Name ______________________________ Date ______________

Geometric and Numerical Patterns

Example 1 **Find the pattern. Then write the next two numbers.**

a. 2, 5, 8, 11, . . .

b. 2, 6, 18, 54, . . .

a. Use a table to organize the terms and find the next two numbers.

Position	1	2	3	4	5	6
Term	2	5	8	11	14	17
		+3	+3	+3	+3	+3

Each term is 3 more than the previous term.

Add 3 to a term to find the next term.

▶ The next two numbers are 14 and 17.

b. Use a table to organize the terms and find the next two numbers.

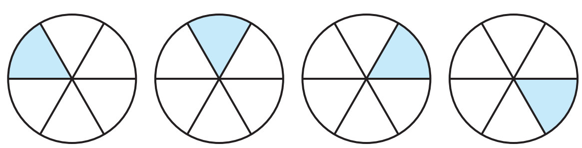
Position	1	2	3	4	5	6
Term	2	6	18	54	162	486
		×3	×3	×3	×3	×3

Each term is 3 times the previous term.

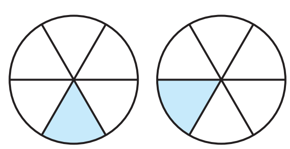
Multiply a term by 3 to find the next term.

▶ The next two numbers are 162 and 486.

Example 2 **Find the pattern. Then draw the next two figures in the sequence.**



The shaded region moves one region clockwise in each figure. The next two figures in the sequence are shown below.



Practice

Check your answers at BigIdeasMath.com.

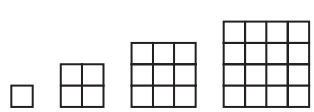
Find the pattern. Then write the next two numbers.

1. 7, 14, 28, 56, . . .

2. 4, 9, 14, 19, . . .

3. 4, −2, −8, −14, . . .

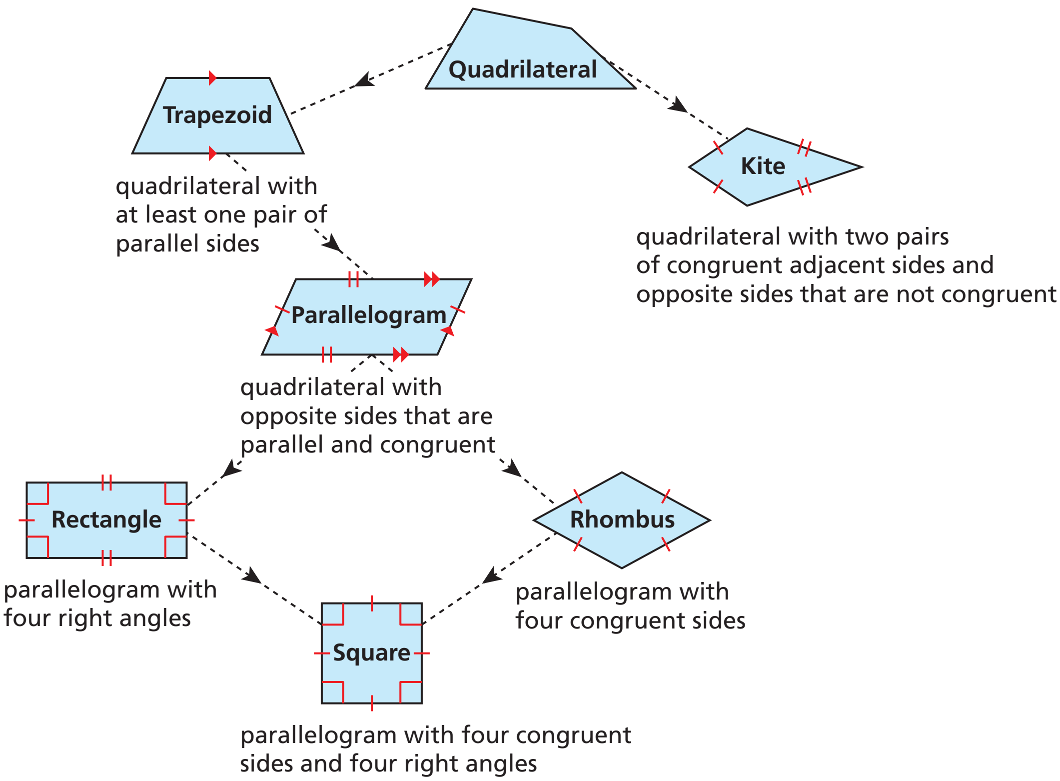
4. **PATTERN** Find the pattern. Then draw the next two figures in the sequence.



Name ____________________ Date ____________

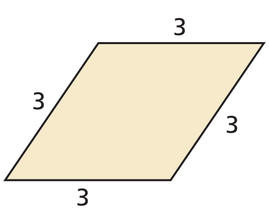
Classifying Quadrilaterals

A **quadrilateral** is a polygon with four sides. The diagram shows properties of different types of quadrilaterals and how they are related. When identifying a quadrilateral, use the name that is most specific.



Example 1 **Classify the quadrilateral.**

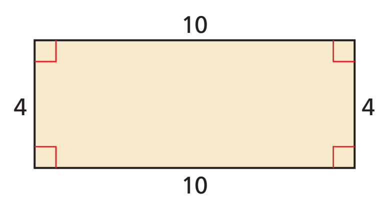
a.



The quadrilateral has four congruent sides.

▶ So, it is a rhombus.

b.



The quadrilateral has four right angles.

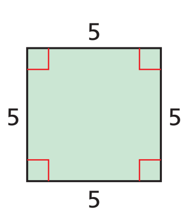
▶ So, it is a rectangle.

Practice

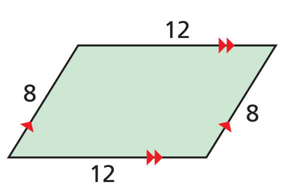
Check your answers at BigIdeasMath.com.

Classify the quadrilateral.

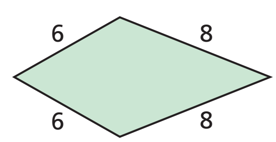
1.



2.



3.



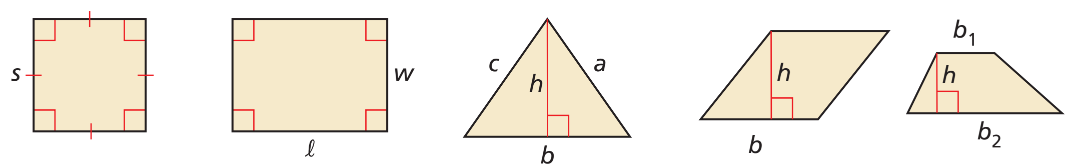
Name ____ Date ______________

Perimeter and Area of Figures

Perimeter and Area of Polygons

The **perimeter** P of a figure is the distance around the figure. The **area** A of a figure is the number of square units enclosed by the figure.

Perimeter and Area



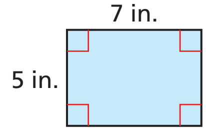
Square	Rectangle	Triangle	Parallelogram	Trapezoid
$P = 4s$	$P = 2\ell + 2w$	$P = a + b + c$	$A = bh$	$A = \frac{1}{2}h(b_1 + b_2)$
$A = s^2$	$A = \ell w$	$A = \frac{1}{2}bh$		

Example 1 **Find the perimeter and area of the figure.**

$P = 2\ell + 2w$ $\quad$ $A = \ell w$

$= 2(7) + 2(5)$ $\quad$ $= 7(5)$

$= 24$ in. $\quad$ $= 35 \text{ in.}^2$

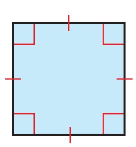


Practice

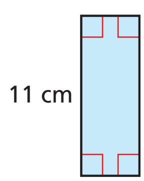
Check your answers at BigIdeasMath.com.

Find the perimeter and area of the figure.

1.



2.



