Grade 3 B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

Benchmark
focal point for instruction within lesson or task

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment
in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment
across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

Strategies to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Mathematics Standards. For more information about tiered instruction, please see the Effective Tiered Instruction for Mathematics: ALL Means ALL document.

Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Mathematical Thinking and Reasoning Standards
MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction, but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:
Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students’ ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Student asks questions to self, others and teacher when necessary. • Student stays engaged in the task and helps others during the completion of the task. • Student analyzes the task in a way that makes sense to themselves. • Student builds perseverance in self by staying engaged and modifying methods as they solve a problem.	• Teacher builds a classroom community by allowing students to build their own set of "norms." • Teacher creates a culture in which students are encouraged to ask questions, including questioning the accuracy within a real-world context. • Teacher chooses differentiated, challenging tasks that fit the students' needs to help build perseverance in students. • Teacher builds community of learners by encouraging students and recognizing their effort in staying engaged in the task and celebrating errors as an opportunity for learning.
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Student chooses their preferred method of representation. • Student represents a problem in more than one way and is able to make connections between the representations.	• Teacher plans ahead to allow students to choose their tools. • While sharing student work, teacher purposefully shows various representations to make connections between different strategies or methods. • Teacher helps make connections for students between different representations (i.e., table, equation or written description).
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Student uses feedback from teacher and peers to improve efficiency.	• Teacher provides opportunity for students to reflect on the method they used, determining if there is a more efficient way depending on the context.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Student effectively justifies their reasoning for their methods. • Student can identify errors within their own work and create possible explanations. • When working in small groups, student recognizes errors of their peers and offers suggestions. • Student communicates mathematical vocabulary efficiently to others.	• Teacher purposefully groups students together to provide opportunities for discussion. • Teacher chooses sequential representation of methods to help students explain their reasoning.
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Student determines what information is needed and logically follows a plan to solve problems piece by piece. • Student is able to make connections from previous knowledge.	• Teacher allows for students to engage with information to connect current understanding to new methods. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. • Teacher provides opportunities for students to develop their own steps in solving a problem.
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Student provides explanation of results. • Student continually checks their calculations. • Student estimates a solution before performing calculations.	• Teacher encourages students to check and revise solutions and provide explanations for results. • Teacher allows opportunities for students to verify their solutions by providing justifications to self and others.
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Student relates their real-world experience to the context provided by the teacher during instruction. • Student performs investigations to determine if a scenario can represent a real-world context.	• Teacher provides real-world context in mathematical problems to support students in making connections using models and investigations.

Grade 3 Areas of Emphasis

In grade 3, instructional time will emphasize four areas:

(1) adding and subtracting multi-digit whole numbers, including using a standard algorithm;
(2) building an understanding of multiplication and division, the relationship between them and the connection to area of rectangles;
(3) developing an understanding of fractions; and
(4) extending geometric reasoning to lines and attributes of quadrilaterals.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following.

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of the major mathematical topics to all stakeholders.
- Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

		Addition and Subtraction Fluency	**Building Understanding of Multiplication and Division**	**Building Understanding of Fractions**	**Geometric Reasoning with Lines and Quadrilaterals**
Number Sense and Operations	MA.3.NSO.1.1	X			
	MA.3.NSO.1.2	X			
	MA.3.NSO.1.3	X			
	MA.3.NSO.1.4	X			
	MA.3.NSO.2.1	X	X		
	MA.3.NSO.2.2		X		
	MA.3.NSO.2.3		X		
	MA.3.NSO.2.4		X		

