

# Geometry

# **Version Description**

In Geometry, instructional time will emphasize five areas:

- (1) proving and applying relationships and theorems involving two-dimensional figures using Euclidean geometry and coordinate geometry;
- (2) establishing congruence and similarity using criteria from Euclidean geometry and using rigid transformations;
- (3) extending knowledge of geometric measurement to two-dimensional figures and three-dimensional figures;
- (4) creating and applying equations of circles in the coordinate plane and
- (5) developing an understanding of right triangle trigonometry.

Curricular content for all subjects must integrate critical-thinking, problem-solving, and workforce-literacy skills; communication, reading, and writing skills; mathematics skills; collaboration skills; contextual and applied-learning skills; technology-literacy skills; information and media-literacy skills; and civic-engagement skills.

All clarifications stated, whether general or specific to Geometry, are expectations for instruction of that benchmark.

# **General Notes**

Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards: This course includes Florida's B.E.S.T. ELA Expectations (EE) and Mathematical Thinking and Reasoning Standards (MTRs) for students. Florida educators should intentionally embed these standards within the content and their instruction as applicable. For guidance on the implementation of the EEs and MTRs, please visit <u>https://www.cpalms.org/Standards/BEST\_Standards.aspx</u> and select the appropriate B.E.S.T. Standards package.

English Language Development ELD Standards Special Notes Section: Teachers are required to provide listening, speaking, reading and writing instruction that allows English language learners (ELL) to communicate information, ideas and concepts for academic success in the content area of Mathematics. For the given level of English language proficiency and with visual, graphic, or interactive support, students will interact with grade level words, expressions, sentences and discourse to process or produce language necessary for academic success. The ELD standard should specify a relevant content area concept or topic of study chosen by curriculum developers and teachers which maximizes an ELL's need for communication and social skills. To access an ELL supporting document which delineates performance definitions and descriptors, please click on the following link:

https://cpalmsmediaprod.blob.core.windows.net/uploads/docs/standards/eld/ma.pdf.

# **General Information**

Course Number: 1206310	Course Type: Core Academic Course
Course Length: Year (Y)	Course Level: 2
Course Attributes: Class Size Core Required	Grade Level(s): 9, 10, 11, 12
Graduation Requirement: Geometry	Number of Credits: One (1) credit
Course Path: Section   Grades PreK to 12 Education Courses > Grade Group   Grades 9 to 12	
and Adult Education Courses > Subject   Mathematics > SubSubject   Geometry >	
Abbreviated Title   GEO	
Educator Certification: Mathematics (Grades 6-12) or	
Middle Grades Mathematics (Middle Grades 5-9)	

# **Course Standards and Benchmarks**

# Mathematical Thinking and Reasoning

# MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

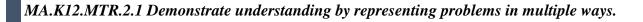
Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

#### **Clarifications:**

Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.



Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

#### Clarifications:

Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

# MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

#### Clarifications:

Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

# MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

#### Clarifications:

Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

# MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

#### Clarifications:

Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

# MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

#### **Clarifications:**

Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

# MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

#### Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

#### **ELA Expectations**

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

ELA.K12.EE.3.1 Make inferences to support comprehension.

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

# **English Language Development**

# ELD.K12.ELL.MA Language of Mathematics

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

### **Geometric Reasoning**

MA.912.GR.1 Prove and apply geometric theorems to solve problems.

Prove relationships and theorems about lines and angles. Solve mathematical MA.912.GR.1.1 and real-world problems involving postulates, relationships and theorems of lines and angles.

#### Benchmark Clarifications:

*Clarification 1:* Postulates, relationships and theorems include vertical angles are congruent; when a transversal crosses parallel lines, the consecutive angles are supplementary and alternate (interior and exterior) angles and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

*Clarification 2:* Instruction includes constructing two-column proofs, pictorial proofs, paragraph and narrative proofs, flow chart proofs or informal proofs.

*Clarification 3:* Instruction focuses on helping a student choose a method they can use reliably.

MA.912.GR.1.2 Prove triangle congruence or similarity using Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes constructing two-column proofs, pictorial proofs, paragraph and narrative proofs, flow chart proofs or informal proofs.

Clarification 2: Instruction focuses on helping a student choose a method they can use reliably.