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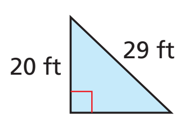
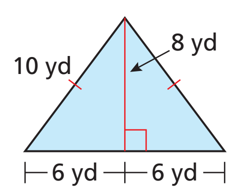
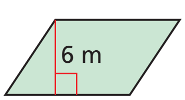


4.

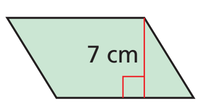


Find the area of the figure.

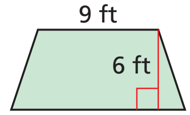
5.



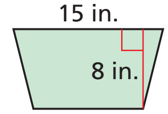
6.



7.



8.



Use a geometric formula to solve the problem.

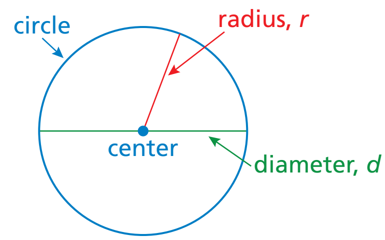
9. A triangle has a base of 7 feet and an area of 63 square feet. Find the height.

10. A rectangle has a length of 6 inches and a perimeter of 28 inches. Find the width.

Name ______________________________ Date ______________

Circumference and Area of a Circle

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**. The distance from the center to any point on the circle is the **radius**. The distance across the circle through the center is the **diameter**. The diameter is twice the radius.



The **circumference** of a circle is the distance around the circle. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for every circle and is represented by the Greek letter π, called **pi**. Pi is an irrational number whose value is approximately 3.14 or $\frac{22}{7}$.

Circumference of a Circle	Area of a Circle
The circumference C of a circle is equal to π times the diameter d or π times twice the radius r. $C = \pi d$ or $C = 2\pi r$	The area A of a circle is the product of π and the square of the radius. $A = \pi r^2$

Example 1 **The diameter of a circle is 8.5 meters. Find the radius.**

$r = \frac{d}{2}$ Radius of a circle

$= \frac{8.5}{2}$ Substitute 8.5 for d.

$= 4.25$ Divide.

▶ The radius is 4.25 meters.

Example 2 **The radius of a circle is $5\frac{3}{4}$ feet. Find the diameter.**

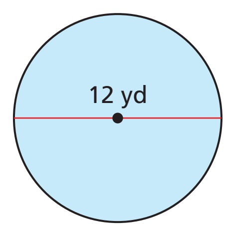
$d = 2r$ Diameter of a circle

$= 2\left(5\frac{3}{4}\right)$ Substitute $5\frac{3}{4}$ for r.

$= 11\frac{1}{2}$

▶ The diameter is $11\frac{1}{2}$ feet.

Example 3 **Find (a) the circumference C and (b) the area A of the circle.**



a. $C = \pi d$

$= \pi(12)$

≈ 37.7

▶ The circumference is about 37.7 yards.

b. $A = \pi r^2$

$= \pi \cdot (6)^2$

$= 36\pi$

≈ 113.1

▶ The area is about 113.1 square yards.

Practice

Check your answers at BigIdeasMath.com.

11. The radius of a circle is 4.6 millimeters. Find the diameter.

12. The diameter of a circle is $2\frac{1}{4}$ miles. Find the radius.

Find the circumference and area of the circle with the given radius or diameter.

13. $r = 16$ inches **14.** $d = 10$ centimeters **15.** $r = 7$ meters **16.** $d = 2.4$ yards

17. The area of a circle is 81π square feet. Find the radius.

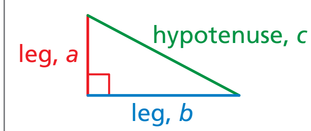
Name ______________________________ Date ______________

The Pythagorean Theorem

In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the two sides that form the right angle.

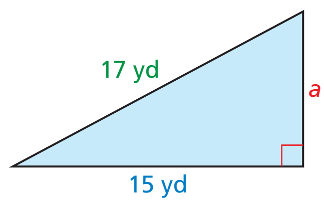
The **Pythagorean Theorem** states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Pythagorean Theorem



$a^2 + b^2 = c^2$

Example 1 **Find the missing length of the triangle.**

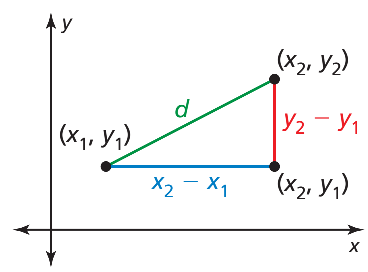


$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$a^2 + 15^2 = 17^2$	Substitute 15 for *b* and 17 for *c*.
$a^2 + 225 = 289$	Evaluate powers.
$a^2 = 64$	Subtract 225 from each side.
$a = 8$	Take positive square root of each side.

▶ The missing length is 8 yards.

You can use the Pythagorean Theorem to develop the *Distance Formula*. You can use the **Distance Formula** to find the distance d between any two points (x_1, y_1) and (x_2, y_2) in a coordinate plane.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 2 **Find the distance between the two points.**

a. (3, 6), (−2, 4)

Let $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (-2, 4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2 - 3)^2 + (4 - 6)^2}$$
$$= \sqrt{25 + 4}$$
$$= \sqrt{29}$$

b. (0, 5), (4, −1)

Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (4, -1)$.

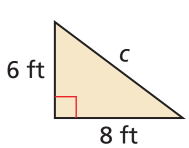
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 0)^2 + (-1 - 5)^2}$$
$$= \sqrt{16 + 36}$$
$$= 2\sqrt{13}$$

Practice

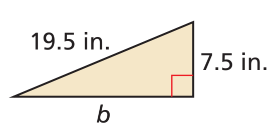
Check your answers at BigIdeasMath.com.

Find the missing length of the triangle.

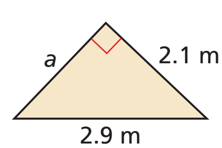
1.



2.



3.



Find the distance between the two points.

4. (0, 0), (4, 3)

5. (0, −7), (5, 5)

6. (4, 2), (−1, 5)

7. (−5, 6), (−7, −2)

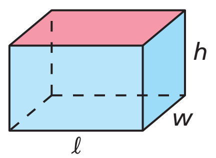
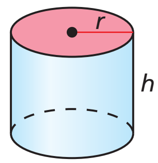
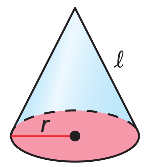
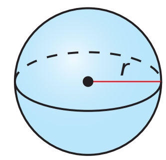
8. (−1, −3), (9, 0)

9. (−4, −4), (−1, −1)

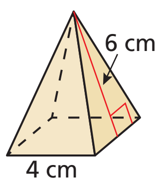
Name ______________________________ Date ______________

Surface Area

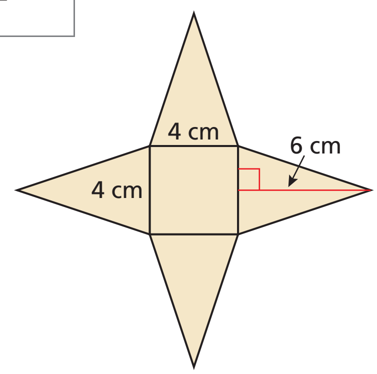
A **solid** is a three-dimensional figure that encloses a space. The **surface area** of a solid is the sum of the areas of all of its faces. Surface area is measured in *square units*. You can use a two-dimensional representation of a solid, called a **net**, to find the surface area of a solid. You can also use the following formulas to find surface areas.

Rectangular Prism	Cylinder	Cone	Sphere
			
$S = 2\ell w + 2\ell h + 2wh$	$S = 2\pi r^2 + 2\pi rh$	$S = \pi r^2 + \pi r\ell$	$S = 4\pi r^2$

Example 1 **Find the surface area of the regular pyramid.**



Draw a net.

