and y-intercepts, domain, range and vertex.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9 *Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.*

MA.912.AR.9.1

Benchmark

MA.912.AR.9.1 Given a mathematical or real-world context, write and solve a system of two- variable linear equations algebraically or graphically.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is to solve systems using elimination, substitution and graphing.
Clarification 2: Within the Algebra I course, the system is limited to two equations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.3, MA.912.AR.2.4

Terms from the K-12 Glossary

- Linear Equation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

Next Benchmarks

- MA.912.NSO.4.2
- MA.912.AR.9.2, MA.912.AR.9.3, MA.912.AR.9.9

Purpose and Instructional Strategies

In grade 8, students determined whether a system of linear equations had one solution, no solution or infinitely many solutions and solved such systems graphically. In Algebra I, students solve systems of linear equations in two variables algebraically and graphically. In later courses, students will solve systems of linear equations in three variables and systems of nonlinear equations in two variables.

- For students to have full understanding of systems, instruction should include MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout instruction.
- Instruction allows students to solve using any method (substitution, elimination or graphing) but recognizing that one method may be more efficient than another

(*MTR.3.1*).
  - If both equations are presented in slope-intercept form, then either graphing or substitution may be most efficient.
  - If one equation is given in slope-intercept form or solved for x, then substitution may be easiest.
  - If both equations are given in standard form, then elimination, or linear combination, may be most efficient.

Consider presenting a system that favors one of these methods and having students divide into three groups to solve them using different methods. Have students share their work and discuss which method was more efficient than the others (*MTR.3.1*, *MTR.4.1*).

- Include cases where students must interpret solutions to systems of equations.
- Various forms of linear equations can be used to write a system of equations.
  - Standard Form
    Can be described by the equation $Ax + By = C$, where A, B and C are any rational number.
  - Slope-Intercept Form
    Can be described by the equation $y = mx + b$, where m is the slope and b is the y-intercept.
  - Point-Slope Form
    Can be described by the equation $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line.

When introducing the elimination method, students may express confusion when considering adding equations together. Historically, students have used the properties of equality to create equivalent equations to solve for a variable of interest. In most of these efforts, operations performed on both sides of the original equation have been identical. With the introduction of the elimination method, students can now see that operations performed on each side of an equation must be *equivalent* (not necessarily *identical*) for the property to hold. Guide students to explore forming equivalent equations with simpler equations by adding or subtracting equivalent values. Lead them to see that the new equations they generate have the same solutions. Have them discuss why the method works: equations are simply pairs of equivalent expressions, which is why they can be added/subtracted with each other.

Common Misconceptions or Errors

- Students may not understand linear systems of equations can only have more than one solution if there are infinitely many solutions.
- Students may not understand linear systems of equations can have no solution.
- Students may have difficulty making connections between graphic and algebraic representations of systems of equations.
- Students may have difficulty choosing the best method of finding the solution to a system of equations.
- Students may have difficulty translating word problems into systems of equations and inequalities.
- Students using the elimination method may alter the original equations in a way that creates like terms that can be subtracted. When subtracting across the two equations students may have difficulty remembering to apply the subtraction to the remaining terms and constants.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to use graphing software to visualize the possible solutions for a system of equations. Systems of equations only produce three different types of solutions: one solution, infinite solutions and no solutions. Each type of system can be graphed for analysis of each type of solution set.
- Teacher models through a think-aloud how a system of equations can have no solutions.
  - For example, "I can algebraically solve a system with no solutions. The solution will reveal that the left and right sides of the equation cannot be equal, causing a no solution set. In addition, if I rearrange both equations to the slope-intercept form, the equations will have the same slope. I can utilize my knowledge of parallel lines to understand that the system cannot have any solutions."
- Teacher provides step-by-step process for solving systems.
  - For example, when solving the system below, students can use the method of elimination.

$$2x + 4y = -10$$
$$3x + 5y = 8$$

If the student chooses to eliminate the y-variable, they can multiply the first equation by 5 and the second by 4 so that both coefficients of y are 20.

$$5(2x + 4y = -10) \text{ to } 10x + 20y = -50$$
$$4(3x + 5y = 8) \text{ to } 12x + 20y = 32$$

The student either subtracts the two new equations or creates additive inverses by multiplying one of the equations by -1 (as shown) and then adds the equations.

$$-1(10x + 20y = -50) \text{ to } -10x - 20y = +50$$
$$-10x - 20y = +50$$
$$\underline{12x + 20y = 32}$$
$$2x = 82$$
$$x = 41$$

Once students determine one of the values (x in this case), then they can substitute this back into one of the given equations to find the other value (y in this case).

$$2(41) + 4y = -10$$
$$4y = -10 - 82$$
$$y = -23$$

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

You and a friend go to Tacos Galore for lunch. You order three soft tacos and three burritos, and your total bill is \$11.25. Your friend's bill is \$10.00 for four soft tacos and two burritos.

Part A. Write a system of two-variable linear equations to represent this situation.

Part B. Solve the system both algebraically and graphically to determine the cost of each burrito and each soft taco.

Part C. Is one method more efficient than the other? Why or why not?

Instructional Task 2 (MTR.3.1, MTR.4.1)

Part A. Determine the solution to the system of linear equations below using your method of choice.

$$0.5x - 1.4y = 5.8$$
$$y = -0.3x - \frac{1}{5}$$

Part B. Discuss with a partner why you chose that method.

Instructional Items

Instructional Item 1

Determine the exact solution of the system of linear equations below.

$$-\frac{1}{10}\text{x} + \frac{1}{2}y = \frac{4}{5}$$

$$\frac{1}{7}\text{x} + \frac{1}{3}y = -\frac{2}{21}$$

Instructional Item 2

Carla volunteered to make pies for a bake sale. She bought two pounds of apples and six pounds of peaches and spent $19. After baking the pies, she decided they looked so good she would make more. She went back to the store and bought another pound of apples and five more pounds of peaches and spent $15. Write a system of linear equations that describes her purchases, where a represents the cost per pound of the apples and p represents the cost per pound of the peaches.

Instructional Item 3

Which two equations form a system of linear equations that has no solution?

$y = \frac{2}{3}x + 2$	$4x - 6y = 12$	$(y - 2) = -\frac{2}{3}(x - 4)$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.4

Benchmark

MA.912.AR.9.4 Graph the solution set of a system of two-variable linear inequalities.

Benchmark Clarifications:

Clarification 1: Instruction includes cases where one variable has a coefficient of zero.

Clarification 2: Within the Algebra I course, the system is limited to two inequalities.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.7, MA.912.AR.2.8

Terms from the K-12 Glossary

- Inequality

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.2
- MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

Next Benchmarks

- MA.912.AR.9.5, MA.912.AR.9.7, MA.912.AR.9.8

Purpose and Instructional Strategies

In grade 8, students solved determined graphically whether a system of linear equations had one determined graphically whether a system of linear equations had one solution, no solution or infinitely many solutions. In Algebra I, students solve systems of linear inequalities by graphing the solution set. In later courses, students will solve problems involving linear programming.

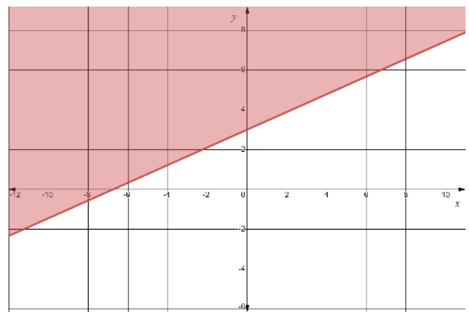
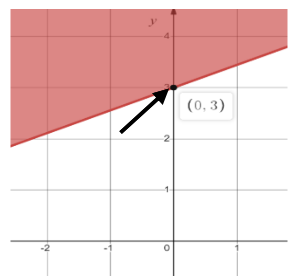
- For students to have full understanding of systems, instruction includes MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction.
- Instruction includes the use of linear inequalities in standard form, slope-intercept form and point-slope form. Include examples in which one variable has a coefficient of zero such as $x < -\frac{17}{5}$.
- Instruction includes the connection to graphing solution sets of one-variable inequalities on a number line, recognizing whether the boundary line should be dotted (exclusive - meaning not included in the solution set) or solid (inclusive – meaning included in the solution set). Additionally, have students use a test point to confirm which side of the line should be shaded (*MTR.6.1*).
- Students should recognize that the inequality symbol only directs where the line is shaded (above or below) for inequalities when in slope-intercept form. Students shading inequalities in other forms will need to use a test point to determine the correct half-plane to shade.
- The solution to a system of inequalities is the area where all the shading overlaps. If the areas do not overlap, it has *no solution*.
- Instruction includes determining whether the point of intersection of the boundary lines of the linear inequalities is within the solution set.
  - For example, if either or both of the two boundary lines are dashed ($<$ or $>$), then the point of intersection is not in the solution set.
- Instruction allows students to make connections between the algebraic and graphical representations of inequalities in two variables (*MTR.2.1*).

Common Misconceptions or Errors

- Students may have difficulties making connections between graphic and algebraic representations of systems of inequalities.
- Students may confuse which points are in the solution set of a system that includes inequalities (including points on the lines in a system of inequalities).
- Students may shade the wrong half-plane or graph the incorrect boundary line (solid vs. dashed).

Strategies to Support Tiered Instruction

- Teacher Instruction includes making the connection between the algebraic and graphical representations of a two-variable linear inequality and its key features.
    - For example, teacher can provide a graphic organizer such as the one below.

Algebraic Representation	**Graphical Representation**
$y =\geq 3 + \frac{4}{9}x$	
The y-intercept is located at the point $(0, 3)$.	
The slope of the boundary line is $\frac{4}{9}$.	From any point on the boundary line, the next point can be found by moving up/down 4 units and then moving right/left 9 units.
$\geq$	The boundary line is solid h the solution set shaded above the boundary line.