MA.912.GR.1.3 Prove relationships and theorems about triangles. Solve mathematical and realworld problems involving postulates, relationships and theorems of triangles.

### Benchmark Clarifications:

*Clarification 1:* Postulates, relationships and theorems include measures of interior angles of a triangle sum to 180°; measures of a set of exterior angles of a triangle sum to 360°; triangle inequality theorem; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

*Clarification 2:* Instruction includes constructing two-column proofs, pictorial proofs, paragraph and narrative proofs, flow chart proofs or informal proofs.

*Clarification 3:* Instruction focuses on helping a student choose a method they can use reliably.

Prove relationships and theorems about parallelograms. Solve mathematical MA.912.GR.1.4 and real-world problems involving postulates, relationships and theorems of parallelograms.

### Benchmark Clarifications:

*Clarification 1:* Postulates, relationships and theorems include opposite sides are congruent, consecutive angles are supplementary, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.

*Clarification 2:* Instruction includes constructing two-column proofs, pictorial proofs, paragraph and narrative proofs, flow chart proofs or informal proofs.

*Clarification 3:* Instruction focuses on helping a student choose a method they can use reliably.

Prove relationships and theorems about trapezoids. Solve mathematical and MA.912.GR.1.5 real-world problems involving postulates, relationships and theorems of trapezoids.

### Benchmark Clarifications:

*Clarification 1:* Postulates, relationships and theorems include the Trapezoid Midsegment Theorem and for isosceles trapezoids: base angles are congruent, opposite angles are supplementary and diagonals are congruent.

*Clarification 2:* Instruction includes constructing two-column proofs, pictorial proofs, paragraph and narrative proofs, flow chart proofs or informal proofs.

*Clarification 3:* Instruction focuses on helping a student choose a method they can use reliably.

MA.912.GR.1.6 Solve mathematical and real-world problems involving congruence or similarity in two-dimensional figures.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes demonstrating that two-dimensional figures are congruent or similar based on given information.



MA.912.GR.2.1 Given a preimage and image, describe the transformation and represent the transformation algebraically using coordinates.

*Example:* Given a triangle whose vertices have the coordinates (-3, 4), (2, 1.7) and (-0.4, -3). If this triangle is reflected across the y-axis the transformation can be described using coordinates as  $(x, y) \rightarrow (-x, y)$  resulting in the image whose vertices have the coordinates (3, 4), (-2, 1.7) and (0.4, -3).

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes the connection of transformations to functions that take points in the plane as inputs and give other points in the plane as outputs.

*Clarification 2:* Transformations include translations, dilations, rotations and reflections described using words or using coordinates.

*Clarification 3:* Within the Geometry course, rotations are limited to 90°, 180° and 270° counterclockwise or clockwise about the center of rotation, and the centers of rotations and dilations are limited to the origin or a point on the figure.

MA.912.GR.2.2 Identify transformations that do or do not preserve distance.

#### Benchmark Clarifications:

*Clarification 1:* Transformations include translations, dilations, rotations and reflections described using words or using coordinates.

Clarification 2: Instruction includes recognizing that these transformations preserve angle measure.

MA.912.GR.2.3 Identify a sequence of transformations that will map a given figure onto itself or onto another congruent or similar figure.

#### Benchmark Clarifications:

*Clarification 1:* Transformations include translations, dilations, rotations and reflections described using words or using coordinates.

*Clarification 2:* Within the Geometry course, figures are limited to triangles and quadrilaterals and rotations are limited to 90°, 180° and 270° counterclockwise or clockwise about the center of rotation. *Clarification 3:* Instruction includes the understanding that when a figure is mapped onto itself using a reflection, it occurs over a line of symmetry.

MA.912.GR.2.5 Given a geometric figure and a sequence of transformations, draw the transformed figure on a coordinate plane.

#### Benchmark Clarifications:

*Clarification 1:* Transformations include translations, dilations, rotations and reflections described using words or using coordinates.

Clarification 2: Instruction includes two or more transformations.

# MA.912.GR.2.6 Apply rigid transformations to map one figure onto another to justify that the two figures are congruent.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes showing that the corresponding sides and the corresponding angles are congruent.

MA.912.GR.2.8 Apply an appropriate transformation to map one figure onto another to justify that the two figures are similar.