			Addition and Subtraction Fluency	Building Understanding of Multiplication and Division	Building Understanding of Fractions	Geometric Reasoning with Lines and Quadrilaterals
	Fractions	MA.3.FR.1.1			X	
		MA.3.FR.1.2			X	
		MA.3.FR.1.3			X	
		MA.3.FR.2.1			X	
		MA.3.FR.2.2			X	
	Algebraic Reasoning	MA.3.AR.1.1		X		
		MA.3.AR.1.2	X	X		
		MA.3.AR.2.1		X		
		MA.3.AR.2.2		X		
		MA.3.AR.2.3		X		
		MA.3.AR.3.1		X		
		MA.3.AR.3.2		X		
		MA.3.AR.3.3		X		
	Measurement	MA.3.M.1.1			X	
		MA.3.M.1.2	X	X		
		MA.3.M.2.1	X			
		MA.3.M.2.2	X			
	Geometric Reasoning	MA.3.GR.1.1				X
		MA.3.GR.1.2				X
		MA.3.GR.1.3				X
		MA.3.GR.2.1	X			X
		MA.3.GR.2.2		X		X
		MA.3.GR.2.3	X	X		X
		MA.3.GR.2.4	X	X		X
	Data Analysis & Probability	MA.3.DP.1.1		X		
		MA.3.DP.1.2		X		

MA.3.NSO.1 *Understand the place value of four-digit numbers.*

MA.3.NSO.1.1

Benchmark

MA.3.NSO.1.1 Read and write numbers from 0 to 10,000 using standard form, expanded form and word form.

Example: The number two thousand five hundred thirty written in standard form is 2,530 and in expanded form is $2{,}000 + 500 + 30$.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.2

Terms from the K-12 Glossary

- Expression
- Whole number

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.1

Next Benchmarks

- MA.4.NSO.1.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to express numbers in standard form, expanded form, and word form. This work extends from the Grade 2 expectation to read and write numbers within 1,000 using standard form, expanded form and word form (MA.2.NSO.2.1).

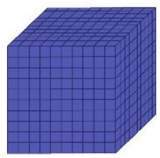
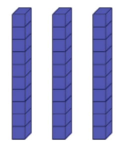
- Students learn to express multi-digit whole numbers using the place value of digits to name them in words. For example, $two\ thousand\ five\ hundred\ thirty$ is named after the 2 in the $thousands$ place, the 5 in the $hundreds$ place, and the 3 in the $tens$ place.
- Students express multi-digit whole numbers by decomposing them by place value and showing them as an addition expression with the value of each nonzero digit. For example, 2,530 is decomposed as $2{,}000 + 500 + 30$.
- Decomposing numbers in expanded form helps students understand how addition and subtraction algorithms work, as well as helps them use the distributive property when multiplying multi-digit numbers *(MTR.2.1)*.
- Throughout instruction, teachers should ensure students have practice with problems that include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. This will provide students opportunities to explain their thinking and show their work by using place-value strategies and algorithms. In addition to verifying that their answer is reasonable *(MTR.3.1, MTR.6.1)*.

Common Misconceptions or Errors

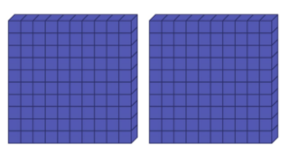
- When the value of a digit in a multi-digit whole number is 0, students can misunderstand that it represents 0 of that place value. For example, in the number 2,530, there are 0 $ones$. In the number 1,008, there are 0 $hundreds$ and 0 $tens$.

Strategies to Support Tiered Instruction

- Instruction includes using models and writing three- and four-digit numbers with a zero in various place values. A place value chart and models such as base-ten blocks or place value disks can be used to help students understand that when the value of a digit in a multi-digit whole number is 0, it represents a 0 of that place value.
    - For example, in the number 1,030 there are 0 *hundreds* (beyond the ten *hundreds* represented by the 1 in the *thousands* place) and 0 *ones* (beyond the *ones* represented by the other digits).

	thousands	***hundreds***	***tens***	***ones***
Standard Form	1	0	3	0
Place Value Disks				
Word Form	*one thousand*		*thirty*	
Expanded Form	1,000		30	
	1,000 + 30			

- For example, in the number 203, there are 0 *tens*.

	hundreds	***tens***	***ones***
Standard Form	2	0	3
Base-Ten Blocks			
Word Form	*two hundred*		*three*
Expanded Form	200		3
	200 + 3		

Instructional Tasks

Instructional Task 1

Henry says that the number 9,300 is read as nine thousand three. Noelle says that 9,300 is read as nine thousand thirty. Do you agree with either Henry or Noelle? Why or why not? Use expanded form to prove your thinking.