- Instruction includes using different colors or shapes to identify each of the solution sets of the linear inequalities.
    - For example, a student can "shade" the solution of the first inequality by highlighting in yellow on its half-plane and can "shade" the solution of the second inequality by highlighting in blue on its half-plane. Where the yellow and blue overlap represent the solution set of the system of linear inequalities.

- Teacher creates connections to solving a system of linear inequalities to determining the solution set to a single two-variable linear inequality, building on students' knowledge from MA.912.AR.2.8.
    - For example, a student can focus first on finding the solution set of one of the inequalities by graphing the boundary line and then choosing a test point to determine where to shade. Next, the student can focus on finding the solution set of the second inequality in the same way. Students should understand that where the two shaded regions overlap is the solution set of the system.

- Instruction includes using transparencies to lay the separately graphed inequalities on top of one another to visualize the solution set of the system.
- Teacher co-creates a graphic organizer to scaffold graphing an inequality and its solution. The steps can be repeated for each inequality.
  - Example:

Symbol (s)	***Graph***
$<$ and $>$	Dashed line, Solution does not include points on the line
$\leq$ and $\geq$	Solid line, Solution includes points on the line

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Part A. Graph the solution set to the system of inequalities:

Part B. What is one point that is a solution to the system above?

Instructional Task 2 (MTR.7.1)

Devonte is throwing a party to watch the Stanley Cup Finals. He orders pizza that cost $11 each and cartons of wings that cost $9.99 each. With at least 34 people coming over, Devonte spends at least $72.96 and orders a minimum of 7 pizzas and cartons of wings.

Part A. Write a system of inequalities that describes this situation.

Part B. Graph the solution set and determine a possible number of pizza and cartons of wings he ordered for his party.

Instructional Items

Instructional Item 1

Graph the solution set to the system of inequalities below.

$$x \geq 3 \qquad \frac{3}{5}x + y < -3$$

Instructional Item 2

Graph the solution set to the system of inequalities below.

$$-y - 3x \leq -1 \qquad 2y \geq 8x - 6$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.6

Benchmark

MA.912.AR.9.6 **Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.**

Benchmark Clarifications:

Clarification 1: Instruction focuses on analyzing a given function that models a real-world situation and writing constraints that are represented as linear equations or linear inequalities.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.7, MA.912.AR.2.8

Terms from the K-12 Glossary

- Inequality
- Linear Equation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.2
- MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

Next Benchmarks

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.6.6
- MA.912.AR.7.3, MA.912.AR.7.4
- MA.912.AR.8.3
- MA.912.AR.9.7, MA.912.AR.9.10
- MA.912.T.3.3

Purpose and Instructional Strategies

in grade 8, students worked with linear equations and inequalities, and graphically solved systems of linear equations. In Algebra I, students represent constraints as systems of linear equations or inequalities and interpret solutions as viable or non-viable options. In later courses, students will solve problems involving linear programming and work with constraints within various function types.

- For students to have a full understanding of systems, instruction includes MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction.
- Allow for both inequalities and equations as constraints. Include cases where students must determine a valid model of a function.
  - Students often use inequalities to represent constraints throughout Algebra I. Equations can be thought of as constraints as well. Solving a system of equations requires students to find a point that is constrained to lie on specific lines simultaneously.
- Instruction includes the use of various forms of linear equations and inequalities.
  - Standard Form
    Can be described by the equation $Ax + By = C$, where A, B and C are any rational number and any equal or inequality symbol can be used.
  - Slope-intercept form
    Can be described by the inequality $y \geq mx + b$, where m is the slope and b is the y-intercept and any equal or inequality symbol can be used.
  - Point-slope form
    Can be described by the inequality $y - y_1 > m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line and any equal or inequality symbol can be used.

Common Misconceptions or Errors

- Students may have difficulty translating word problems into systems of equations and inequalities.

- Students may shade the wrong half-plane for an inequality.
- Students may graph an incorrect boundary line (dashed versus solid) due to incorrect translation of the word problem.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation.

Strategies to Support Tiered Instruction

- Instruction provides opportunities to translate systems of equations or inequalities from word problems by first creating equations, then by identifying keywords to determine the inequality symbol (i.e., no more than, less than, at least, etc.). The appropriate inequality symbols can then replace the equal signs. Students can separate and organize information for each equation or inequality.
  - Separate given information for each equation or inequality.
  - Determine the appropriate form of equation or inequality based on givens.
  - Define a variable to represent the item wanted in the equation or inequality.
  - Determine what values are constants or should be placed with the variables.
  - Write the equation or inequality.
- Teacher co-creates a graphic organizer to scaffold graphing an inequality and its solution. The steps can be repeated for each inequality.

Symbol (s)	***Graph***
$<$ and $>$	Dashed line, Solution does not include points on the line
$\leq$ and $\geq$	Solid line, Solution includes points on the line

- Instruction includes opportunities to identify a test point to substitute into an inequality to determine which symbol should be used when writing the inequality. It is usually easiest to use the origin $(0,0)$ as it makes mental calculations easier. If the point selected creates a true statement, the half plane that includes the test point should be shaded. If it creates a false statement, the half plane that does not include the test point should be shaded.
- Teacher makes connections back to students' understanding of MA.912.AR.2.5 and MA.912.AR.3.8 and writing constraints based on a real-world context.
  - For example, Dani is planning her wedding, and the venue charges a flat rate of $8250 for four hours. The venue can provide meals for each of the guests and charges $21.25 per plate for adults and $13.75 per plate for children if she has a minimum of 75 guests. If Dani's budget is $38,000, students can describe this situation using the inequalities $a + c \geq 75$ and $21.25a + 13.75c + 8250 \leq 38000$. Depending on the number of adults and children she wants to invite and the capacity of the venue, students can determine various other constraints.
- Teacher provides questions to be answered by students to aid in the identification of domain and range restrictions:
  - Does the problem involve humans, animals or things that cannot or are normally not broken into parts? If yes, you are restricted by integers.
  - Do negative numbers not make sense? If yes, you are restricted by positive numbers.
  - Was a maximum or minimum value given? If yes, the solution must not exceed

the maximum or drop lower than the minimum.

- Instruction includes identifying which variable(s) the constraints apply to.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A baker has 16 eggs and 15 cups of flour. One batch of chocolate chip cookies requires 4 eggs and 3 cups of flour. One batch of oatmeal raisin cookies requires 2 eggs and 3 cups of flour. The baker makes $3 profit for each batch of chocolate chip cookies and $2 profit for each batch of oatmeal raisin cookies. How many batches of each cookie should she make to maximize profit?

Instructional Task 2 (MTR.4.1)

Amy and Anthony are starting a pet sitter business. To make sure they have enough time to properly care for the animals, they create a feeding and pampering plan. Anthony can spend up to 8 hours a day taking care of the feeding and cleaning, and Amy can spend up to 8 hours each day pampering the pets.

Feeding/Cleaning Time: Amy and Anthony estimate they need to allot 6 minutes twice a day, morning and evening, to feed and clean litter boxes for each cat, a total of 12 minutes a day per cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog.

Pampering Time: Sixteen minutes per day will be allotted for brushing and petting each cat and 20 minutes each day for bathing and playing with each dog.

Part A. Write an inequality for feeding/cleaning time needed for the pets. Represent all time in the same unit (minutes or hours).

Part B. Write an inequality for pampering time needed for the pets. Represent all time in the same unit (minutes or hours).

Part C. Graph the two inequalities.

Part D. In term of this scenario, explain the meanings of the following points: $(0,24)$ and $(30,0)$.

Part E. What is the greatest number of dogs they can watch if they are watching 19 cats?

Part F. List two viable combinations of pets that can be watched.

Possibility 1: _cats ___dogs
Possibility 2: _cats ___dogs

Instructional Items

Instructional Item 1

There are several elevators in the Sandy Beach Hotel. Each elevator can hold at most 12 people. Additionally, each elevator can only carry 1600 pounds of people and baggage for safety reasons. Assume on average an adult weighs 175 pounds and a child weighs 70 pounds. Also assume each group will have 150 pounds of baggage plus 10 additional pounds of personal items per person.

Part A. Write a system of linear equations or inequalities that describes the weight limit for one group of adults and children on a Sandy Beach Hotel elevator and that represents the total number of passengers in a Sandy Beach Hotel elevator.

Part B. Several groups of people want to share the same elevator. Group 1 has 4 adults and 3 children. Group 2 has 1 adult and 11 children. Group 3 has 9 adults. Which of the groups, if any, can safely travel in a Sandy Beach elevator?

Instructional Item 2

A farmer is planning to plant two types of crops, Crop X and Crop Y. The maximum amount of field space is 6 acres, and the maximum water supply is 10 units. Each acre of Crop X requires 1 unit of water, and each acre of Crop Y requires 2 units of water. Write a system of equations or inequalities that represents the situation.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Functions

MA.912.F.1 *Understand, compare and analyze properties of functions.*

MA.912.F.1.1

Benchmark

MA.912.F.1.1	**Given an equation or graph that defines a function, classify the function type. Given an input-output table, determine a function type that could represent it.**

Benchmark Clarifications:
Clarification 1: Within the Algebra I course, functions represented as tables are limited to linear, quadratic and exponential.
Clarification 2: Within the Algebra I course, functions represented as equations or graphs are limited to vertical or horizontal translations or reflections over the x-axis of the following parent functions:

$f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = \sqrt{x}$, $f(x) = \sqrt[3]{x}$, $f(x) = |x|$, $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2, MA.912.AR.3, MA.912.AR.4
- MA.912.DP.2.6

Terms from the K-12 Glossary

- Exponential Function
- Function
- Linear Function
- Quadratic Function

Vertical Alignment

Previous Benchmarks
- MA.8.F.1.1, MA.8.F.1.2, MA.8.F.1.3

Next Benchmarks
- MA.912.AR.5, MA.912.AR.6, MA.912.AR.7
- MA.912.AR.8
- MA.912.GR.7
- MA.912.T.2

Purpose and Instructional Strategies

In grade 8, students identified the domain and range of a relation and determined whether it is a function or not. In Algebra I-A, students classify function types limited to simple linear and absolute value. In Algebra I-B, students will continue to classify functions including quadratic, cubic, square root, cube root and exponential. In later courses, students will classify other function types.