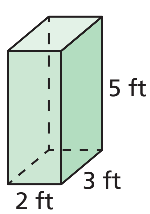


Area of Base	*Area of a Lateral Face*
$4 \bullet 4 = 16$	$\frac{1}{2} \bullet 4 \bullet 6 = 12$

▶ There are four identical lateral faces. So, the surface area is $16 + 4(12) = 64$ square centimeters.

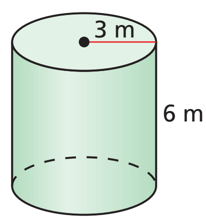
Example 2 **Find the surface area of each solid.**

a.



$$S = 2\ell w + 2\ell h + 2wh$$
$$= 2(2)(3) + 2(2)(5) + 2(3)(5)$$
$$= 12 + 20 + 30$$
$$= 62 \text{ ft}^2$$

b.



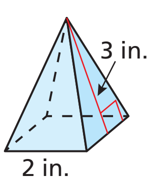
$$S = 2\pi r^2 + 2\pi rh$$
$$= 2\pi(3)^2 + 2\pi(3)(6)$$
$$= 18\pi + 36\pi$$
$$= 54\pi \approx 170 \text{ m}^2$$

Practice

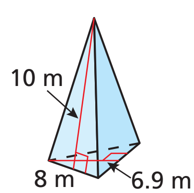
Check your answers at BigIdeasMath.com.

Find the surface area of the regular pyramid.

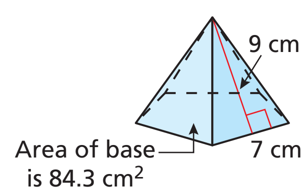
1.



2.

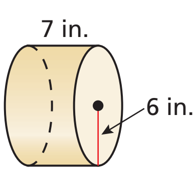


3.

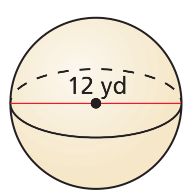


Find the surface area of the solid.

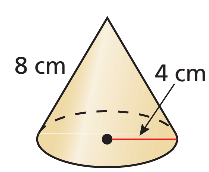
4.



5.



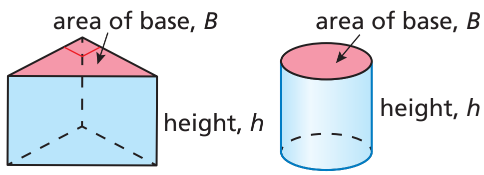
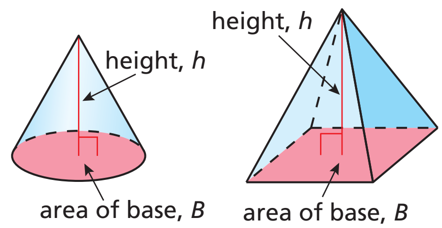
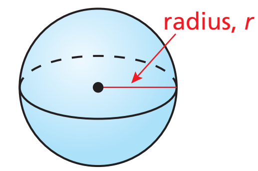
6.



Name ______________________________ Date ______________

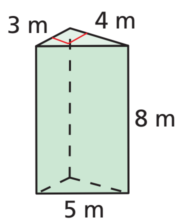
Volume

A **volume** of a solid is a measure of the amount of space that it occupies. Volume is measured in *cubic units*. You can use the following formulas to find volumes.

Prism and Cylinder	Cone and Pyramid	Sphere
 $V = Bh$	 $V = \frac{1}{3}Bh$	 $V = \frac{4}{3}\pi r^3$

Example 1 Find the volume of each solid.

a.



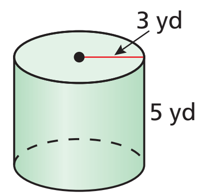
$V = Bh$

$= \frac{1}{2}(3)(4) \cdot 8$

$= 6 \cdot 8$

$= 48 \text{ m}^3$

b.

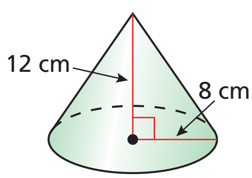


$V = Bh$

$= \pi(3)^2 \cdot 5$

$= 45\pi \approx 141 \text{ yd}^3$

c.

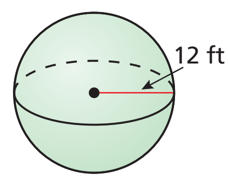


$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(8)^2 \cdot 12$

$= 256\pi \approx 804 \text{ cm}^3$

d.



$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(12)^3$

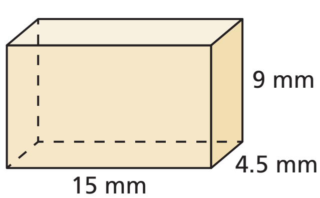
$= 2304\pi \approx 7238 \text{ ft}^3$

Practice

Check your answers at BigIdeasMath.com.

Find the volume of the solid.

1.



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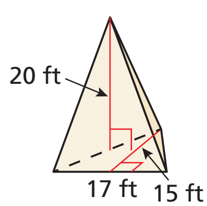

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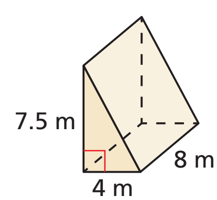

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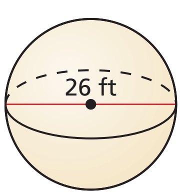

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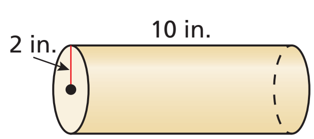

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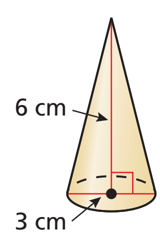


Name ______________________________ Date ______________

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Example 1 **Use inductive reasoning to make a conjecture about the result when the expression $n^2 - n + 11$ is evaluated at any natural number n.**

Step 1 Find a pattern using the first few natural numbers.

$1^2 - 1 + 11 = 11$ $\quad$ $3^2 - 3 + 11 = 17$ $\quad$ $5^2 - 5 + 11 = 31$

$2^2 - 2 + 11 = 13$ $\quad$ $4^2 - 4 + 11 = 23$ $\quad$ $6^2 - 6 + 11 = 41$

Step 2 Make a conjecture.

▶ **Conjecture** The expression $n^2 - n + 11$ gives a prime number when evaluated at any natural number n.

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

Example 2 **Find a counterexample to show that the conjecture in Example 1 is false.**

To find a counterexample, you need to find a natural number n for which $n^2 - n + 11$ gives a composite number.

$11^2 - 11 + 11 = 121$ $\quad$ Substitute 11 for n.

Because the factors of 121 are 1, 11, and 121, it is composite.

▶ Because a counterexample exists, the conjecture is false.

Practice

Check your answers at BigIdeasMath.com.

Make a conjecture about the given quantity.

1. the product of a nonzero number and its opposite
2. the sum of the first n positive odd integers
3. the ones digit of 4^{200}

Find a counterexample to show that the conjecture is false.

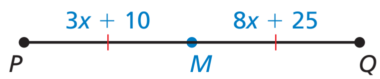
4. If x is an integer, then $-x < x$.
5. A nonzero number is always greater than its reciprocal.
6. If the product of two natural numbers is even, then both of the natural numbers are even.
7. If the sum of two natural numbers is even, then the product of the two natural numbers is even.

Name ______________________________ Date ______________

Using Midpoints

The **midpoint** of a segment is the point that divides the segment into two congruent segments. In the figure, M is the midpoint of $\overline{AB}$. So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

Example 1 **Point M is the midpoint of $\overline{PQ}$. Find the length of $\overline{PQ}$.**



First write and solve an equation. Use the fact that $PM = MQ$.

$PM = MQ$	Write the equation.
$3x + 10 = 8x + 25$	Substitute.
$-3 = x$	Solve for x.

Then evaluate the expression for PM when $x = -3$ to obtain $PM = 1$. By the Segment Addition Postulate and the definition of midpoint, $PQ = PM + MQ = 1 + 1 = 2$.