Instructional Items

Instructional Item 1

Which shows three thousand seventy-nine in expanded form?

a. $300 + 70 + 9$
b. $3{,}000 + 70 + 9$
c. $3{,}000 + 70 + 90$
d. $3{,}000 + 700 + 90$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.1.2

Benchmark

MA.3.NSO.1.2	Compose and decompose four-digit numbers in multiple ways using thousands, hundreds, tens and ones. Demonstrate each composition or decomposition using objects, drawings and expressions or equations.

Example: The number 5,783 can be expressed as $5\ thousands + 7\ hundreds + 8\ tens + 3\ ones$ or as $56\ hundreds + 183\ ones$.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.1
- MA.3.NSO.2.1

Terms from the K-12 Glossary

- Expression
- Whole numbers

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.2

Next Benchmarks

- MA.4.NSO.1.1
- MA.4.NSO.1.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to identify ways numbers can be written flexibly using decomposition. In addition to students knowing that number sense and computational understanding is built on a firm understanding of place value. This work extends from the Grade 2 expectation to compose and decompose three-digit numbers in multiple ways using hundreds, tens and ones (MA.2.NSO.1.2).

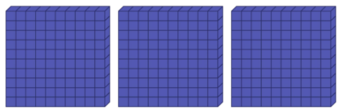
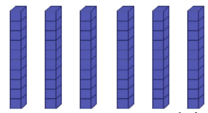
- Multiple representations of multi-digit whole numbers allow students to identify opportunities for regrouping while adding and subtracting. For example, when subtracting $5{,}783 - 892$, we can represent $5{,}783$ as $5\ thousands + 6\ hundreds + 18\ tens + 3\ ones$ by regrouping $1\ hundred$ as $10\ tens$, allowing us to subtract $9\ tens$ *(MTR.2.1, MTR.3.1)*.
- Students should use objects (e.g., base ten blocks), drawings, and expressions or equations side-by-side to see compare and contrast the representations. Model to show how multiple representations relate to the original number. For example, use base ten blocks to show how in the number $5{,}783$, $1\ hundred$ can be regrouped as $10\ tens$ to express it as $5\ thousands + 6\ hundreds + 18\ tens + 3\ ones$, while asking students how they are the same *(MTR.2.1)*.
- Allow students to decompose numbers in as many ways as possible. Have students compare and contrast the representations shared *(MTR.4.1)*.
- Students should see examples of numbers within $10{,}000$ where zero is a digit and make sense of its value.
- Flexibility of place value is a prerequisite for conceptual understanding of a standard algorithm for addition and subtraction with regrouping.

Common Misconceptions or Errors

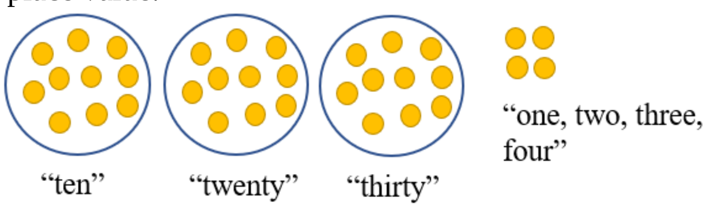
- Students can misunderstand that the 5 in 57 represents 5, not 50 or $5\ tens$. Students need practice with representing two and three-digit numbers with manipulatives that group (base ten blocks) and those that do NOT group, such as counters, etc.
- Students can misunderstand that when decomposing a number in multiple ways, the value of the number does not change. 879 is the same as $87\ tens + 9\ ones\ and\ 8\ hundreds + 79\ ones$.

Strategies to Support Tiered Instruction

- Instruction includes decomposing numbers using manipulatives that group (base ten blocks) and those that do not group such as counters. When decomposing a number, students focus on the value of each digit based on its place value. To reinforce this concept, students may count by units based on the place value.
    - For example, decompose 362 using base ten blocks and explain the value of each digit.

