## Fluency with Arithmetic Operations and Automaticity with Basic Arithmetic Facts

Throughout this document, benchmark expectations regarding arithmetic operations within the Number Sense and Operations (NSO) strand have been developed with a hierarchy in mind consisting of three stages: exploration, procedural reliability and procedural fluency. Students will first explore arithmetic operations with no fluency expectations, then will be able to show procedural reliability and finally they will carry out these operations with procedural fluency. Interwoven into this hierarchy is the development of direct recall of basic arithmetic facts. Basic arithmetic facts are first derived, then utilized while becoming procedurally reliable or fluent and finally recalled with automaticity. Refer to Appendix B: Proficiency and Procedural Fluency Chart.

## Stage 1: Exploration

The expectation is to develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings. An example of an "exploration" benchmark is shown below.

MA.1.NSO.2.4
Explore the addition of a two-digit number and a one-digit number with sums to 100 .
Benchmark Clarifications:
Clarification 1: Instruction focuses on combining ones and tens and composing new tens from ones, when needed.
Clarification 2: Instruction includes the use of manipulatives, number lines, drawings or models.

## Stage 2: Procedural reliability

The expectation is to utilize skills from the exploration stage to develop an accurate, reliable method that aligns with the student's understanding and learning style. Students may need the teacher's help to choose a method, and they will learn how to use a method without help. An example of a "procedural reliability" benchmark is shown below.

Add two whole numbers with sums up to 100 with procedural reliability.
MA.2.NSO.2.3 Subtract a whole number from a whole number, each no larger than 100, with procedural reliability.
Example: The sum $41+23$ can be found by using a number line and "jumping up" by two tens and then by three ones to "land" at 64 .
Example: The difference $87-25$ can be found by subtracting 20 from 80 to get 60 and then 5 from 7 to get 2 . Then add 60 and 2 to obtain 62 .
Benchmark Clarifications:
Clarification 1: Instruction focuses on helping a student choose a method they can use reliably.

## Stage 3: Procedural fluency

The expectation is to utilize skills from the procedural reliability stage to become fluent with an efficient and accurate procedure, including a standard algorithm. An example of a "procedural fluency" benchmark is shown below.

Add and subtract multi-digit whole numbers including using a standard MA.3.NSO.2.1 algorithm with procedural fluency.

## Embedded within Stage 1 - Stage 3: Automaticity

The expectation is to directly recall basic arithmetic facts from memory. Automaticity is the ability to act according to an automatic response which is easily retrieved from long-term memory. It usually results from repetition and practice. An example of an "automaticity" benchmark is shown below.

Recall addition facts with sums up to 20 and related subtraction facts with MA.2.NSO.2.1 automaticity.

## Fluency and Recall with Automaticity throughout Grade Levels

The purpose of this table is to provide educators with an overview of procedural fluencies and recall with automaticity within number sense and operations and measurement from Kindergarten to Grade 8. This crosswalk should not drive instruction or curriculum. Please refer to your specific course description that can be found on CPALMS.

| Grade <br> Level | Required Procedural Reliability, Procedural Fluency and Basic Fact Automaticity |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Number Sense: Counting and Place Value | Operations: Addition and Subtraction | Operations: <br> Multiplication and Division | Measurement |
| K | Recite numbers to 100 by ones and tens <br> Count backward within 20 <br> Locate, order and compare whole numbers up to 20 | Procedural Reliability: <br> Two one-digit whole numbers with sums from 0 to 10 and related subtraction facts |  |  |
| 1 | Count forward and backward within 120 by ones <br> Skip count by 2 s to 20 and by 5 s to 100 . <br> Plot, order and compare whole numbers up to 100 | Recall: <br> Two whole numbers with sums from 0 to 10 and related subtraction facts <br> Procedural Reliability: Two whole numbers with sums from 0 to 20 and related subtraction facts |  | Length of an object to the nearest inch or centimeter |
| 2 | Round whole numbers from 0 to 100 to the nearest 10 <br> Plot, order and compare whole numbers up to 1,000 | Recall: <br> Two whole numbers with sums from 0 to 20 and related subtraction facts <br> Procedural Reliability: Two whole numbers with sums up to 100 and subtract a whole number from a whole number, each no larger than 100 |  | Length of an object to the nearest inch, foot, yard, centimeter or meter |


|  | Required Procedural Reliability, Procedural Fluency and Basic Fact Automaticity |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade Level | Number Sense: Counting and Place Value | Operations: Addition and Subtraction | Operations: Multiplication and Division | Measurement |
| 3 | Round whole numbers from 0 to 1,000 to the nearest 10 or 100 <br> Plot, order and compare: <br> - whole numbers up to 10,000 <br> - fractional numbers with the same numerator or the same denominator | Procedural Fluency: Multi-digit whole numbers, including using a standard algorithm | Procedural Reliability: Multiplication of a onedigit whole number by a multiple of 10 up to 90 or a multiple of 100 up to 900 <br> Procedural Reliability: Two whole numbers with factors from 0 to 12 and related division facts | Length of an object to the nearest centimeter and half or quarter inch <br> Volume of a liquid within a beaker to the nearest milliliter and half or quarter cup <br> Temperature to the nearest degree |
| 4 | Round whole numbers from 0 to 10,000 to the nearest 10,100 or 1,000 . <br> Plot, order and compare: <br> - multi-digit whole numbers up to 1,000,000 <br> - decimals up to the hundredths <br> - fractions with different numerators and different denominators, including mixed numbers and fractions greater than 1 | Procedural Reliability: Two fractions with like denominators, including mixed numbers and fractions greater than 1 | Recall: <br> Two whole numbers with factors up to 12 and related division facts <br> Procedural Reliability: Multiplication of a whole number up to three digits by a whole number up to two digits <br> Procedural Fluency: Multiplication of a twodigit whole number by a two-digit whole number, including using a standard algorithm <br> Procedural Reliability: Division of a whole number up to four digits by a one-digit whole number | Length of an object <br> Volume of a liquid within a beaker <br> Weight of an object <br> Mass of an object <br> Temperature of an object |


| Grade <br> Level | Required Procedural Reliability, Procedural Fluency and Basic Fact Automaticity |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Number Sense: Counting and Place Value | Operations: Addition and Subtraction | Operations: <br> Multiplication and Division | Measurement |
| 5 | Round multi-digit numbers with decimals to the nearest hundredth, tenth or whole number <br> Plot, order and compare multi-digit numbers with decimals up to the thousandths | Procedural Fluency: Multi-digit numbers with decimals to the thousandths, including using a standard algorithm <br> Procedural Reliability: Fractions with unlike denominators, including mixed numbers and fractions greater than 1 | Procedural Fluency: Multiplication of multidigit whole numbers, including using a standard algorithm <br> Procedural Fluency: Division of a whole number up to five digits by two digits, including using a standard algorithm <br> Procedural Reliability: Multiply a multi-digit number with decimals to the tenths by one-tenth or by one-hundredth <br> Procedural Reliability: Multiplication of a fraction by a fraction, including mixed numbers and fractions greater than 1 |  |
| 6 | Plot, order and compare rational numbers | Procedural Fluency: <br> Positive multi-digit decimals, including using a standard algorithm <br> Procedural Fluency: <br> Positive fractions, including mixed numbers and fractions greater than 1 <br> Procedural Fluency: Integers |  |  |
| 7 |  | Procedur Rational | l Fluency: numbers |  |
| 8 | Plot, order and compare rational and irrational numbers | Procedur Numbers expressed <br> Procedur <br> Laws of | al Fluency: <br> in scientific notation <br> al Fluency: <br> Exponents |  |