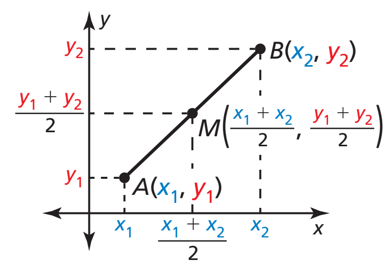
▶ So, the length of $\overline{PQ}$ is 2.

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Example 2 **The endpoints of $\overline{AB}$ are $A(-4, 7)$ and $B(10, -8)$. Find the coordinates of the midpoint M.**

Use the Midpoint Formula.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-4 + 10}{2}, \frac{7 + (-8)}{2}\right) = M\left(3, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(3, -\frac{1}{2}\right)$.

Practice

Check your answers at BigIdeasMath.com.

1. Point M bisects $\overline{JK}$ such that $JM = 4x - 5$ and $MK = 3x + 2$. Find the length of $\overline{JK}$.

2. Point M bisects $\overline{EF}$ such that $EM = 7x + 11$ and $MF = 8x$. Find the length of $\overline{EF}$.

The endpoints of $\overline{RS}$ are given. Find the coordinates of the midpoint M.

3. $R(-6, 3)$, $S(-4, 1)$

4. $R(0, -2)$, $S(6, -8)$

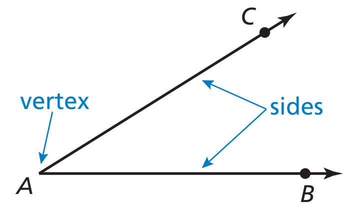
5. $R(13, 2)$, $S(7, 6)$

6. $R(-2, 9)$, $S(4, 0)$

Name ______________________________ Date ______________

Naming and Bisecting Angles

You can name an angle by its vertex, such as $\angle A$, or by a point on each ray and the vertex, such as $\angle BAC$ or $\angle CAB$.

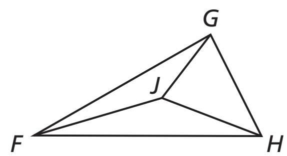


When a point is the vertex of more than one angle, you cannot use the vertex alone to name the angles, as shown in the following example.

Example 1 **Name the included angle between each given pair of sides.**

a. $\overline{FJ}$ and $\overline{FH}$

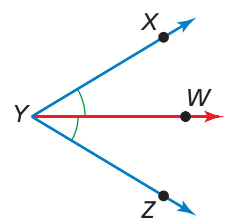
▶ $\angle JFH$ or $\angle HFJ$



b. $\overline{JH}$ and $\overline{GH}$

▶ $\angle JHG$ or $\angle GHJ$

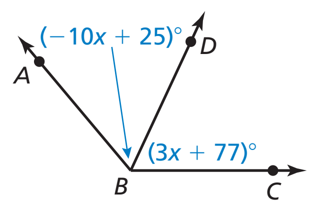
An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the figure, $\overrightarrow{YW}$ bisects $\angle XYZ$, so $\angle XYW \cong \angle ZYW$.



Example 2 $\overrightarrow{BD}$ **bisects** $\angle ABC$**. Find** $m\angle ABC$**.**

First write and solve an equation. Use the fact that $m\angle ABD = m\angle CBD$.

$m\angle ABD = m\angle CBD$	Write the equation.
$-10x + 25 = 3x + 77$	Substitute.
$-4 = x$	Solve for x.



Then evaluate the expression for $m\angle ABD$ when $x = -4$ to obtain $m\angle ABD = 65°$. By the Angle Addition Postulate and the definition of angle bisector, $m\angle ABC = m\angle ABD + m\angle CBD = 65° + 65° = 130°$.

▶ So, the measure of $\angle ABC$ is 130°.

Practice

Check your answers at BigIdeasMath.com.

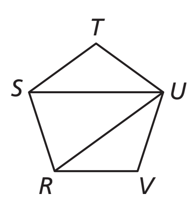
In Exercises 1–4, use the figure to name the included angle between the given pair of sides.

1. $\overline{ST}$ and $\overline{UT}$

2. $\overline{SU}$ and $\overline{VU}$

3. $\overline{UR}$ and $\overline{UT}$

4. $\overline{RV}$ and $\overline{RS}$



5. $\overrightarrow{QS}$ bisects $\angle PQR$ such that $m\angle PQS = (5x + 9)°$ and $m\angle RQS = (9x - 3)°$. Find the value of x and $m\angle PQR$.

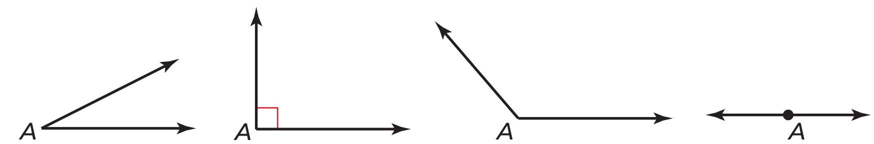
6. $\overrightarrow{KM}$ bisects $\angle JKL$ such that $m\angle JKM = (6x + 33)°$ and $m\angle LKM = (13x - 2)°$. Find the value of x and $m\angle JKL$.

Name ______________________ Date ____________

Measuring and Classifying Angles

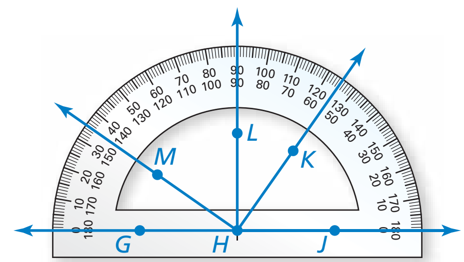
A protractor helps you approximate the measure of an angle. You can classify angles according to their measures.

Types of Angles



acute angle	right angle	obtuse angle	straight angle
Measures greater than 0° and less than 90°	Measures 90°	Measures greater than 90° and less than 180°	Measures 180°

Example 1 **Find the measure of each angle. Then classify the angle.**



a. $\angle GHK$

b. $\angle JHL$

c. $\angle LHK$

a. $\overrightarrow{HG}$ lines up with 0° on the outer scale of the protractor. $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an *obtuse* angle.

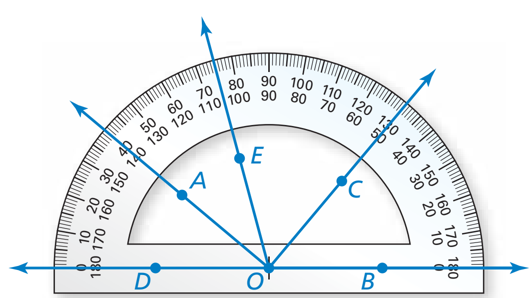
b. $\overrightarrow{HJ}$ lines up with 0° on the inner scale of the protractor. $\overrightarrow{HL}$ passes through 90°. So, $m\angle JHL = 90°$. It is a *right* angle.

c. $\overrightarrow{HL}$ passes through 90°. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle LHK = |90 - 55| = 35°$. It is an *acute* angle.

Practice

Check your answers at BigIdeasMath.com.

Use the diagram to find the angle measure. Then classify the angle.



1. $\angle BOC$
2. $\angle AOB$
3. $\angle DOB$
4. $\angle DOE$
5. $\angle AOC$
6. $\angle BOE$
7. $\angle EOC$
8. $\angle COD$

Name ______________________________ Date ______________

Parallel Lines and Transversals

Using Properties of Parallel Lines

Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the right, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Alternate Interior Angles Theorem

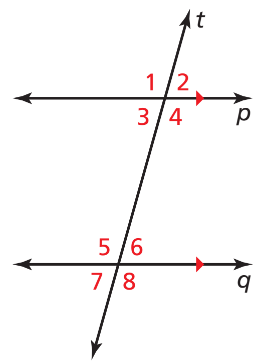
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the right, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the right, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

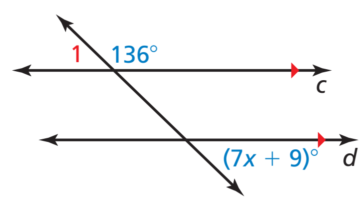


Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the right, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Example 1 **Find the value of x.**



By the Linear Pair Postulate, $m\angle 1 = 180° - 136° = 44°$. Lines c and d are parallel, so you can use the theorems about parallel lines.