3	**6**	**2**
 "one hundred, two hundreds, three hundreds"	 "ten, twenty, thirty, forty, fifty, sixty"	 "one, two"
3 hundreds	6 tens	2 ones
300	60	2

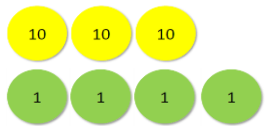
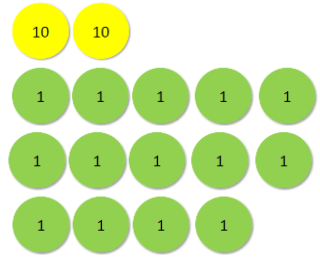
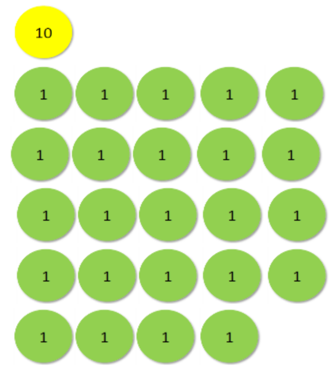
-
    - For example, represent 34 using counters and explain the value of each digit. Students group 10 ones as a group of ten and focus on the value of each digit based on its place value. To reinforce this concept, students count by units based on the place value.



- Teacher provides opportunities to decompose numbers in multiple ways using manipulatives and a chart to organize their thinking and asks students to name/identify the different ways to name the values (regrouping the hundreds into tens and the tens into the ones, e.g., 36 *tens* and 2 *ones* or 3 *hundreds* and 62 *ones*, etc.)
    - For example, students decompose 362 in multiple ways using hundreds, tens, and ones.

362			
	Example 1	**Example 2**	**Example 3**
Ones only	362 *ones*		
Tens and ones	36 *tens* + 2 *ones*	35 *tens* + 12 *ones*	29 *tens* + 72 *ones*
Hundreds and ones	3 *hundreds* + 62 *ones*	2 *hundreds* + 162 *ones*	1 *hundred* + 262 *ones*
Hundreds and tens	Not applicable for this example		
Hundreds, tens and ones	3 *hundreds* + 6 *tens* + 2 *ones*	2 *hundreds* + 15 *tens* + 12 *ones*	1 *hundred* + 24 *tens* + 22 *ones*

- For example, students decompose 34 in multiple ways using tens and ones.

34			
Tens and ones	3 *tens* + 4 *ones* 	2 *tens* + 14 *ones* 	1 *tens* + 24 *ones* 

Instructional Tasks

Instructional Task 1

Express the number 5,783 using only hundreds and ones.

Instructional Task 2

Express the number 5,783 using only thousands and hundreds.

Instructional Task 3

Express the number 5,783 using only tens and ones.

Instructional Items

Instructional Item 1

Select all the ways that express the number 8,709.

a. $8{,}000 + 600 + 19$
b. $8{,}000 + 700 + 9$
c. $879\ ones$
d. $8\ thousands + 6\ hundreds + 10\ tens + 9\ ones$
e. $8\ thousands + 7\ tens + 9\ ones$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.1.3

Benchmark

MA.3.NSO.1.3 Plot, order and compare whole numbers up to 10,000.

Example: The numbers 3,475; 4,743 and 4,753 can be arranged in ascending order as 3,475; 4,743 and 4,753.

Benchmark Clarifications:
Clarification 1: When comparing numbers, instruction includes using an appropriately scaled number line and using place values of the thousands, hundreds, tens and ones digits.
Clarification 2: Number lines, scaled by 50s, 100s or 1,000s, must be provided and can be a representation of any range of numbers.
Clarification 3: Within this benchmark, the expectation is to use symbols ($<$, $>$ or $=$).

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.1
- MA.3.NSO.1.2
- MA.3.AR.2.2

Terms from the K-12 Glossary

- Number Line
- Whole Number

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.3

Next Benchmarks

- MA.4.NSO.1.3

Purpose and Instructional Strategies

This purpose of this benchmark is for students to compare two numbers by examining the place values of thousands, hundreds, tens and ones in each number. This work extends from the Grade 2 expectation to plot, order and compare up to 1,000 (MA.2.NSO.1.2).