- The purpose of this benchmark is to lay the groundwork for students to be able to choose appropriate functions to model real-world data.
- Instruction includes the connection of the graph to its parent function. See Clarification 1 for specifics of the Algebra I course.

- Students will work extensively with linear and absolute value models in the Algebra I-A course. Strong attention should be given to the other function types so that students can build familiarity with them. As new function types are introduced, take time to allow students to produce a rough graph of the parent function from a table of values they develop. Lead student discussion to build connections with why these function types produce their corresponding graphs (*MTR.4.1*).
- Instruction develops the understanding that if given a table of values, unless stated, one cannot absolutely determine the function type, but state which function the table of values could represent.
    - For example, if given the function $y = |x|$ and only positive values were given in a table, one could say that table of values could represent a linear or absolute value function.

Common Misconceptions or Errors

- Some students may miscalculate first and second differences that deal with negative values, especially if they perform them mentally.

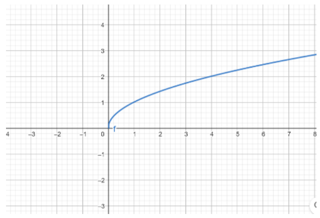
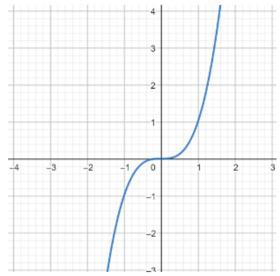
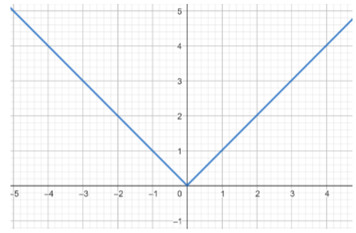
Strategies to Support Tiered Instruction

- Teacher provides opportunities to write out subtraction sentences next to each line of the table when determining first and second differences.
- Instructions are provided to determine the type of function the graph represents. Knowledge on the end behavior of different types of functions may provide students with additional information to identify different types of functions. The teacher co-creates an anchor chart showing different types of functions and their end behavior.
- Teacher provides methods for calculating and/or interpreting the first and second differences given a table of values.

Exponential			Linear			Quadratic			
−3	2	× 2	3	−20	+3	1	7		
−2	4	× 2	4	−17	+3	2	13	+6	
−1	8	× 2	5	−14	+3	3	23	+10	+4
0	16	× 2	6	−11	+3	4	37	+14	+4
1	32		7	−8		5	55	+18	+4

- Instruction includes opportunities to use graphing software to graph parent functions of different equations (i.e., square root, cubic, absolute value, etc.).

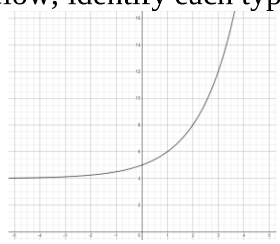
$y = \sqrt{x}$	$y = x^3$	$y = \lvert x \rvert$

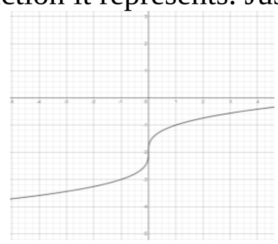
		

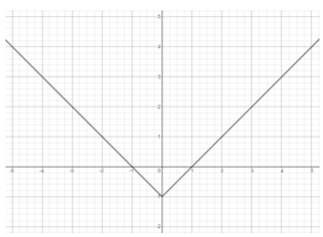
Instructional Tasks

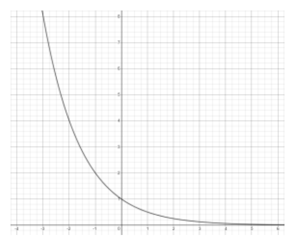
Instructional Task 1 (MTR.3.1)

Given the graphs below, identify each type of function it represents. Justify your answer.









Instructional Items

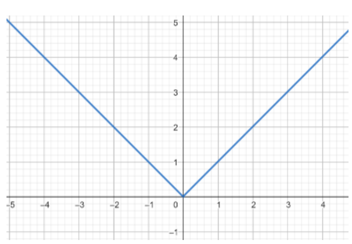
Instructional Item 1

Given the table below, determine the function type that could represent it.

x	6	8	10	12	14
y	-1.5	0	2.5	6	10.5

Instructional Item 2

Determine the function type of the graph below.



Instructional Item 3

Determine the function type of the equation, $f(x) = 5x + 2$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.2

Benchmark

MA.912.F.1.2	**Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.**

Benchmark Clarifications:

Clarification 1: Problems include simple functions in two-variables, such as $f(x, y) = 3x - 2y$.

Clarification 2: Within the Algebra I course, functions are limited to one-variable such as $f(x) = 3x$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2
- MA.912.AR.4.3

Terms from the K-12 Glossary

- Function Notation

Vertical Alignment

Previous Benchmarks

- MA.6.AR.1.2

Next Benchmarks

- MA.912.AR.3.8
- MA.912.AR.5.6
- MA.912.F.3

Purpose and Instructional Strategies

In middle grades, students worked with x-y notation and substituted values in expressions and equations. In Algebra I-A, students work with x-y notation and function notation throughout instruction of linear and absolute value functions. In Algebra I-B, students will focus on quadratic and exponential functions. In later courses, students will continue to use function notation with other function types and perform operations that combine functions, including compositions of functions.

- Instruction leads students to understand that $f(x)$ reads as "f of x" and represents an output of a function equivalent to that of the variable y in x-y notation.
- Instruction includes a series of functions with random inputs so that students can see the pattern that emerges (*MTR.5.1*).
  - For example,

$$f(x) = 2x^2 + 5x - 7$$
$$f(k) = 2k^2 + 5k - 7$$
$$f(-2) = 2(-2)^2 + 5(-2) - 7$$

- Students should discover that the number in parenthesis corresponds to the input or x-value on the graph and the number to the right of the equal sign corresponds to the output or y-value.
- Although not conventional, instruction includes using function notation flexibly.

- For example, function notation can be written as $h(x) = 4x + 7$ or $4x + 7 = h(x)$.
- Instruction leads students to consider the practicality that function notation presents to mathematicians. In several contexts, multiple functions can exist that we want to consider simultaneously. If each of these functions is written in x-y notation, it can lead to confusion in discussions.
  - For example, representing the equations, $y = -2x + 4$ and $y = 3x + 7$, in function notation allows mathematicians to distinguish them from each other more easily (i.e., $f(x) = -2x + 4$ and $g(x) = 3x + 7$).
- Function notation also allows for the use of different symbols for the variables, which can add meaning to the function.
  - For example, $h(t) = -16t^2 + 49t + 4$ could be used to represent the height, h, of a ball in feet over time, t, in seconds.
- Function notation allows mathematicians to express the output and input of a function simultaneously.
  - For example, $h(3) = 7$ would represent a ball 7 feet in the air after 3 seconds of elapsed time. This is equivalent to the ordered pair $(3, 7)$ but with the added
  - benefit of knowing which function it is associated with.

Common Misconceptions or Errors

- Throughout students' prior experience, two variables written next to one another indicate they are being multiplied. This changes in function notation and will likely cause confusion for some of your students.
- Students may need additional support in the order of operations.
  - For example, students may think that multiplication is always performed before division.

Strategies to Support Tiered Instruction

- Instruction includes discussing the meaning of function notation with students with understanding that $f(x)$ does not mean $f \cdot x$.
- Instruction is provided to determine the order of operations required once a given input is placed into the function for evaluation. Students may need additional support determining the correct order of operations to perform.
- Teacher models using parentheses to help organize order of operations when evaluating functions.
  - When evaluating $f(x) = 4x^2$ for $x = -1$, teacher can model the use of parentheses by writing the expression $4(-1)^2$ rather than without using parentheses writing $4 \cdot -1^2$. This will help students visualize the operations.
- Teacher provides instruction for identifying the operations in various functions as they relate to the order of operations using a graphic organizer.

Find $f(5)$	$f(x) = 4x^3 - 3.5x^2 + 10$
Substitute 5 for x.	$f(5) = 4(5)^3 - 3.5(5)^2 + 10$
Evaluate the exponents.	$f(5) = 4(125) - 3.5(25) + 10$

Multiply the factors within each term.	$f(5) = 500 - 87.5 + 10$
Perform addition and subtraction of terms from left to right.	$f(5) = 412.5 + 10$ $f(5) = 422.5$

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Jazlyn's grandmother started a savings account for Jazlyn. She deposited $250 into the account which pays a simple interest rate of 7% each year. The value of the account can be described by the function $V(t) = 250(1 + 0.07t)$, where t is the time in years and $V(t)$ is the value in the account.

Part A. Create a table of values that corresponds to this function.

Part B. Graph the function.

Instructional Items

Instructional Item 1

Evaluate $f(24)$, when $f(x) = \frac{3}{2}x + 9$.

Instructional Item 2

Given $f(x) = 2x^4 - 0.24x^2 + 6.17x - 7$, find $f(2)$.

Instructional Item 3

A shipping company charges a base fee of $20 plus an additional $5 for each package shipped. The total cost (C) of shipping p packages can be modeled by the function $C(p) = 20 + 5p$ where p is the number of packages.