### Benchmark Clarifications:

*Clarification 1:* Instruction includes showing that the corresponding sides are proportional, and the corresponding angles are congruent.

MA.912.GR.3 Use coordinate geometry to solve problems or prove relationships.

MA.912.GR.3.1 Determine the weighted average of two or more points on a line.

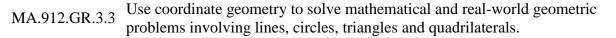
# Benchmark Clarifications:

*Clarification 1:* Instruction includes using a number line and determining how changing the weights moves the weighted average of points on the number line.

MA.912.GR.3.2	Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.
	<i>Example:</i> Given Triangle ABC has vertices located at $(-2, 2)$ , $(3, 3)$ and $(1, -3)$ , respectively, classify the type of triangle ABC is based on its angle measures and side lengths.
	<i>Example:</i> If a square has a diagonal with vertices $(-1, 1)$ and $(-4, -3)$ , find the coordinate values of the vertices of the other diagonal and show that the two diagonals are perpendicular.

Benchmark Clarifications:

*Clarification 1:* Instruction includes using the distance or midpoint formulas and knowledge of slope to classify or justify definitions, properties and theorems.



*Example:* The line x + 2y = 10 is tangent to a circle whose center is located at (2, -1). Find the tangent point and a second tangent point of a line with the same slope as the given line.

*Example:* Given M(-4, 7) and N(12, -1), find the coordinates of point P on  $\overline{MN}$  so that P partitions  $\overline{MN}$  in the ratio 2:3.

Benchmark Clarifications:

*Clarification 1:* Problems involving lines include the coordinates of a point on a line segment including the midpoint.

*Clarification 2:* Problems involving circles include determining points on a given circle and finding tangent lines.

Clarification 3: Problems involving triangles include median and centroid.

Clarification 4: Problems involving quadrilaterals include using parallel and perpendicular slope criteria.

# MA.912.GR.3.4 Use coordinate geometry to solve mathematical and real-world problems on the coordinate plane involving perimeter or area of polygons.

*Example:* A new community garden has four corners. Starting at the first corner and working counterclockwise, the second corner is 200 feet east, the third corner is 150 feet north of the second corner and the fourth corner is 100 feet west of the third corner. Represent the garden in the coordinate plane, and determine how much fence is needed for the perimeter of the garden and determine the total area of the garden.

# MA.912.GR.4 Use geometric measurement and dimensions to solve problems.

MA.912.GR.4.1 Identify the shapes of two-dimensional cross-sections of three-dimensional figures.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes the use of manipulatives and models to visualize cross-sections. *Clarification 2:* Instruction focuses on cross-sections of right cylinders, right prisms, right pyramids and right cones that are parallel or perpendicular to the base.

MA.912.GR.4.2 Identify three-dimensional objects generated by rotations of two-dimensional figures.

#### Benchmark Clarifications:

*Clarification 1:* The axis of rotation must be within the same plane but outside of the given twodimensional figure.

	Extend previous understanding of scale drawings and scale factors to	
MA.912.GR.4.3	determine how dilations affect the area of two-dimensional figures and the surface area or volume of three-dimensional figures.	
	<i>Example:</i> Mike is having a graduation party and wants to make sure he has enough pizza. Which option would provide more pizza for his guests: one 12-inch pizza or three 6-inch pizzas?	
MA.912.GR.4.4	Solve mathematical and real-world problems involving the area of two- dimensional figures.	
	<i>Example:</i> A town has 23 city blocks, each of which has dimensions of 1 quarter mile by 1 quarter mile, and there are 4500 people in the town. What is the population density of the town?	
Benchmark Clarifications: Clarification 1: Instruction includes concepts of population density based on area.		
MA.912.GR.4.5	Solve mathematical and real-world problems involving the volume of three- dimensional figures limited to cylinders, pyramids, prisms, cones and spheres.	
	<i>Example:</i> A cylindrical swimming pool is filled with water and has a diameter of 10 feet and height of 4 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank to the nearest pound?	
Benchmark Clarifications:		
<i>Clarification 1:</i> Instruction includes concepts of density based on volume. <i>Clarification 2:</i> Instruction includes using Cavalieri's Principle to give informal arguments about the formulas for the volumes of right and non-right cylinders, pyramids, prisms and cones.		
MA.912.GR.4.6	Solve mathematical and real-world problems involving the surface area of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres.	
MA.912.GR.5 Make formal geometric constructions with a variety of tools and methods.		
MA.912.GR.5.1	Construct a copy of a segment or an angle.	