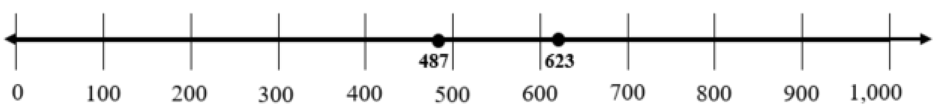
- Instruction should use the terms greater than, less, than, and equal. Students should use place value strategies and number lines (horizontal and vertical) to justify how they compare numbers and explain their reasoning. Instruction should not rely on tricks for determining the direction of the inequality symbols. Students should read entire statements (e.g., read $7{,}309 > 7{,}039$, "7,309 is greater than 7,039" and vice versa) *(MTR.2.1, MTR.3.1)*.
- It is imperative for teachers to define the meaning of the $\neq$ symbol through instruction. It is recommend that students use $=$ and $\neq$ symbols first. Once students have determined that numbers are not equal, then they can determine "how" they are not equal, with the understanding now the number is either $< or >$. If students cannot determine if amounts are $\neq$ or $=$ then they will struggle with $< or >$. This will build understanding of statements of inequality and help students determine differences between inequalities and equations *(MTR.6.1)*.

Common Misconceptions or Errors

- Often students think of these relational symbols as operational symbols instead. In order to address this misconception, allow students to have practice using the number line and/or place value blocks to see the relationship between one number and the other.

Strategies to Support Tiered Instruction

- Teacher uses a number line, base-ten blocks, place value charts and relational symbols to demonstrate the relationship between one number and the other.
    - For example, the teacher uses a number line and relational symbols to compare 487 and 623, labeling the endpoints of the number line 0 and 1,000. The teacher asks students to place 487 and 623 on the number line, discussing the placement of the numbers and distance from zero. Next, the teacher uses the number line to demonstrate that 487 is closer to zero than 623 so $487 < 623$ and that 623 is farther from zero so $623 > 487$. Then, the teacher explains that 487 and 623 are not the same point on the number line so $487 \neq 623$ and asks students to identify numbers that are greater than... and less than.... Finally, the teacher repeats with two four-digit numbers (number line endpoints of 0 and 10,000) and discusses the placement of the other numbers on the number line and if their values are greater than or less than other numbers.



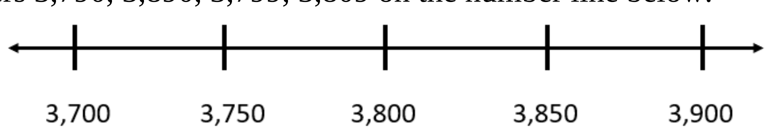
    - For example, the teacher uses base-ten blocks, a place value chart and relational symbols to compare 274 and 312. The teacher begins by having students represent 274 and 312 using base-ten blocks and a place value chart and asking students to compare these numbers, beginning with the greatest place value. Next, the teacher explains that the number 274 has $2\ hundreds$ and the number 312 has $3\ hundreds$ so $274 < 312$ and $312 > 274$ and that 274 and 312 have different digits in the hundreds place so $274 \neq 312$.

hundreds	**tens**	**ones**
2	**7**	**4**
3	**1**	**2**

Instructional Tasks

Instructional Task 1

Plot the numbers 3,790, 3,890, 3,799, 3,809 on the number line below.



Choose two values from the list and compare them using $>$, $<$, or $=$.
Choose a number between 3,799 and 3,809 and plot it on the number line.
Use evidence from your number line to justify which number is greatest.

Instructional Items

Instructional Item 1

Which of the following correctly compares 6,909 and 6,099?

a. $6{,}909 < 6{,}099$, because the value of the 9 in the tens place of 6,099 is greater than the value of the 0 in the tens place of 6,909.
b. $6{,}909 > 6{,}099$, because the value of the 9 in the tens place of 6,099 is greater than the value of the 0 in the tens place of 6,909.
c. $6{,}909 < 6{,}099$, because the value of the 9 in the hundreds place of 6,909 is greater than the value of the 0 in the hundreds place of 6,099.
d. $6{,}909 > 6{,}099$, because the value of the 9 in the hundreds place of 6,909 is greater than the value of the 0 in the hundreds place of 6,099.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.1.4

Benchmark

MA.3.NSO.1.4 Round whole numbers from 0 to 1,000 to the nearest 10 or 100.