$m\angle 1 = (7x + 9)°$	Alternate Exterior Angles Theorem
$44° = (7x + 9)°$	Substitute 44° for $m\angle 1$.
$35 = 7x$	Subtract 9 from each side.
$5 = x$	Divide each side by 7.

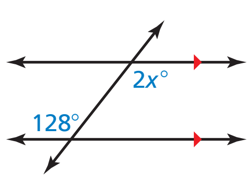
So, the value of x is 5.

Practice

Check your answers at BigIdeasMath.com.

Find the value of x.

1.



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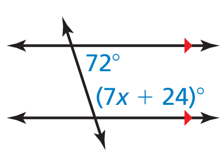

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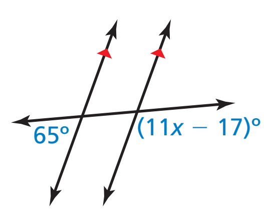

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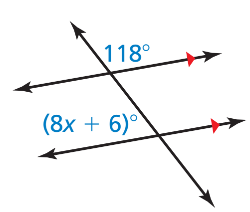


Name ______________________________ Date ______________

Parallel Lines and Transversals

Determining Whether Lines are Parallel

The theorems about angles formed when parallel lines are cut by a transversal have true converses.

Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

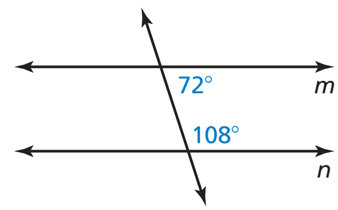
Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Example 1 **Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use.**



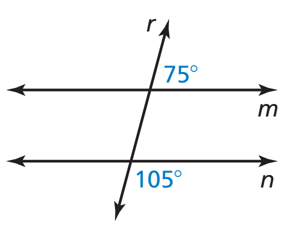
The sum of the marked consecutive interior angles is 180°. Lines *m* and *n* are parallel when the consecutive interior angles are supplementary. So, by the Consecutive Interior Angles Converse, $m \parallel n$.

Practice

Check your answers at BigIdeasMath.com.

Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use.

1.



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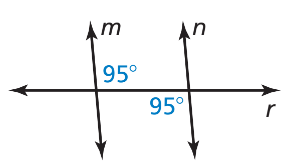

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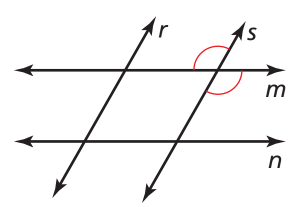

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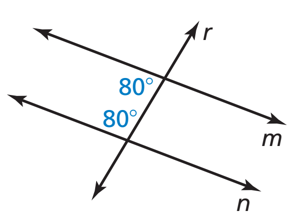

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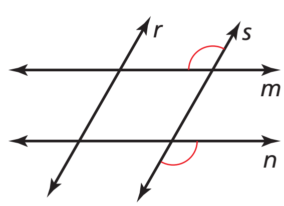

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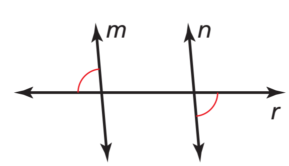


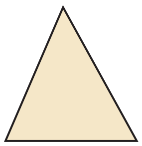
Name ______________________________ Date ______________

Classifying Triangles

You can use angle measures and side lengths to classify triangles.

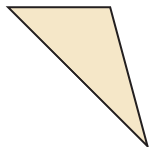
Classifying Triangles Using Angles

acute triangle



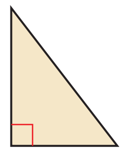
all acute angles

obtuse triangle



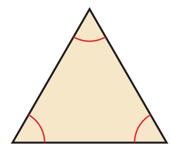
1 obtuse angle

right triangle



1 right angle

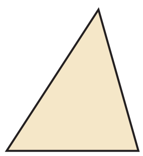
equiangular triangle



3 congruent angles

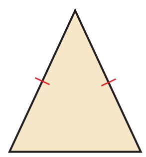
Classifying Triangles Using Sides

scalene triangle



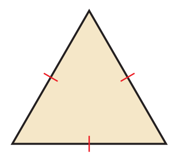
no congruent sides

isosceles triangle



at least 2 congruent sides

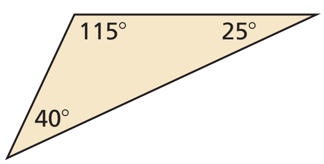
equilateral triangle



3 congruent sides

Example 1 **Classify each triangle by its angles and by its sides.**

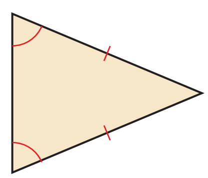
a.



The triangle has one obtuse angle and no congruent sides.

▶ So, the triangle is an obtuse scalene triangle.

b.



The triangle has all acute angles and two congruent sides.

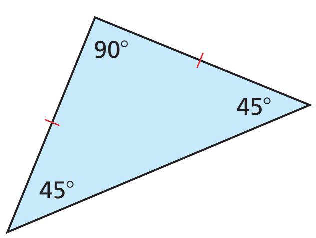
▶ So, the triangle is an acute isosceles triangle.

Practice

Check your answers at BigIdeasMath.com.

Classify the triangle by its angles and by its sides.

1.



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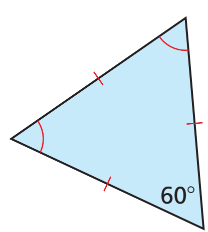

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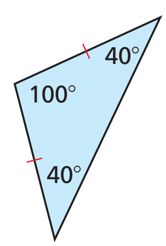

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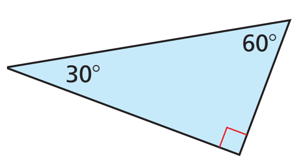

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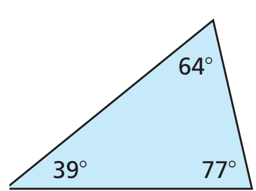

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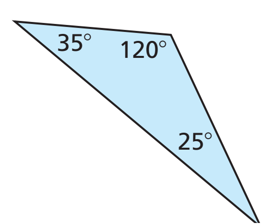


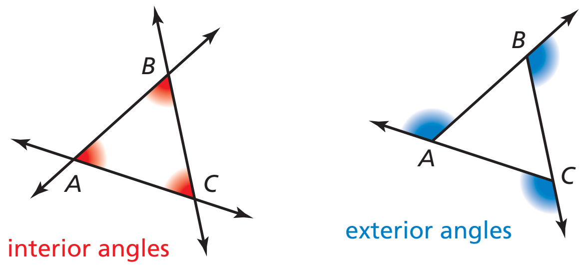
Name ______________________ Date ______________

Finding Angles of Triangles

Using Interior and Exterior Angles

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

The theorems given below show how the angle measures of a triangle are related. You can use these theorems to find angle measures.



Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

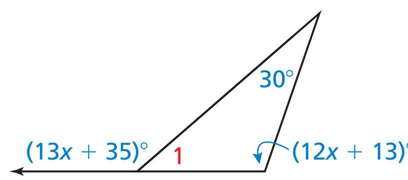
Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Example 1 **Find $m\angle 1$.**



First write and solve an equation to find the value of x.

$(13x + 35)° = 30° + (12x + 13)°$ — Apply the Exterior Angle Theorem.

$x = 8$ — Solve for x.

Substitute 8 for x in $(12x + 13)°$ to find the obtuse angle measure, 109°. Then write and solve an equation to find $m\angle 1$.