You are a small business owner looking to send out 10 packages to customers. Evaluate the function C for $p = 10$ to find the total cost of shipping the packages.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.3

Benchmark

MA.912.F.1.3	**Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.**

Benchmark Clarifications:

Clarification 1: Instruction includes making the connection to determining the slope of a particular line segment.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.2
- MA.912.FL.3.4

Terms from the K-12 Glossary

- Rate of Change
- Slope (of a graph)

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.2
- MA.8.F.1.3

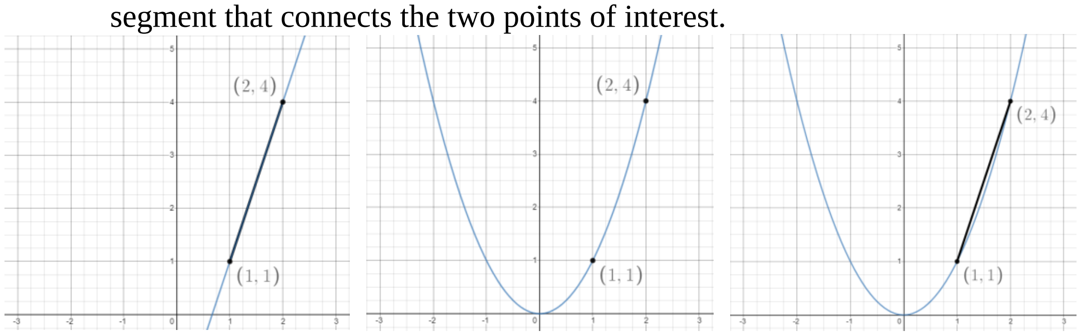
Next Benchmarks

- MA.912.F.1.4
- MA.912.C.3.8, MA.912.C.3.9, MA.912.C.3.10

Purpose and Instructional Strategies

In grade 8, students determined the slope, constant rate of change, of a linear equation in two variables and analyzed graphical representations of functional relationships. In Algebra I-A, students are introduced to the concept of the average rate of change. The instruction will focus on constant rate of change due to the linear focus of this course. In Algebra I-B, students calculate the average rate of change in real-world situations represented in various ways. In later courses, this concept leads to the difference quotient and differential calculus.

- The purpose of this benchmark is to extend students' understanding of rate of change to allow them to apply it in non-linear contexts.
- Instruction emphasizes a graphical context so students can see the meaning of the average rate of change. Students can use graphing technology to help visualize this.
    - Starting with the linear function $f(x) = 3x - 2$, shown below, ask students to calculate the rate of change between two points using the slope formula. Lead students to verify their calculations visually.
    - Once students have successfully used the formula, transition to the graph of $f(x) = x^2$.
    - Highlight the same two points and ask students to discuss what the rate of change might be between them (*MTR.4.1*). Lead students to realize that while there is not a constant rate of change, they can calculate an *average* rate of change for an interval. Show students that this is equivalent to calculating the slope of the line segment that connects the two points of interest.



    - Once students have an understanding, ask them to find the average rate of change for other intervals, such as $-2 \leq x \leq -1$ or $0 \leq x \leq 2$. As each of these calculations produce different values, reinforce the concept that non-linear functions do not have constant rates of change (*MTR.5.1*).
    - Look for opportunities to continue students' work with function notation. Ask students to find the average rate of change between $f(1)$ and $f(4)$ for $f(x)$.

Common Misconceptions or Errors

- Students may confuse the average rate of change and the constant rate of change.

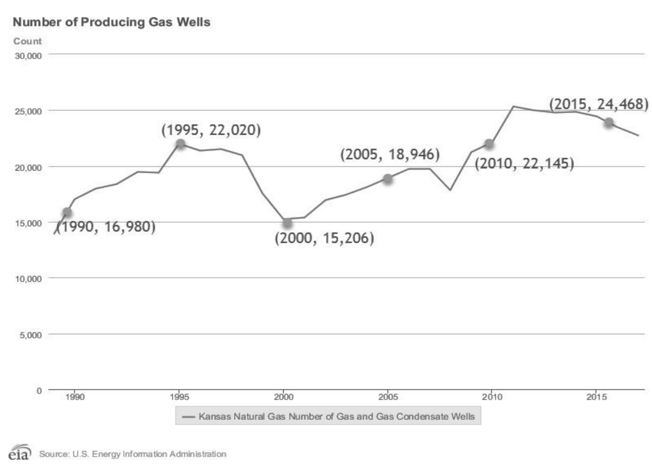
Strategies to Support Tiered Instruction

- When determining an average rate of change on an interval where the function only increases or only decreases, instruction includes directions to highlight the domain and range for the interval so that the change can be seen more clearly.
- Instruction includes providing the graph to visualize the change in the x- and y-values by drawing a line from the leftmost point on the graph within the interval to the rightmost point on the graph within the interval. Discuss with students how the average rate of change is the slope of the line between the two points.
    - For example, for the function $f(x) = x^2$, on the interval $[-2,2]$, the average rate of change is zero. This can be visualized by drawing a line from the point $(-2,4)$ to $(2,4)$ which has slope of zero (horizontal line).
- Instruction includes assistance recognizing the connection between slope (MA.912.AR.2) and rate of change for a linear function. A linear function has a constant rate of change. Regardless of the interval, the rate of change is the same (constant). For nonlinear functions rate of change is not constant, so it is not considered slope. However, the formula for slope can be used to calculate the average rate of change over an interval.
- Teacher provides instruction on assigning the ordered pairs when calculating average rate of change. The formula for slope is the change in y divided by the change in x. The designation of the interval points as the first or second pair of coordinates does not matter.
- Teacher co-creates an x-y chart to organize information when solving real-world problems involving a graphical representation. The x-column should be labeled with the input description used for the x-axis. The y-column should be labeled with the output description used for the y-axis.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR. 7.1)

The graph below represents the number of producing gas wells in Kansas from 1989 to 2017. Discuss with a partner whether you would use constant rate of change or average rate of change for the interval between 2000 and 2015?



Instructional Items

Instructional Item 1

A student is taking a science test. The table shows the number of questions they have remaining and the time that has passed.

Time (minutes)	Remaining questions
0	30
10	25
20	18
30	12
40	5
50	0

What is the average rate of change, to the nearest tenth, from 10 minutes to 40 minutes, and what does it mean?

A. On average, the student answered 0.7 questions per minute during the first 30 minutes.

B. On average, the student answered 1.5 questions per minute during the first 30 minutes.

C. On average, the student answered 0.7 questions per minute during those 30 minutes.

D. On average, the student answered 1.5 questions per minute during those 30 minutes.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.5

Benchmark

MA.912.F.1.5 **Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.**

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; slope and end behavior.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5

Terms from the K-12 Glossary

- Domain
- Intercept
- Range
- Slope

Vertical Alignment

Previous Benchmarks
- MA.8.AR.3.5

Next Benchmarks
- MA.912.F.1.7

Purpose and Instructional Strategies

In grade 8, students interpreted the slope and y-intercept of a linear equation in two variables. In Algebra I-A, students compare key features of two or more linear functions. In later courses, students will compare key features of linear and nonlinear functions.

- Instruction includes the use of various forms of linear equations. Additionally, linear functions can be represented as a table of values or graphically.
  - Standard Form
    Can be described by the equation $Ax + By = C$, where A, B and C are any rational number.
  - Slope-Intercept Form
    Can be described by the equation $y = mx + b$, where m is the slope and b is the y-intercept.
  - Point-Slope Form
    Can be described by the equation $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line.
- Problem types include comparing linear functions presented in similar forms and in different forms, and comparing more than two linear functions.
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the domain is all values of x less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."

Common Misconceptions or Errors

- When describing domain or range, students may assign their constraints to the incorrect variable.
- Students may miss the need for compound inequalities when describing domain or range.
- Students may confuse a negative rate of change as a lesser rate of change than a positive rate of change while comparing functions. For example, students may think that a slope of –3 is less than a slope of 2 due to comparing integers in prior grades.

Strategies to Support Tiered Instruction

- Teacher provides a laminated cue card to aid in the identification of domain and range restrictions:
  - Is the constraint on the independent or dependent variable in the context of the problem?
  - Does the constraint restrict the input or output value in the context of the problem?
  - Was the constraint shown or highlighted on the x- or y-axis?
- The use of a graph of the function to point out areas of constraints in a real-world context can help students understand the need for compound inequalities when describing the domain and range.
- Teacher provides a chart to show different terminology associated with domain and range.

Domain	***Range***
x-values	y-values
Input	Output

- Instruction includes opportunities to use a graphic organizer to chart and provide specific examples of domain and range as compound inequalities.

Compound Inequality	**Example(s)**	**Notation**
AND	Between 3 pm and 6 pm	$3 \leq x \leq 6$
OR	Under 10 years or at least 70 years	$x < 10 \text{ or } x \geq 70$

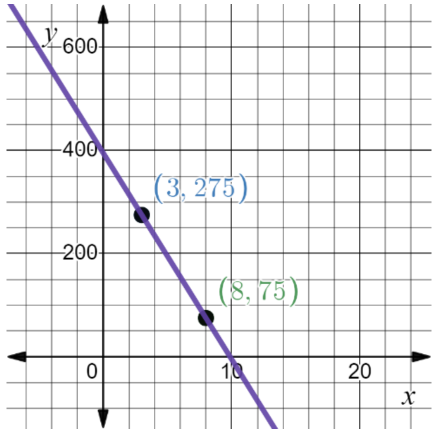
- Instruction includes clarification on using magnitude when comparing slopes of functions and integer comparisons for y-intercepts.
- Teacher provides a graphic organizer for each of the three forms of linear equations (standard, slope-intercept and point-slope form) that can be co-created to highlight key similarities and differences.

Teacher and student write the equation that shows this form chosen.	Teacher can ask student to describe different features of the selected form or ways this form is different from other forms. Teacher asks student what are some advantages or disadvantages of the selected form.
Teacher and student select one of the three forms and place name here.	
Teacher and student show several examples. Be sure to show examples where a coefficient of x or y is zero, negative or one.	Teacher and student write linear equations in other forms or in none of the forms.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.5.1)

Callie and her friend Elena are reading through different novels they checked out from their school library last Tuesday. Callie's progress through her novel can be modeled by the function $p(d) = -25d + 318$, where $p(d)$ represents the number of pages remaining to be read and d is the number of days since receiving the book. Elena's progress through her novel is modeled by the graph below.