Example: The number 775 is rounded to 780 when rounded to the nearest 10.
Example: The number 745 is rounded to 700 when rounded to the nearest 100.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.3
- MA.3.NSO.2.1

Terms from the K-12 Glossary

- Number Line
- Whole Number

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.4

Next Benchmarks

- MA.4.NSO1.4

Purpose and Instructional Strategies

The purpose of this benchmark is for students to use place value understanding to explain and reason about rounding. It is important for students to have numerous experiences using a number line, a place-value chart and a hundred chart to support their work with rounding to assist with their understanding of knowing when and why to round numbers providing opportunities to investigate and explore place value *(MTR.2.1)*. This benchmark continues instruction of rounding from Grade 2, where students rounded numbers from 0 to 100 to the nearest 10 (MA.2.NSO.1.4).

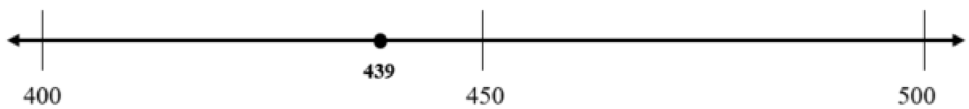
- Instruction of rounding should include place value representations (e.g., base ten blocks) and number lines (both horizontal and vertical) *(MTR.2.1)*.
- Students should identify benchmarks based on place value to justify rounding. For example, when rounding 643 to the nearest ten, students should use place value to determine that 643 falls between the benchmark tens, 640 and 650. Between 640 and 650, a number line shows that 643 is closer to 640 than 650. When rounding 643 to the nearest hundred, students should use place value to determine that 643 is between the benchmark hundreds, 600 and 700. Between 600 and 700, 643 is closer to 600 than 700 *(MTR.2.1, MTR.3.1)*.
- During instruction have students practice identifying possible answers and halfway points. In addition to, understanding that, by rule, if a number is exactly at the halfway point of two possible answers, the number is rounded up. For example, students learn the convention that when the value to the right of the rounded place value is 5, they round up to the greater of the two benchmark values. For example, when rounding 765 to the nearest ten, 765 is the same distance between 760 and 770. The rounding convention tells us to round up to 770 *(MTR.2.1, MTR.3.1)*.
- Rounding numbers is a skill that helps students estimate reasonable solutions when adding or subtracting. Instruction of rounding skills should be taught within the context of estimating while adding or subtracting. Rounding numbers in an expression should be done before performing operations to estimate reasonable sums or differences. Rounding sums and differences should not be done after students have already performed operations.
- Instruction should not focus on tricks for rounding that do not focus on place value understanding or the use of number lines.

Common Misconceptions or Errors

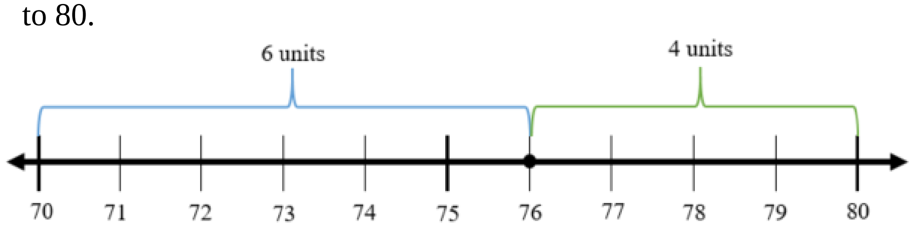
- Students can confuse the place value to which they are rounding. For example, students mistakenly round 923 to 900 when rounding to the nearest ten because they observe 2 tens and round to 900, instead of using the ones value of 3 to help them determine that 923 is closest to 920. The use of benchmarks numbers and number lines helps students understand rounding conceptually.
- Students assume numbers that are already located at benchmarks cannot be rounded. For example, students think that 920 cannot be rounded to the nearest ten.
- It is imperative for students to develop a conceptual understanding of rounding, such as what the benchmarks are, using place value understanding to round numbers without instruction of mnemonics, rhymes or songs.