$m\angle 1 + 30° + 109° = 180°$ — Apply the Triangle Sum Theorem.

$m\angle 1 = 41°$ — Solve for $m\angle 1$.

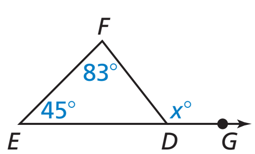
▶ So, the measure of $\angle 1$ is 41°.

Practice

Check your answers at BigIdeasMath.com.

Find the value of x.

1.



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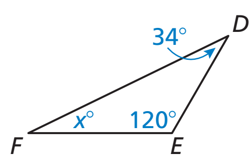

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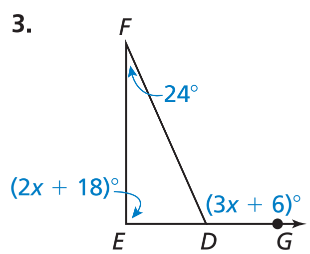

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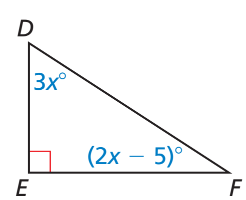
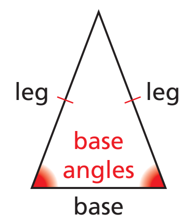


Name ____ Date ________________

Finding Angles of Triangles

Using Isosceles and Equilateral Triangles

When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



You can use the theorems given below to find angle measures and side lengths.

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

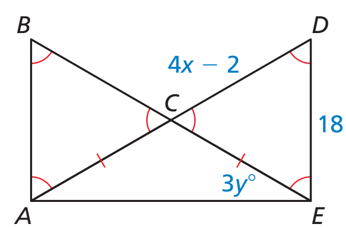
Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

Corollary to the Converse of the Base Angles Theorem

If a triangle is equiangular, then it is equilateral.

Example 1 **Find the values of *x* and *y* in the diagram.**



Step 1 Find the value of x. Because $\triangle CDE$ is equiangular, it is also equilateral by the Corollary to the Converse of the Base Angles Theorem. So, $\overline{CD} \cong \overline{DE}$.

$CD = DE$	Definition of congruent segments
$4x - 2 = 18$	Substitute.
$x = 5$	Solve for x.

Step 2 Find the value of y. By the Triangle Sum Theorem, $3(m\angle DCE) = 180°$, so $m\angle DCE = 60°$. Because $\angle ACE$ and $\angle DCE$ form a linear pair, they are supplementary angles and $m\angle ACE = 180° - 60° = 120°$. The diagram shows that $\triangle ACE$ is isosceles. By the Base Angles Theorem, $\angle CAE \cong \angle CEA$. So, $m\angle CAE = m\angle CEA$.

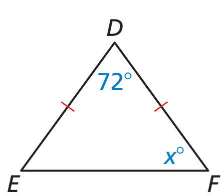
$120° + 3y° + 3y° = 180°$	Apply the Triangle Sum Theorem.
$y = 10$	Solve for y.

Practice

Check your answers at BigIdeasMath.com.

Find the value(s) of the variable(s).

1.



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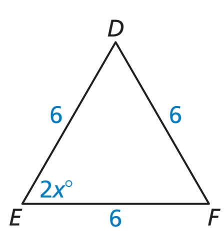

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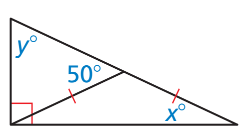

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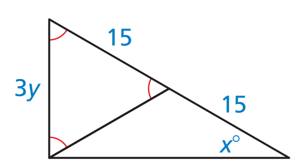

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Transformations

Translations and Reflections

A **transformation** changes a figure into another figure. The new figure is called the **image**.

A **translation** is a transformation in which a figure slides but does not turn. Every point of the figure moves the same distance and in the same direction. Translating a figure a units horizontally and b units vertically in a coordinate plane changes the coordinates of the figure as follows.

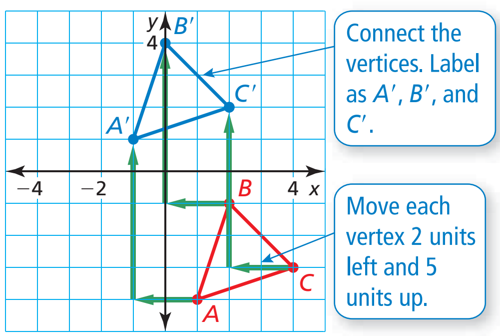
$$(x, y) \rightarrow (x + a, y + b)$$

A **reflection** is a transformation in which a figure is reflected in a line called the **line of reflection**. A reflection creates a mirror image of the original figure. Reflecting a figure in the x-axis or the y-axis changes the coordinates of the figure as follows.

x-axis: $(x, y) \rightarrow (x, -y)$ **y-axis:** $(x, y) \rightarrow (-x, y)$

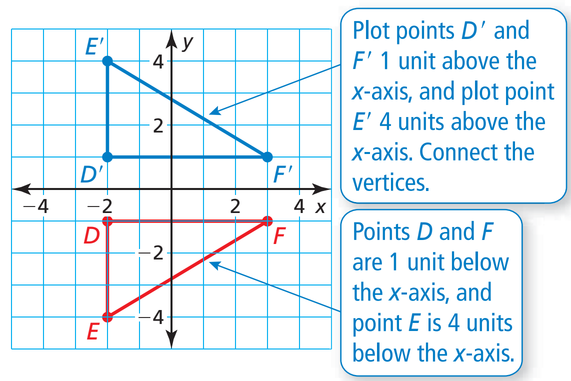
In a translation or reflection, the original figure and its image are congruent.

Example 1 **Translate the red triangle 2 units left and 5 units up. What are the coordinates of the image?**



▶ The coordinates of the image are $A'(-1, 1)$, $B'(0, 4)$, and $C'(2, 2)$.

Example 2 **Reflect the red triangle in the x-axis. What are the coordinates of the image?**



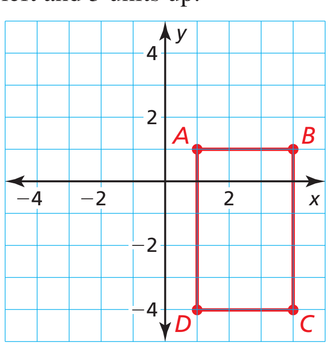
▶ The coordinates of the image are $D'(-2, 1)$, $E'(-2, 4)$, and $F'(3, 1)$.

Practice

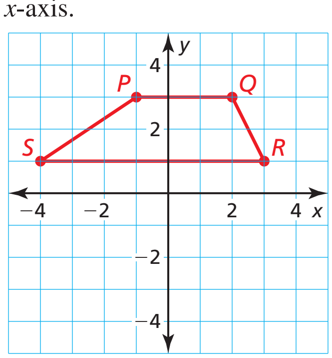
Check your answers at BigIdeasMath.com.

Find the coordinates of the figure after the transformation.

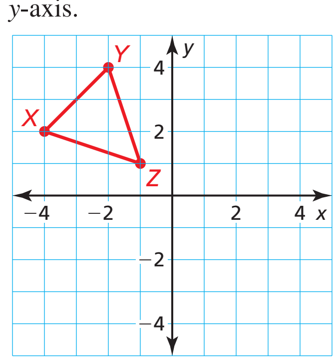
1. Translate the rectangle 2 units left and 3 units up.



2. Reflect the trapezoid in the x-axis.



3. Reflect the triangle in the y-axis.



Name ______________________________ Date ______________

Transformations

Rotations and Dilations

A **rotation** is a transformation in which a figure is rotated about a point called the **center of rotation**. The number of degrees a figure rotates is the **angle of rotation**.