Part A. Which student's novel has more pages to read?
Part B. Assuming they both continue to read at a constant rate, which student will complete their novel first?

Instructional Items

Instructional Item 1

Two linear functions, $f(x)$ and $g(x)$, are represented below. Compare the functions by stating which has a greater y-intercept, x-intercept and rate of change.

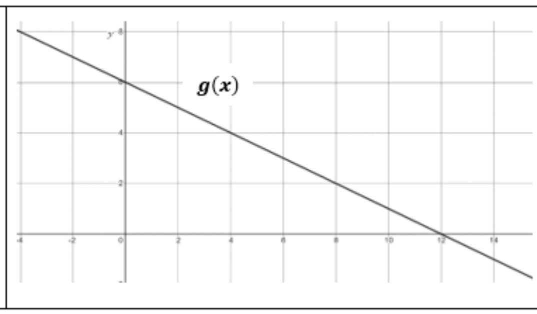
$$f(x) = 4x - 3$$

x	-3	-1	1	3
$g(x)$	13	8	3	-2

Instructional Item 2

Consider three linear functions, f(x), g(x) and h(x):

$f(x) = 2x + 3$



x	$h(x)$
−2	−10
−1	−6
0	−2
1	2
2	6

Compare the three linear functions and draw conclusions about their behavior and characteristics.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.8

Benchmark

MA.912.F.1.8 **Determine whether a linear, quadratic or exponential function best models a given real-world situation.**

Benchmark Clarifications:
Clarification 1: Instruction includes recognizing that linear functions model situations in which a quantity changes by a constant amount per unit interval; that quadratic functions model situations in which a quantity increases to a maximum, then begins to decrease or a quantity decreases to a minimum, then begins to increase; and that exponential functions model situations in which a quantity grows or decays by a constant percent per unit interval.
Clarification 2: Within this benchmark, the expectation is to identify the type of function from a written description or table.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.1, MA.912.DP.2.4

Terms from the K-12 Glossary

- Exponential Function
- Linear Function
- Quadratic Function

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.1

Next Benchmarks

- MA.912.DP.2.8
- MA.912.DP.2.9

Purpose and Instructional Strategies

In grade 8, students determined whether a linear relationship is also a proportional relationship. In Algebra I-A, students determine whether a linear or nonlinear function best models a situation. In Algebra I-B, students will determine whether a linear, quadratic or exponential function best models a situation. In later grades, students will fit linear, quadratic, and exponential functions to statistical data.

- Instruction should include identifying function types from tables and from written descriptions.
    - When examining written descriptions, guide students to see that linear functions model situations in which a quantity changes by a constant amount per unit interval.
- When considering tables, instruction guides students to understand that linear relationships have a common difference per unit interval (or a constant rate of change) (*MTR.5.1*).

    - Considering tables like the one below, lead students to discover that there is a common difference of 0.4 between successive y-values. Plotting these points using graphing software will verify that they are all points on the same line.

x	-4	-3	-2	-1	0
y	1.6	2.0	2.4	2.8	3.2
1st Difference	0.4	0.4	0.4	0.4	

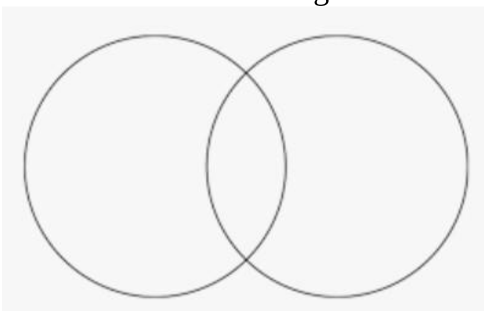
- Students should note that the search for common differences only works when the x-values are equidistant from each other. Lead them to check for this when presented with tables of values to consider.
- It is important to note that other function types could produce these relationships, making the connection to classifying different function types in MA.912.F.1.1.

Common Misconceptions or Errors

- Some students may miscalculate first differences that deal with negative values, especially if they perform them mentally.

Strategies to Support Tiered Instruction

- Instruction includes verifying an exponential relationship by looking for common ratios. When interpreting a written description, make a sample table of values from the context to examine the type of function.
- Teacher co-creates a graphic organizer to compare exponential and quadratic functions.
  - For example, a Venn Diagram can be used with the common middle section including the non-constant rate of change.



- Teacher provides opportunities to write out subtraction sentences next to each line of the table when determining first and second differences. Have students write out the subtraction expression [i.e., $-14 - (-2)$] so they can see that they are subtracting a negative value and should convert it to adding a positive value.
  - It is often helpful to have these students draw a blank number line with a mark for 0 to use for their calculations. Students who solve $-14 + 2$ to equal -16 could place their pencil tip to the left of 0 on the number line in a position that could represent -14. Ask them which direction they would move to represent adding 2. When students see movement to the right, toward zero, they should understand that the magnitude of the negative number decreases, resulting in -12 rather than -16.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A scientist is monitoring cell division and notes that a single cell divides into 4 cells within one hour. During the next hour, each of these cells divides into 4 cells. This process continues at the same rate every hour.

Part A. What type of function could be used to represent this situation?

Part B. Justify your reasoning.

Instructional Items

Instructional Item 1

Sarah is spending the summer at her grandmother's house. The table below shows the amount of money in her bank account at the end of each week. What type of function could be used to model the total amount of money in Sarah's bank account as a function of time?

Week #	Total $
1	$3428
2	$3276
3	$3124
4	$2972

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2 *Identify and describe the effects of transformations on functions. Create new functions given transformations.*

MA.912.F.2.1

Benchmark

MA.912.F.2.1	**Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.**

Benchmark Clarifications:

Clarification 1: Within the Algebra I course, functions are limited to linear, quadratic and absolute value.

Clarification 2: Instruction focuses on including positive and negative values for k.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4
- MA.912.AR.4.3
- MA.912. F.1.1

Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2

Next Benchmarks

- MA.912.AR.3.7
- MA.912.GR.2

Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I-A,

students identify the effects of single transformations on linear and absolute value functions.

In Algebra I-B students will continue the learning from the previous course and identify the effects of single transformations on quadratic functions.

In Geometry, students will perform multiple transformations on two-dimensional figures. In later courses, students will work with transformations of many types of functions.

- In this benchmark, students will examine the impact of transformations on linear and absolute value functions. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
    - Have students use graphing technology to explore different parent functions.
        - In each graph, toggle on/off the graphs for $f(x) + k, kf(x)$, $f(kx)$ and $f(x + k)$ to examine their impacts on the function. Use the slider to change the value of k (be sure to examine the impacts when k is positive and negative).
        - As students explore, prompt discussion (*MTR.4.1*) among them about the patterns they see as they adjust the slider (*MTR.5.1*).
- For $f(x) + k$, students should discover that k is being added to the output of the function (equivalent to the y-value) and will therefore result in a *vertical translation* of the function by k units.
    - Ask students to describe what values of k cause the graph to shift up. Which values cause it to shift down?
- For $kf(x)$, students should discover that k is being multiplied by the output of the function (equivalent to the y-value) and will therefore result in a *vertical dilation* (stretch/compression) of the function by a factor of k.
    - Ask students to describe what values of k cause the graph to stretch up vertically. Which values cause it to compress? Which values for k cause the graph to reflect over the x-axis? What is the significance of $k = -1$?
- For $f(x + k)$, students should discover that k is being added to the input of the function and will therefore result in a *horizontal translation* of the function by $-k$ units.
    - Ask students to describe what values of k cause the graph to shift left. Which values cause it to shift right?
- For $f(kx)$, students should discover that k is being multiplied by the input of the function and will therefore result in a *horizontal dilation* (stretch/compression) of the function by a factor of k.
    - Ask students to describe what values of k cause the graph to stretch horizontally. Which values cause it to compress? Which values for k cause the graph to reflect over the y-axis? What is the significance of $k = -1$?
- After students have a good understanding of the impact of $f(x) + k, kf(x), f(kx)$ and $f(x + k)$ on graphs of functions, connect that knowledge to tables of values for a function.
    - For $f(x) + k$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 4$. Guide students to form a table and discuss its connection to the vertical translation observed on the graph.

x	$f(x)$	$f(x) + 4$
1	6	10
2	3	7
3	2	6
4	3	7

- For $kf(x)$, use graphing technology to display a graph of an absolute value function (like the one below) and set $k = 0.5$. Guide students to form a table and discuss its connection to the vertical compression observed on the graph.

x	$f(x)$	$0.5[f(x)]$
1	4	2
2	3	1.5
3	2	1
4	3	1.5

- For $f(x + k)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 2$. Guide students to form a table and discuss its connection to the horizontal translation observed on the graph using the highlighted values. For the table shown, consider $x = 5$. For $f(x)$, $f(5) = 6$. But for $g(x) = f(x + 2)$, $g(5) = f(5 + 2)$ which is equivalent to 18, which is equivalent to shifting $f(7)$ two units to the left on the graph. Bridge this conversation with a graph of the two functions to help them understand the connection.

x	$f(x)$
1	6
2	3
3	2
4	3
5	6
6	11

→

$x + 2$	$g(x) = f(x + 2)$
3	2
4	3
5	6
6	11
7	18
8	27

- For $f(kx)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 3$. Guide students to form a table and discuss its connection to the horizontal compression observed on the graph using the highlighted values.

x	$f(x)$
1	6
2	3
3	2
4	3
5	6
6	11
7	18
8	27
9	38

→

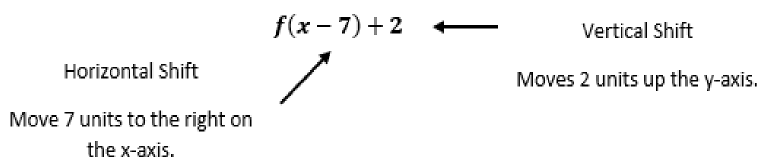
$3x$	$f(3x)$
3	2
6	11
9	38
12	83
15	146
18	258
21	326
24	443
27	578

Common Misconceptions or Errors

- Similar to writing functions in vertex form, students may confuse effect of the sign of k in $f(x+k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of $kf(x)$ and $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same.