Strategies to Support Tiered Instruction

- Instruction includes using number lines, benchmark numbers and place value understanding to round numbers to the nearest ten and hundred. To develop a conceptual understanding of rounding, such as what the benchmarks are, students use place value understanding to round numbers without instruction using mnemonics, rhymes or songs.
    - For example, students round 439 to the nearest hundred using a number line and place value understanding. The teacher explains that the endpoints of our number line will be represented using hundreds, because we are rounding to the nearest hundred. The teacher then explains that there are 4 hundreds in the number 439 and one more hundred would be 5 hundreds and represents these endpoints on the number line as 4 $hundreds$ (400) and 5 $hundreds$ (500). Next, the teacher explains that the mid-point on the number line can be labeled as 4 hundreds and 5 tens (450). This midpoint is halfway between 400 and 500. The teachers ask students to plot 439 on the number line and discuss if it is closer to 400 or 500. Then, explains that 439 rounds to 400 because it is 61 units away from 500 and only 39 units away from 400.



    - For example, students round 76 to the nearest ten using a number line and place value understanding. The teacher explains that the endpoints of the number line will be represented using tens, because we are rounding to the nearest ten. Then, the teacher explains that there are 7 $tens$ in the number 76 and one more ten would be 8 $tens$ and represents these endpoints on the number line as 7 $tens$ (70) and 8 $tens$ (80). The mid-point on the number line can be labeled as 7 $tens$ and 5 $ones$ (75). This midpoint is halfway between 70 and 80. The teacher asks students to plot 76 on the number line and discuss if it is closer to 70 or 80. Then, explains that 76 rounds to 80 because it is 6 $units$ away from 70 and only 4 $units$ away from 80. Once students master this concept, there should be a discussion about rounding the number 75 where the choice is made to round it up to 80.



Instructional Tasks

Instructional Task 1

Part A. Emily is thinking of a mystery number that rounds to 350 when rounded to the nearest ten and 300 when rounded to the nearest hundred. Could Emily's number be 344? Why or why not?

Part B. Give two numbers that could be Emily's mystery number. Justify your answer using a number line.

Instructional Items

Instructional Item 1

Identify all the true statements.

a. 302 rounded to the nearest ten is 300.
b. 302 rounded to the nearest ten is 310.
c. 302 rounded to the nearest hundred is 300.
d. 493 rounded to the nearest ten is 500.
e. 493 rounded to the nearest ten is 490.
f. 493 rounded to the nearest hundred is 500.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.2 *Add and subtract multi-digit whole numbers. Build an understanding of multiplication and division operations.*

MA.3.NSO.2.1

Benchmark

MA.3.NSO.2.1 Add and subtract multi-digit whole numbers including using a standard algorithm with procedural fluency.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.4
- MA.3.AR.1.2
- MA.3.M.1.2

Terms from the K-12 Glossary

- Expression
- Equation
- Whole Number

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.2.1
- MA.2.NSO.2.2
- MA.2.NSO.2.3
- MA.2.NSO.2.4

Next Benchmarks

- MA.4.NSO.2.7

Purpose and Instructional Strategies

The purpose of this benchmark is for students to add and subtract multi-digit whole numbers with procedural fluency. Students use skills from the procedural reliability stage in Grade 2 to become fluent with efficient and accurate procedures, including standard algorithms for addition and subtraction.

- A standard algorithm is defined as any efficient and accurate procedure that allows students to add and subtract whole numbers. Students' choices of standard algorithms for addition and subtraction do not need to be the same *(MTR.5.1)*.
- Students should be able to justify their use of a standard algorithm for adding and subtracting by explaining the steps mathematically. Each student should be able to explain if and when regrouping is needed, and how regrouping is computed using their chosen algorithm. During instruction, teachers and students should work together to relate place value understanding to algorithms *(MTR.3.1, MTR.4.1, MTR.5.1)*.
- Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties.
- Instruction of this benchmark should be taught with MA.3.NSO.1.4. Students should use rounding as a means to estimate reasonable solutions of sums and differences before calculating *(MTR.6.1)*.