<u>Benchmark Clarifications:</u> *Clarification 1:* Instruction includes using compass and straightedge, string, reflective devices, paper folding or dynamic geometric software.

MA.912.GR.5.2 Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes using compass and straightedge, string, reflective devices, paper folding or dynamic geometric software.

MA.912.GR.5.3 Construct the inscribed and circumscribed circles of a triangle.

### Benchmark Clarifications:

*Clarification 1:* Instruction includes using compass and straightedge, string, reflective devices, paper folding or dynamic geometric software.

MA.912.GR.6 Use properties and theorems related to circles.

MA.912.GR.6.1 Solve mathematical and real-world problems involving the length of a secant, tangent, segment or chord in a given circle.

### Benchmark Clarifications:

*Clarification 1:* Problems include relationships between two chords; two secants; a secant and a tangent; and the length of the tangent from a point to a circle.

MA.912.GR.6.2 Solve mathematical and real-world problems involving the measures of arcs and related angles.

# Benchmark Clarifications:

*Clarification 1:* Within the Geometry course, problems are limited to relationships between inscribed angles; central angles; and angles formed by the following intersections: a tangent and a secant through the center, two tangents, and a chord and its perpendicular bisector.

MA.912.GR.6.3 Solve mathematical problems involving triangles and quadrilaterals inscribed in a circle.

# Benchmark Clarifications:

*Clarification 1:* Instruction includes cases in which a triangle inscribed in a circle has a side that is the diameter.

MA.912.GR.6.4 Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle.

#### Benchmark Clarifications:

*Clarification 1:* Instruction focuses on the conceptual understanding that for a given angle measure the length of the intercepted arc is proportional to the radius, and for a given radius the length of the intercepted arc is proportional is the angle measure.



# MA.912.GR.7 Apply geometric and algebraic representations of conic sections.

MA.912.GR.7.2 Given a mathematical or real-world context, derive and create the equation of a circle using key features.

#### Benchmark Clarifications:

*Clarification 1:* Instruction includes using the Pythagorean Theorem and completing the square. *Clarification 2:* Within the Geometry course, key features are limited to the radius, diameter and the center.

MA.912.GR.7.3 Graph and solve mathematical and real-world problems that are modeled with an equation of a circle. Determine and interpret key features in terms of the context.

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain, range, eccentricity, center and radius. *Clarification 2:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

*Clarification 3:* Within the Geometry course, notations for domain and range are limited to inequality and set-builder.

# Trigonometry

MA.912.T.1 Define and use trigonometric ratios, identities or functions to solve problems.

MA.912.T.1.1 Define trigonometric ratios for acute angles in right triangles.

# Benchmark Clarifications:

*Clarification 1:* Instruction includes using the Pythagorean Theorem and using similar triangles to demonstrate that trigonometric ratios stay the same for similar right triangles.

*Clarification 2:* Within the Geometry course, instruction includes using the coordinate plane to make connections to the unit circle.

Clarification 3: Within the Geometry course, trigonometric ratios are limited to sine, cosine and tangent.

MA.912.T.1.2 Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem.

# Benchmark Clarifications:

*Clarification 1:* Instruction includes procedural fluency with the relationships of side lengths in special right triangles having angle measures of 30°-60°-90° and 45°-45°-90°.



# **Logic and Discrete Theory**

# MA.912.LT.4 Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof.

MA.912.LT.4.3 Identify and accurately interpret "if…then," "if and only if," "all" and "not" statements. Find the converse, inverse and contrapositive of a statement.

#### Benchmark Clarifications:

*Clarification 1:* Instruction focuses on recognizing the relationships between an "if...then" statement and the converse, inverse and contrapositive of that statement.

*Clarification 2:* Within the Geometry course, instruction focuses on the connection to proofs within the course.

MA.912.LT.4.10 Judge the validity of arguments and give counterexamples to disprove statements.

#### Benchmark Clarifications:

*Clarification 1:* Within the Geometry course, instruction focuses on the connection to proofs within the course.