Strategies to Support Tiered Instruction

- Instruction includes explaining to students that horizontal shifts are “inside” of the function. Additionally, the teacher provides instruction to ensure understanding that the movement of the function is opposite of the sign that effects the horizontal shift.
    - For example, teacher can provide the identification of the type of transformation and its effects to the below function.



- Teacher provides instruction that includes the use of a graph that displays stretch and compression (shrink) scaling. Including a visual representation will allow students to categorize their thinking.
    - For example, have students copy the graphs into their notebooks. Give students an opportunity to identify changes in both types of transformations before giving students the transformations.
    - Teachers can also introduce the effects of $kf(x)$ and $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Instruction includes providing a grid with a parent function and horizontal and vertical stretch on one grid, using different colors to distinguish both types of stretches (vertical and horizontal).
- Instruction includes directing students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Instruction includes having a non-zero y-intercept to visualize the difference between scaling in the horizontal direction, $f(kx)$, and scaling in the vertical direction, $kf(x)$.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Part A. Given the function $f(x) = x^2$, determine the vertex, domain and range.

Part B. If the function $f(x)$ is translated to the right 6 units, predict what may happen to the vertex, domain and range.

Part C. How does the graph of the function $f(x) = x^2 - 7$, compare to the graph of the function in Part A?

Instructional Items

Instructional Item 1

How does the graph of $g(x) = f(x - 2)$ compare to the graph of $f(x) = |x + 3|$?

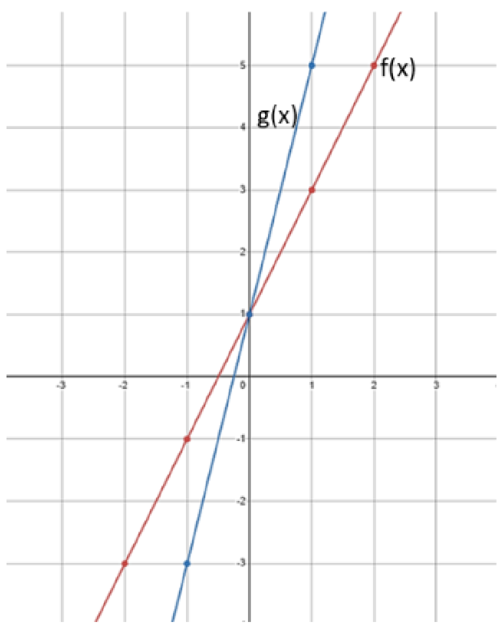
Instructional Item 2

Describe the effect of the transformation $f(x) + 2$ on the function table below.

x	$f(x)$
-2	4
0	0
2	4
4	16
6	36

Instructional Item 3

The graph of two functions is shown below.



Which of the following describes the effect on f(x) after a transformation was performed to produce g(x)?

a. The graph of g(x) is reflected over the x-axis of the graph f(x)

b. The graph of g(x) is a vertical shift of 2 from the graph f(x)
c. The graph of g(x) is a horizontal shift of 1/2 from the graph of f(x)
d. The graph of g(x) is a horizontal stretch by a factor of 2 from the graph of f(x)

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Financial Literacy

MA.912.FL.3 *Describe the advantages and disadvantages of short-term and long-term purchases.*

MA.912.FL.3.2

Benchmark

MA.912.FL.3.2 **Solve real-world problems involving simple, compound and continuously compounded interest.**

Example: Find the amount of money on deposit at the end of 5 years if you started with $500 and it was compounded quarterly at 6% interest per year.

Example: Joe won $25,000 on a lottery scratch-off ticket. How many years will it take at 6% interest compounded yearly for his money to double?

Benchmark Clarifications:

Clarification 1: Within the Algebra I course, interest is limited to simple and compound.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.5

Terms from the K-12 Glossary

- Simple Interest

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.1

Next Benchmarks

- MA.912.NSO.1.1
- MA.912.AR.5.3
- MA.912.FL.3.3, MA.912.FL.3.4

Purpose and Instructional Strategies

In grade 7, students solved problems involving simple interest. In Algebra I-A, students solve problems involving simple interest, using arithmetic operations, and graphing. In later courses, students will solve compound interest problems using arithmetic operations and graphing to determine lengths of time, including those that require the use of logarithms, and solve continuously compounded interest problems.

- In this benchmark, students will solve simple interest in real-world context.
- Instruction compares the differences between simple interest formula and the Final Amounts under Simple Interest Formula.
  - The Simple Interest Formula ($I = prt$) calculates *only the interest* earned over

time. Each year's interest is calculated from the initial principal, not the total value of the investment of that point in time.
  - The Final Amounts under Simple Interest Formula, $A = P(1 + rt)$, calculates the *total value* of an investment over time.

Common Misconceptions or Errors

- Some problems related to this standard may ask students for the interest earned over a period of time while others may ask for the account balance or total value of the investment over a period of time. Some students may miss this distinction and may always calculate total interest for simple interest problems.
- Students may confuse the frequencies of interest being compounded.
- When forming interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before value before substituting it into the formula.

Strategies to Support Tiered Instruction

- Teacher provides a highlighter to identify if a question is asking for the interest or the total amount. Point students back to the highlighted portions of the problem and help them assess the reasonableness of their answers *(MTR.6.1)* in context.
- Instruction provides a graphic organizer to identify the important information in a problem.
  - For example, given a simple interest problem, students could complete the following table.

Principal (*P*)	Interest (*I*)	Rate (*r*)	Time (*t*)	Total Value

- For students who need extra support in converting a percentage to a decimal, instruction includes students thinking about percent as "per one-hundred."
  - For example, when writing 8% as a decimal, ask "8% is how many per 100?"

  Then write 8% = $\frac{8}{100}$, which is equivalent to 0.08.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Gwen deposits $800 in a savings account in which she earns simple annual interest. After 18 months, she earns $64.80. What is the interest rate for her account?

Instructional Items

Instruction Item 1

Beatrice deposits $525 in an account that pays 4.3% simple annual interest. If she keeps the money in the account for 12 years, how much interest will she earn?

Instructional Item 2

You deposit $500 in a savings account that pays 2.2% compound annual interest. Find your account balance after 15 months.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.4

Benchmark

MA.912.FL.3.4	**Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.**

Benchmark Clarifications:
Clarification 1: Within the Algebra I course; exponential growth is limited to compound interest.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.5
- MA.912.F.1.8

Terms from the K-12 Glossary

- Simple Interest

Vertical Alignment

Previous Benchmarks
- MA.7.AR.3.1

Next Benchmarks
- MA.912.F.1.6
- MA.912.FL.3.1, MA.912.FL.3.3

Purpose and Instructional Strategies

In grade 7, students solved problems involving simple interest. In Algebra I-A, students explain the relationship between simple interest and linear growth. In later courses, students will explain the relationship between compound interest and exponential growth. Students will extend this to include continuously compounded interest.

- In MA.912.FL.3.2, students became familiar with simple interest and how to use the formulas to solve real-world problems. In this benchmark, students will make connections between simple interest and linear growth. To help students discover this relationship, consider guiding them to form a table.
    - For example, Kianna received $1,000 cash from graduation gifts from family and friends. She decided to invest her money in an

investment account. Kianna's investment earns 10% in *simple* interest.
Guide students to create the interest formula below and use it to create the table below to visualize the growth of Kianna's investment over time.

- Kianna's Interest Earned would be represented by $A = 1000 \cdot 0.1 \cdot t$.
- Kianna's Total Value would be represented by $A = 1000(1 + 0.1t)$.

Years Invested	Kianna's Interest Earned ($)	Total Value of Kianna's Investment ($)
1	100	1,100
2	200	1,200
3	300	1,300
4	400	1,400
5	500	1,500
10	1,000	2,000
15	1,500	2,500
20	2,000	3,000
30	3,000	4,000
50	5,000	6,000

- Once completed, ask students what relationship they observe in the behavior of Kianna's investment. Students should discover Kianna's investment exhibits linear growth.
- Solidify this understanding by having students graph the function that represents the total value of Kianna's investment.
- In Algebra I-B, students will compare the relationship between linear and exponential growth and determine which type of interest would be more advantageous for long-term investments.
  - Remember the expectation for this benchmark is for students to explain *why* this relationship occurs. Be sure to discuss the equation formed and that the variation of years is used as a factor in the simple interest formula.

Common Misconceptions or Errors

- When forming compound interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

Strategies to Support Tiered Instruction

- Instruction includes making the connection to determining linear functions (MA.912. F.1.8) from a financial context.
- For students who need extra support in converting a percentage to a decimal, instruction includes students thinking about percent as "per one-hundred."
    - For example, when writing 8% as a decimal, ask "8% is how many per 100?" Then write $8\% = \frac{8}{100}$ which is equivalent to 0.08.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1)

Phoenix invests in a savings account that applies simple interest.

Part A. How will her investment grow? Justify your answer.

Part B. If Phoenix invests $725 and earns an annual rate of 4.2%, write an equation that would represent the total amount she would have at the end of each year.

Part C. How long will it take for her initial investment to double?

Instructional Items

Instructional Item 1

Trevarius invests in a savings account that applies simple interest. Will this investment grow linearly? Justify your answer.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Data Analysis & Probability

MA.912.DP.1 *Summarize, represent and interpret categorical and numerical data with one and two variables.*

MA.912.DP.1.3

Benchmark

MA.912.DP.1.3 **Explain the difference between correlation and causation in the contexts of both numerical and categorical data.**

Algebra I Example: There is a strong positive correlation between the number of Nobel prizes won by country and the per capita chocolate consumption by country. Does this mean that increased chocolate consumption in America will increase the United States of America's chances of a Nobel prize winner?

