Mathematical Thinking and Reasoning Standards Coaching Tool

MTR: *Because Math Matters*

The purpose of this document is to provide educators and leaders with a tool and resources to support their pedagogical knowledge in an effort to ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous and extended learning and apply mathematical concepts to real-world applications.

The state of Florida has defined the Mathematical Thinking and Reasoning Standards (MTRs) as instructional practices, or strategies, which should be authentically and appropriately seen in the classroom on a daily basis. The B.E.S.T. Standards for Mathematics should be learned and taught through the lens of the MTRs. In doing so, educators will be able to help students make their learning visible and be able to reach all learners through the MTRs.

The desired outcome of this document is to provide educators with a resource for implementing the MTRs into the classroom. This document will describe specific moves the teacher and student should be doing as aligned to each of the MTRs. Additionally, this document will provide a sample "Look-for Tool" that educators may use when observing the classroom to provide coaching feedback on the implementation of the MTRs.

Student and Teacher Moves Aligned to the Mathematical Thinking and Reasoning Standards (MTRs)

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction keeping in mind the benchmark(s) that are the focal point of the lesson or task. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. • Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* • Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. • Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance.	• Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. • Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. • Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. • Teacher provides appropriate time for student processing, productive struggle and reflection. • Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. • Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Students represent problems concretely using objects, models and manipulatives. • Students represent problems pictorially using drawings, models, tables and graphs. • Students represent problems abstractly using numerical or algebraic expressions and equations. • Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)*	• Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* • Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. • Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* • Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* • Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology.
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Students complete tasks with flexibility, efficiency and accuracy. • Students use feedback from peers and teachers to reflect on and revise methods used. • Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)*	• Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* • Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. • Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. • Teacher offers multiple opportunities to practice generalizable methods.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Students use content specific language to communicate and justify mathematical ideas and chosen methods. • Students use discussions and reflections to recognize errors and revise their thinking. • Students use discussions to analyze the mathematical thinking of others. • Students identify errors within their own work and then determine possible reasons and potential corrections. • When working in small groups, students recognize errors of their peers and offers suggestions.	• Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* • Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. • Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. • Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)*
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Students identify relevant details in a problem in order to create plans and decompose problems into manageable parts. • Students find similarities and common structures, or patterns, between problems in order to solve related and more complex problems using prior knowledge.	• Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)* • Teacher provides students opportunities to connect prior and current understanding to new concepts. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* • Teacher allows students to develop an appropriate sequence of steps in solving problems. • Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. • Students monitor calculations, procedures and intermediate results during the process of solving problems. • Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* • Students reflect on the accuracy of their estimations and their solutions.	• Teacher provides opportunities for students to estimate or predict solutions prior to solving. • Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* • Teacher prompts students to self-monitor by continually asking, "Does this solution or intermediate result make sense? How do you know?" • Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)*
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Students connect mathematical concepts to everyday experiences. • Students use mathematical models and methods to understand, represent and solve real-world problems. • Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Students re-design models and methods to improve accuracy or efficiency.	• Teacher provides real-world context to help students build understanding of abstract mathematical ideas. • Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. • Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Teacher provides opportunities for students to apply concepts to other content areas.

Mathematical Thinking and Reasoning Standards Look-for Tool

Teacher Name: _____ Course: _____ Date: _____

This document is intended to provide school leaders and teachers with look-fors in the form of guiding questions to support mathematics learning and instruction focused around the Mathematical Thinking and Reasoning Standards (MTRs). It is important to note that not all MTRs will be evident in every lesson and MTRs should be embedded as appropriate for the learning and teaching of the benchmark concepts.

	Are students demonstrating progression towards mastery of benchmark concepts by…	Evident?	Notes
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	viewing mistakes as an opportunity to learn?		
	using and modifying strategies or methods?		
	persevering through challenging tasks?		
	engaging in opportunities for collaboration?		
	providing and using effective feedback?		
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	making connections between concepts and various concrete, visual and abstract representations?		
	using models and manipulatives?		
	showing that various representations are appropriate in different contexts?		
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	demonstrating flexibility while solving problems?		
	engaging in opportunities to practice efficiency and accuracy?		
	analyzing the efficiency of a method for a given situation?		

	Are students demonstrating progression towards mastery of benchmark concepts by…	Evident?	Notes
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	engaging in opportunities to discuss their thinking with the teacher and their peers?		
	viewing error analysis as an opportunity for learning?		
	developing their ability to justify their thinking and compare their responses with others?		
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	recognizing patterns and connecting those patterns to mathematical concepts?		
	engaging in opportunities to develop generalizations based on similarities found among problems?		
	engaging in opportunities to create plans and procedures to solve problems?		
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	estimating and predicting solutions prior to solving?		
	making sense of their solutions and their methods to achieve their solutions?		
	checking their work within and after a task to verify the reasonableness of their solution?		
	justifying their solutions?		
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	creating concrete, visual and abstract models?		
	verifying the accuracy of their models and methods by using context?		
	applying various concepts to other content areas?		