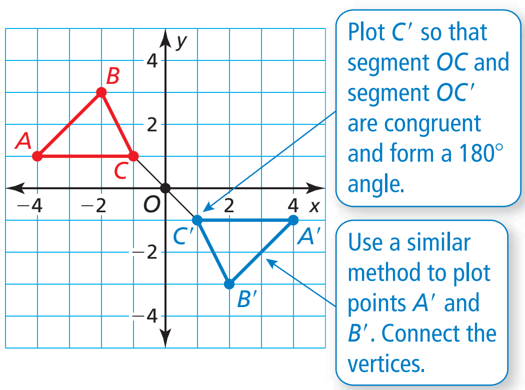
A **dilation** is a transformation in which a figure is made larger or smaller with respect to a point called the **center of dilation**.

In a rotation, the original figure and its image are congruent. In a dilation, the original figure and its image are similar. The ratio of the side lengths of the image to the corresponding side lengths of the original figure is the **scale factor** of the dilation.

Dilating a figure in a coordinate plane with respect to the origin by a scale factor k changes the coordinates of the figure as follows.

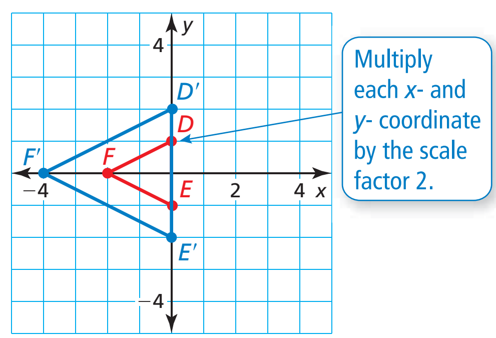
$$(x, y) \rightarrow (kx, ky)$$

Example 1 **Rotate the red triangle 180° about the origin.**



▶ The coordinates of the image are $A'(4, -1)$, $B'(2, -3)$, and $C'(1, -1)$.

Example 2 **Dilate the red triangle with respect respect to the origin using a scale factor of 2.**



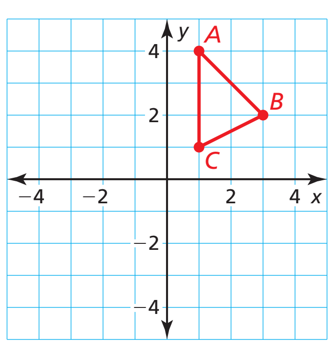
▶ The coordinates of the image are $D'(0, 2)$, $E'(0, -2)$, and $F'(-4, 0)$.

Practice

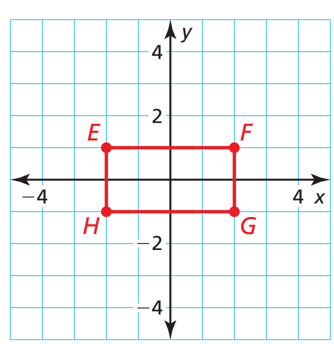
Check your answers at BigIdeasMath.com.

Find the coordinates of the figure after the transformation.

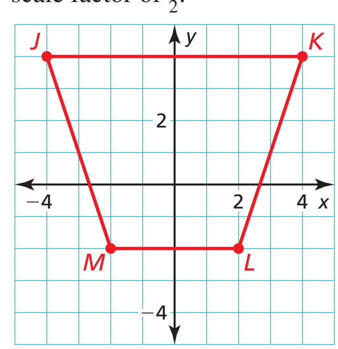
1. Rotate the triangle 90° counterclockwise about the origin.



2. Dilate the rectangle with respect to the origin using a scale factor of 3.



3. Dilate the trapezoid with respect to the origin using a scale factor of $\frac{1}{2}$.



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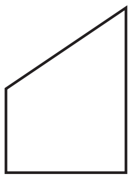
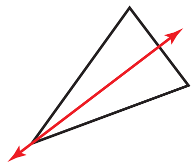
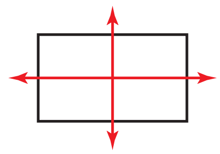
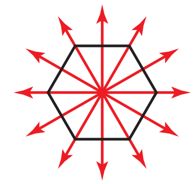
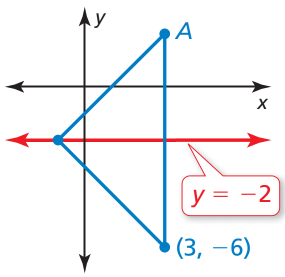
Line Symmetry

A figure has **line symmetry** when a line, called a **line of symmetry**, divides the figure into two parts that are mirror images of each other. Below are four figures with their lines of symmetry shown in red.

			
Trapezoid No lines of symmetry	**Isosceles triangle** One line of symmetry	**Rectangle** Two lines of symmetry	**Regular hexagon** Six lines of symmetry

Example 1 **A line of symmetry for the figure is shown in red. Find the coordinates of point *A*.**



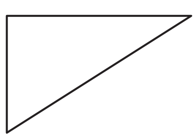
Point A is the mirror image of the point $(3, -6)$ with respect to the line of symmetry $y = -2$. The x-coordinate of A is 3, the same as the x-coordinate of $(3, -6)$. Because -6 is the y-coordinate of $(3, -6)$, and $-2 - (-6) = 4$, the point $(3, -6)$ is *down* 4 units from the line of symmetry. So, point A must be *up* 4 units from the line of symmetry. So, the y-coordinate of A is $-2 + 4 = 2$.

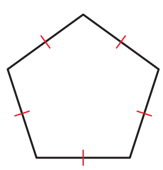
▶ The coordinates of point A are $(3, 2)$.

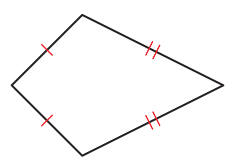
Practice

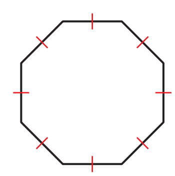
Check your answers at BigIdeasMath.com.

Tell how many lines of symmetry are in the figure.

1. 

2. 

3. 

4. 

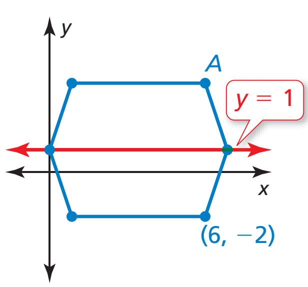
5. square

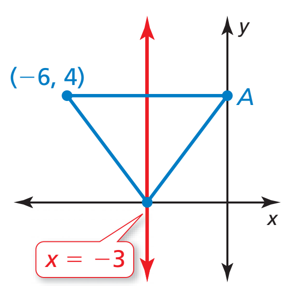
6. equilateral triangle

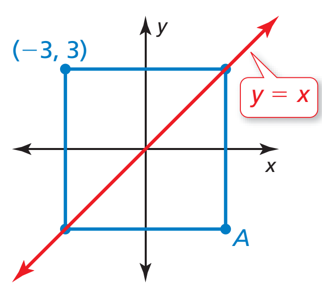
7. regular decagon

8. isosceles trapezoid

A line of symmetry for the figure is shown in red. Find the coordinates of point *A*.

9. 

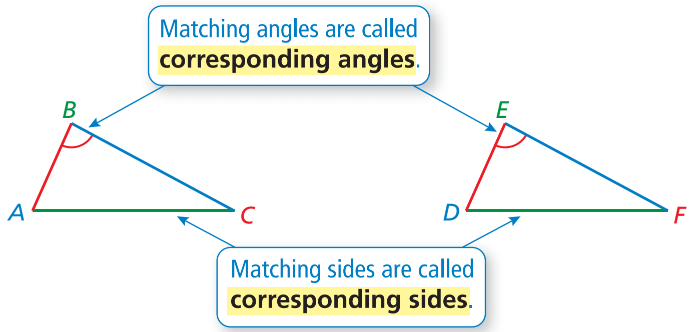
10. 

11. 

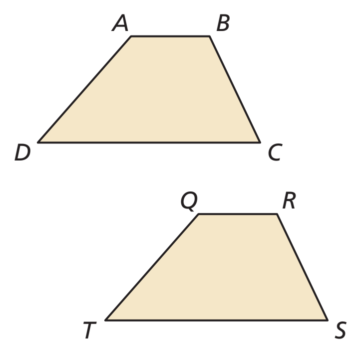
Name ______________________________ Date ______________

Congruent Figures

Figures that have the same size and the same shape are called **congruent figures**. The triangles below are congruent.