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.2.4, MA.912.DP.2.6
- MA.912.DP.3.1

Terms from the K-12 Glossary

- Bivariate Data
- Categorical Data

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1

Next Benchmarks

- MA.912.DP.2.5
- MA.912.DP.3.1, MA.912.DP.3.2, MA.912.DP.3.3
- MA.912.DP.5

Purpose and Instructional Strategies

In grade 8, students first analyzed bivariate numerical data using scatter plots. In Algebra I, students study association between variables in bivariate data and learn that there is a difference between two variables being strongly associated and one of them having a causative effect on the other. In later courses, students will learn how to design statistical experiments that can show causation.

- The intent of this benchmark includes the ability to informally draw conclusions about whether causation is justified when two variables are correlated.
- Correlation and causation are often misunderstood. It is important for students to understand their relationship. Causation and correlation can exist at the same time;

however, correlation does not imply causation. Causation explicitly applies to cases where an action causes an outcome. Correlation is simply a relationship observed in bivariate data. One action may relate to the other, but that action doesn't necessarily cause the other to happen, because both of them may be the result of a third "hidden variable."

- Causation is possible, but it is also possible that correlation occurs from a third variable.
  - For example, if one states, "On days when I drink coffee, I feel more productive." it may be that one feels more productive because of the caffeine (causation) or because they spent time in the coffee shop drinking coffee where there are fewer distractions (third variable). Since one cannot determine whether the causation or the third variable results in correlation, then causation is not confirmed.
- Causation seems unlikely and a third variable seems likely.
  - For example, there is a strong correlation between the number of Nobel prizes won by country and the per capita chocolate consumption by country. However, there are many possibilities a third variable, such as a strong economy, that can result in this correlation so causation can be ruled out.
- Causation is likely because there is a reasonable explanation for the causation.
  - For example, if one states, "After I exercise, I feel physically exhausted." it is reasonable to consider this to be a cause-and-effect. Causation can be confirmed by the explanation that because one is purposefully pushing their body to physical exhaustion when doing exercise, the muscles used to exercise are exhausted (effect) after they exercise (cause).
- When correlation is apparent in a bivariate data set, students are encouraged to seek a reasonable explanation that either identifies a hidden variable or a reasonable explanation for causation. Further investigation may be required to confirm or disconfirm causation.

- In Algebra I, the term correlation is used to describe an association between two variables and does not necessarily imply a linear relationship.
- Instruction includes asking the following questions while students investigate correlation and causation.
  - Does this correlation make sense? Is there a direct connection between these variables?
  - Will the correlation hold if I look at some new data that I haven't used in my current analysis?
  - Is the relationship between these variables direct, or are they both a result of some other variable?

Common Misconceptions or Errors

- Even though students may not be able to reasonably explain why a causal relationship exists, they may assume that correlation implies causation.

Strategies to Support Tiered Instruction

- Instruction includes co-creating and discussing examples and non-examples of causal relationships in numerical and categorical data.
  - For example, a non-causal relationship could be a person's shoe size and approximate number of vocabulary words they know.
  - For example, a causal relationship could be a person's shoe size and their age.
- Teacher provides instruction to increase understanding of the relationship between correlation and causation. Teacher provides students with context that demonstrates when both correlation and causation are present. They may also provide context when only correlation is represented in the given context.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Data from a certain city shows that the size of an individual's home is positively correlated with the individual's life expectancy. Which of the following factors would best explain why this correlation does *not* necessarily imply that the size of an individual's home is the main cause of increased life expectancy?

a. Larger homes have more safety features and amenities, which lead to increased life expectancy.
b. The ability to afford a larger home and better healthcare is a direct effect of having more wealth.
c. The citizens were not selected at random for the study.
d. There are more people living in small homes than large homes in the city. Some responses may have been lost during the data collection process.

Instructional Items

Instructional Item 1

Dr. Larry has noticed that when he carries around his lucky rock, his students seem to be nicer to him. Can one conclude that this positive correlation shows a causal relationship?

a. Yes, because Larry decides whether or not to put his lucky rock in his pocket before he encounters people during the day.
b. Yes, because it is not a negative correlation.
c. No, because lucky rocks only work for children.
d. No, because it is possible that people are nice to Larry because of another factor that also causes him to put the rock in his pocket.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive*

MA.912.DP.2 *Solve problems involving univariate and bivariate numerical data.*

MA.912.DP.2.4

Benchmark

MA.912.DP.2.4	**Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.**

Benchmark Clarifications:
Clarification 1: Instruction includes fitting a linear function both informally and formally with the use of technology.
Clarification 2: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.3

Terms from the K-12 Glossary

- Bivariate Data
- Line of Fit
- Scatter Plot

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1

Next Benchmarks

- MA.912.DP.1.1, MA.912.DP.1.2
- MA.912.DP.2.7, MA.912.DP.2.8, MA.912.DP.2.9

Purpose and Instructional Strategies

In grade 8, students first worked with scatter plots and lines of fit. In Algebra I, students relate the slope and y-intercept of a line of fit to association in bivariate numerical data and interpret these features in real-world contexts. In later courses, students use the correlation coefficient to measure how well a line fits the data in a scatter plot, and they also work with scatter plots that suggest quadratic and exponential models.

- This is an extension of MA.912.DP.1.1, where students are working with numerical bivariate data (scatter plots and line graphs). It is good to review with students that a scatter plot is a display of numerical data sets between two variables.
    - They are good for showing a relationship or association between two variables.

    - They can reveal trends, shape of trend or strength of relationship trend.
    - They are useful for highlighting outliers and understanding the distribution of data.
    - One variable could be the progression of time, like in a line graph.
- In this benchmark, students are fitting a linear function to numerical bivariate data, interpreting the slope and y-intercept based on the context and using that linear function to make predictions about values that correspond to parts of the graph that lie beyond or within the scatter plot.
    - Predictions made outside of the range of data are called extrapolation and predictions made inside the range of data are called interpolation.
- Instruction includes the use of technology for students to understand the difference between a line of fit and a line of best fit. Additionally, instruction of this benchmark should be combined with MA.912.DP.2.6 and MA.912.DP.2.5, as these are extensions of this benchmark.
- During instruction is important to distinguish the difference between a “line of fit” and the “line of best fit.”
    - A “line of fit” is used when students are visually investigating numerical bivariate data that appears to have a linear relationship and can sketch a line (using a writing instrument and straightedge) that appears to “fit” the data. Using this “line of fit” students can *estimate* its slope and y-intercept and use that information to interpret the context of the data.
    - The “line of best fit” (also referred to as a “trend line”) is used when the data is further analyzed using linear regression calculations (the process of minimizing the squared distances from the individual data values to the line), often done with the assistance of technology.

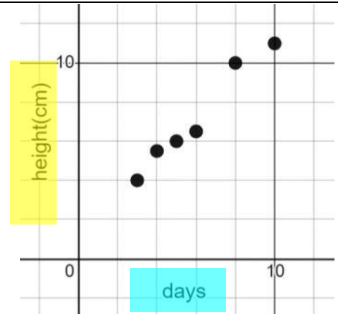
Common Misconceptions or Errors

- Students may not know how to sketch a line of fit.
    - For example, they may always go through the first and last points of data.
- Students may confuse the two variables when interpreting the data as related to the context.
- Students may not know the difference between interpolation (predictions within a data set) and extrapolation (predictions beyond a data set).

Strategies to Support Tiered Instruction

- Teacher provides sketched lines of fit and has students identify the one that best models the data.
- Teacher provides a sentence frame for interpreting the data in the context of the problem using two different colored highlighters to highlight the same variable in the sentence frame and table or graph.

- Example:

<table>
<tr><th>Sentence frame</th><th>Table</th><th>Graph</th></tr>
<tr>
<td>A line of fit can be used to estimate how much the height of the plant increases [__] centimeters when the number of days increases by [__] days.</td>
<td>
<table>
<tr><th>Days</th><th>Height (cm)</th></tr>
<tr><td>3</td><td>4</td></tr>
<tr><td>4</td><td>5.5</td></tr>
<tr><td>5</td><td>6</td></tr>
<tr><td>6</td><td>6.5</td></tr>
<tr><td>8</td><td>10</td></tr>
<tr><td>10</td><td>11</td></tr>
</table>
</td>
<td>
</td>
</tr>
</table>

- Teacher models creating a scatterplot on a piece of graph paper, then has students place a piece of spaghetti on the scatterplot to model the line of best fit. Students could also use a coordinate plane peg board to plot each point creating a scatterplot and then use a rubber band to model the line of best fit.
- Instruction includes vocabulary development by co-creating a graphic organizer for interpolation and extrapolation.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1, MTR.6.1)

Crickets are one of nature's more interesting insects, partly because of their musical ability. In England, the chirping or singing of a cricket was once considered to be a sign of good luck. Crickets will not chirp if the temperature is below 40 degrees Fahrenheit (°F) or above 100 degrees Fahrenheit (°F). A table is given with some data collected.

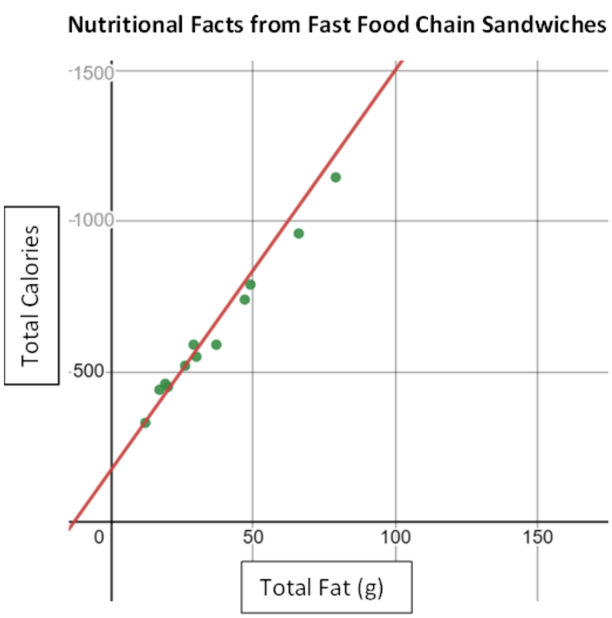
Average Number of Chirps (per minute)	*Temperature (°F)*
45	*40°*
60	*47°*
75	*50°*
80	*45°*
95	*55°*
110	*50°*
125	*60°*
140	*55°*
140	*80°*
150	*65°*
165	*70°*
180	*65°*
185	*70°*

Part A. Create a line of fit based on the data. Compare your line of fit with a partner. Part B. What is the estimated slope and y-intercept of the line?
Part C. What does the slope mean in terms of the context?
Part D. What does the y-intercept mean in terms of the context?
Part E. Using technology, determine the line of best fit. Compare this to the line of fit determined from Part A. What is the difference?
Part F. Based on this line, predict the temperature to be if you recorded 250 chirps per minute?
Part G. Based on this line, estimate the number of chirps per minute at exactly 50°F.

Instructional Items

Instructional Item 1

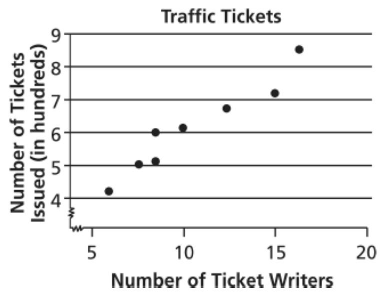
Data was collected from a variety of fast-food chains on their sandwiches and is represented on the scatter plot below. The equation $y = 13.3x + 174$ models the line of fit.



Part A. What do the slope and y-intercept tell us about the relationship of total fat and total calories in these fast-food items?
Part B. If a fast-food item has 10 grams of fat, estimate the total calories of that item.

Instructional Item 2

A police department tracked the number of ticket writers and number of tickets issued for each of the past 8 weeks. The scatter plot shows the results.



Part A. Create an equation for the line of fit based on the data.
Part B. What does the slope y-intercept mean in terms of the context?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.2.6

Benchmark

MA.912.DP.2.6 **Given a scatter plot that represents bivariate numerical data, assess the fit of a given linear function by plotting and analyzing residuals.**

Benchmark Clarifications:
Clarification 1: Within the Algebra I course, instruction includes determining the number of positive and negative residuals; the largest and smallest residuals; and the connection between outliers in the data set and the corresponding residuals.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.3

Terms from the K-12 Glossary

- Bivariate Data
- Line of Fit
- Scatter Plot

Vertical Alignment

Previous Benchmarks
- MA.8.DP.1

Next Benchmarks
- MA.912.DP.1.1
- MA.912.DP.2.8, MA.912.DP.2.9
- MA.912.DP.3.1

Purpose and Instructional Strategies

In grade 8, students informally fitted a line to a scatter plot. In Algebra I, students use the slope and residuals of a line of fit to determine the strength and direction of the correlation. In later courses, students will use the slope and residuals to more quantitatively determine the strength of the correlation.

- A residual is a measure of how well a line predicts an individual data point. It can be illustrated by the vertical distance between a data point and the regression line. Each data point has one residual given by the equation $R = D - P$, where R represents the residual, D represents the y-coordinate of data value and P represents the y-coordinate of predicted value. Residuals are positive if data points are above the regression line and negative if data points are below the regression line. If the regression line actually passes through the point, the residual at that point is zero.
    - The slope of the line and the residuals determine the sign and strength of the correlation. A line with a positive slope indicates a positive correlation. A line with a negative slope indicates a negative correlation. Residuals with smaller absolute values indicate stronger correlations. Residuals with larger absolute values indicate weaker correlations.
    - If the slope is close to 0, then the correlation may be considered weak even when the residuals are all small. A slope near zero indicates that the independent variable has little effect on the dependent variable.
- Instruction focuses on real-world contexts and includes the use of technology.
- A residual plot has the residual values on the vertical axis; the horizontal axis displays the x-variable.
- Instruction includes providing residual values, then students can determine the number of positive and negative residuals and the largest and smallest residuals.
- Outliers, which are observed data points that are far from a line of fit, can be determined from the residual as points whose residuals have a large absolute value.

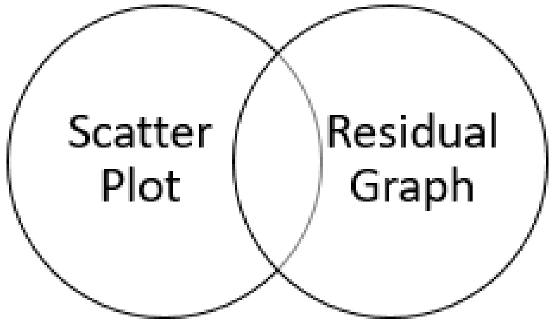
Common Misconceptions or Errors

- Students may not be able to distinguish between a scatter plot and a residual graph.
- Students may forget that residual graphs consist of the ordered pair: independent, residual.
- Students may not be able to determine an appropriate model (linear/nonlinear) from a residual graph.
- Students may think that a correlation is strong when the residuals are small, and the slope is close to zero.

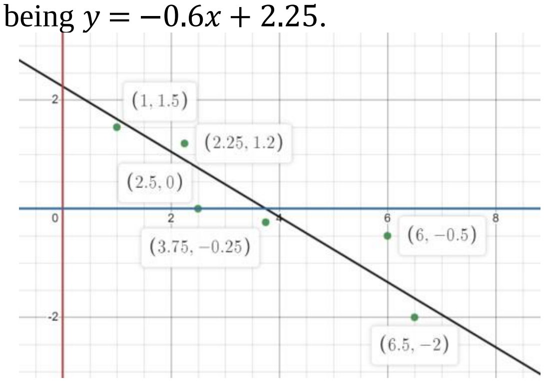
Strategies to Support Tiered Instruction

- Teacher co-creates an anchor chart that provides examples of residual graphs for linear models.
- Instruction includes vocabulary development by co-creating a graphic organizer for

scatter plot and residual graph.



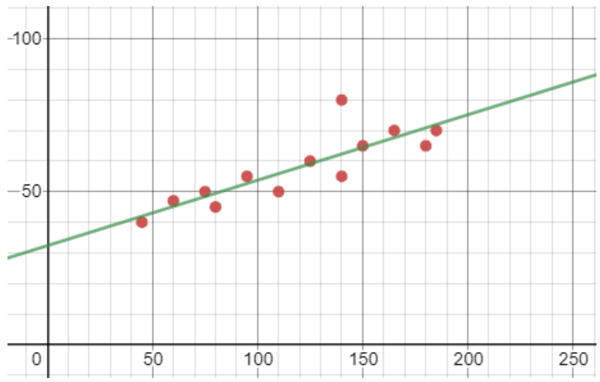
- Instruction includes the opportunity to use colors to identify the x- and y-values in the original data and the x- and y-values in the ordered pairs associated with the residual graph, and highlight the x- and y-axis of the residual graph in the same color in order to see the relationship.
  - For example, for the data point $(6, -0.5)$, the residual point is $(6, 0.85)$ based on the line fit being $y = -0.6x + 2.25$.



Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.7.1)

Crickets are one of nature's more interesting insects, partly because of their musical ability. In England, the chirping or singing of a cricket was once considered to be a sign of good luck. Crickets will not chirp if the temperature is below 40 degrees Fahrenheit (°F) or above 100 degrees Fahrenheit (°F). A scatter plot is shown with the line of best fit, which can be described by the model $y = 0.214x + 32.317$.



The residuals (r) based on the scatter plot are shown.

Average Number of Chirps (per minute)	*Temperature (°F)*	*Residual*
45	*40°*	*-1.95*
60	*47°*	*1.84*
75	*50°*	*1.62*
80	*45°*	*-4.45*
95	*55°*	*2.34*
110	*50°*	*-5.87*
125	*60°*	*0.92*
140	*55°*	*-7.29*
140	*80°*	*17.7*
150	*65°*	*0.57*
165	*70°*	*2.35*
180	*65°*	*-5.86*
185	*70°*	*-1.93*

Part A. Determine if the data has a positive or negative correlation. Part B. Determine the strength of the correlation.
Part C. Compare your answers from Part A and B with a partner.
Part D. Do you notice any possible outliers? Do they affect the judgment of the strength of correlation from Part B?

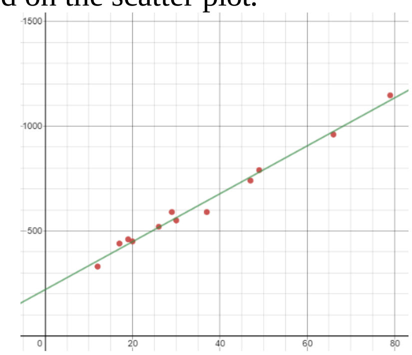
Instructional Items

Instructional Item 1

Based on the data, we know that the line of best fit for the relationship between fat grams and the total calories in fast food using the given data below can be represented by $y = 11.44x + 219.89$. The residuals of this data have been calculated and are represented in the last column.

Sandwich	Total Fat (g)	Total Calories	Residuals
Double Cheeseburger	26	520	2.810
Cheeseburger (1/4 pound)	30	550	-12.93
Cheeseburger (1/3 pound)	47	740	-17.32
Bacon Burger	79	1147	23.78
Bacon Cheeseburger (1/4 pound)	66	960	-14.57
Bacon Cheeseburger (1/3 pound)	49	790	9.814
Hamburger	37	590	-52.97
Fried Chicken Sandwich	29	590	38.51
Grilled Chicken Sandwich	17	440	25.72
Spicy Grilled Chicken Sandwich	19	460	22.85
Roast Beef and Cheese Sandwich	20	450	1.42
Tuna Melt	12	330	-27.11

All of this data is represented on the scatter plot.



Based on this scatter plot and the residuals, interpret the strength and direction of the correlation of the line of fit as it relates to total grams of fat and total calories of fast food.

The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.