Example 1 **The figures are congruent. Name the corresponding angles and the corresponding sides.**



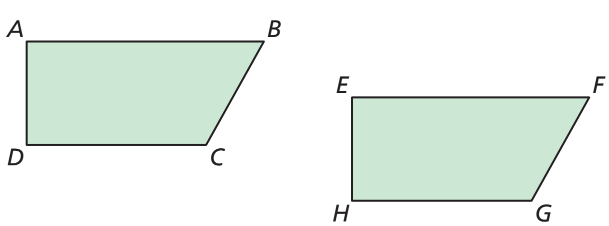
Corresponding Angles	*Corresponding Sides*
$\angle A$ and $\angle Q$	Side $\overline{AB}$ and Side $\overline{QR}$
$\angle B$ and $\angle R$	Side $\overline{BC}$ and Side $\overline{RS}$
$\angle C$ and $\angle S$	Side $\overline{CD}$ and Side $\overline{ST}$
$\angle D$ and $\angle T$	Side $\overline{DA}$ and Side $\overline{TQ}$

Practice

Check your answers at BigIdeasMath.com.

The figures are congruent. Name the corresponding angles and the corresponding sides.

1.



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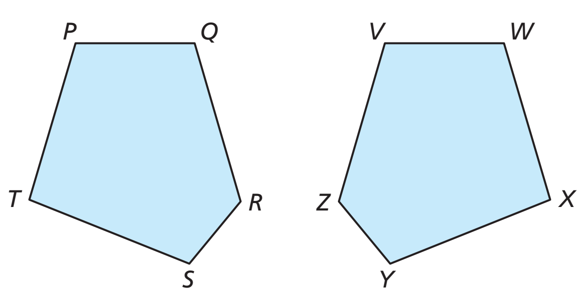


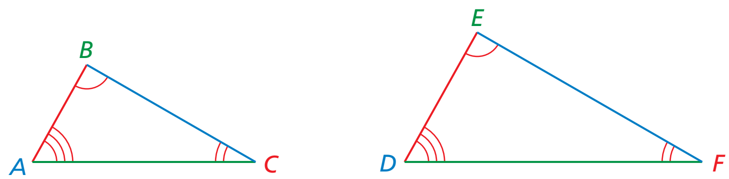
Name ______________________ Date ____________

Similar Figures

Figures that have the same shape but not necessarily the same size are called **similar figures**. Two figures are similar when

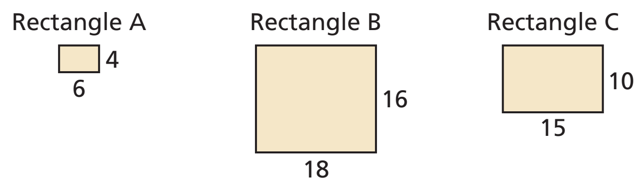
- corresponding side lengths are proportional and
- corresponding angles are congruent.

In the figure below, Triangle *ABC* is similar to Triangle *DEF*.



Side Lengths	*Angles*	*Figures*
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\triangle ABC \sim \triangle DEF$

Example 1 **Which rectangle is similar to Rectangle A?**



Each figure is a rectangle, so corresponding angles are congruent.
Check to see whether corresponding side lengths are proportional.

Rectangle A and Rectangle B

$\frac{\text{Length of A}}{\text{Length of B}} = \frac{6}{18} = \frac{1}{3}$ $\quad \frac{\text{Width of A}}{\text{Width of B}} = \frac{4}{16} = \frac{1}{4}$

not proportional

Rectangle A and Rectangle C

$\frac{\text{Length of A}}{\text{Length of B}} = \frac{6}{15} = \frac{2}{5}$ $\quad \frac{\text{Width of A}}{\text{Width of C}} = \frac{4}{10} = \frac{2}{5}$

proportional

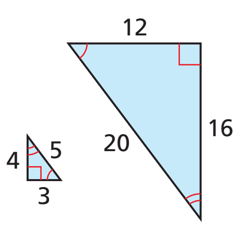
▶ So, Rectangle C is similar to Rectangle A.

Practice

Check your answers at BigIdeasMath.com.

Tell whether the two figures are similar. Explain your reasoning.

1.



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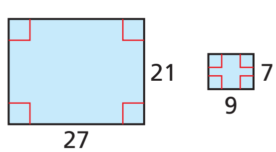

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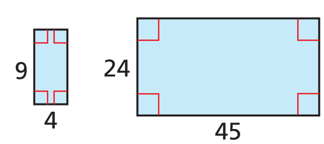
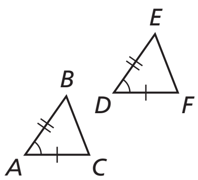
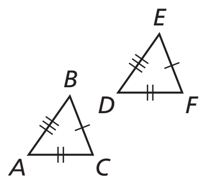
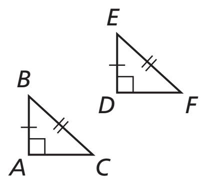
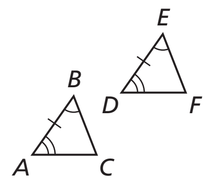
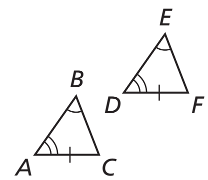


Name ______________________________ Date ______________

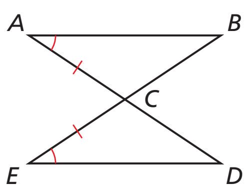
Congruent Triangles

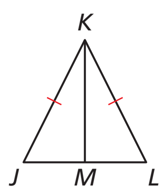
Triangle Congruence Theorems

Five valid methods for proving that triangles are congruent are given below.

SAS	SSS	HL (right triangles only)	ASA	AAS
				
Two sides and the included angle are congruent.	All three sides are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a non-included side are congruent.

Example 1 **Determine whether there is enough information to prove that the triangles are congruent. Explain your reasoning.**

a. 

b. 

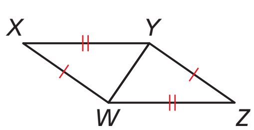
a. You are given that $\angle A \cong \angle E$ and $\overline{AC} \cong \overline{EC}$. By the Vertical Angles Congruence Theorem, $\angle ACB \cong \angle ECD$. So, two pairs of angles and their included sides are congruent. By the ASA Congruence Theorem, $\triangle ABC \cong \triangle EDC$.

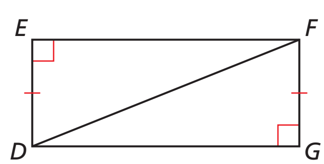
b. You are given that $\overline{JK} \cong \overline{LK}$. You know that $\angle J \cong \angle L$ by the Base Angles Theorem. You also know that $\overline{KM} \cong \overline{KM}$ by the Reflexive Property of Segment Congruence. Because two pairs of sides and their non-included angles are congruent, you cannot conclude that $\triangle JKM \cong \triangle LKM$.

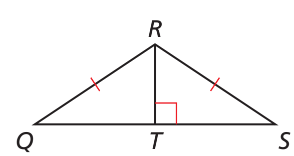
Practice

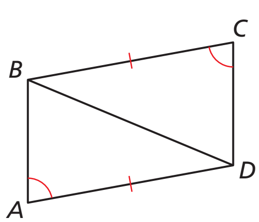
Check your answers at BigIdeasMath.com.

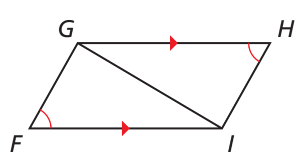
Determine whether there is enough information to prove that the triangles are congruent. If so, state the theorem you would use.

1. 

2. 

3. 

4. 

5. 

6.