Common Misconceptions or Errors

- Students who learn a standard algorithm without being able to explain why it works using place value understanding often make computational errors and/or cannot determine if their solutions are reasonable. To assist students with this misconception, teachers should expect students to justify the algorithm they choose.

Strategies to Support Tiered Instruction

- Instruction includes guiding students through the process of estimating reasonable values for sums and differences using understanding of place value, addition and subtraction.
  - For example, students make reasonable estimates for the sum of $174 + 253$. Instruction includes a prompt such as "Before using an algorithm, we will estimate the sum to make sure that we are using the algorithm correctly and our answer is reasonable. The first addend of 174 is close to the benchmark number 200 and the second addend of 253 is close to the benchmark number 250. So, we can use $200 + 250 = 450$ to estimate that our sum should be close to 450."
- Instruction includes guiding students through the process of explaining and justifying the chosen algorithm and determining if an algorithm was used correctly by reviewing the reasonableness of solutions.
  - For example, students use a standard algorithm to solve $174 + 253$ and explain their thinking using a place value visual representation. Instruction includes a prompt such as "Begin by adding in the ones place. 4 ones plus 3 ones is 7 ones. Because the total number of ones is less than 10 ones, it is not necessary to regroup. Next, add in the tens place. 7 $tens$ plus 5 $tens$ is 12 $tens$. Because I have more than 10 $tens$ it is necessary to regroup the 10 $tens$ to make $one\ hundred$. After composing a group of 10 $tens$ there are 2 $tens$ remaining. Finally, add 1 $hundred$ plus 2 $hundreds$. Add the 1 $hundred$ that was regrouped from the tens place. The sum is 427. Our sum of 427 is close to our

estimate of 450, this helps us determine that our answer is reasonable"

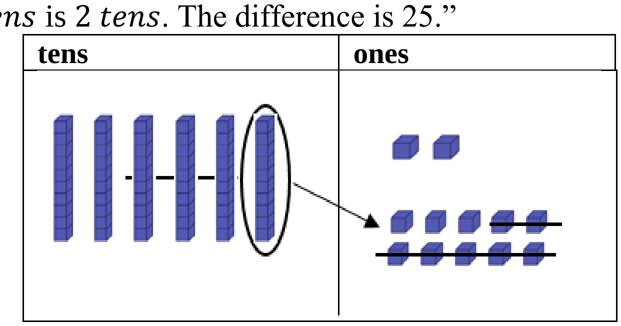
$$\begin{array}{cccc} & 1 & & \\ & 1 & 7 & 4 \\ + & 2 & 5 & 3 \\ \hline & 4 & 2 & 7 \end{array}$$

- For example, students use a standard algorithm to solve $327 - 174$ and explain their thinking using a place value visual representation. Instruction includes prompt such as "Begin subtracting 174 starting in the ones place. 7 *ones* minus 4 *ones* are 3 *ones*. There are not enough tens to subtract 7 *tens* from 2 *tens*. It is necessary to decompose one hundred into 10 *tens*. Now there are 12 *tens*, and there is enough to subtract 7 *tens*. 12 *tens* minus 7 *tens* equals 5 *tens*. Finally, subtract the hundreds: 3 *hundreds* minus 1 *hundred* equals 2 *hundreds*. The difference is 253."

$$\begin{array}{cccc} & 3 & & \\ & \cancel{4} & \boxed{12} & 7 \\ - & 1 & 7 & 4 \\ \hline & 2 & 5 & 3 \end{array}$$

- For example, students use a standard algorithm and base-ten blocks to solve $62 - 37$ and explain their thinking using a place value visual representation. Instruction includes a prompt such as "Begin subtracting 37 starting in the ones place. There are not enough ones to subtract 7 *ones* from 2 *ones*. It is necessary to decompose *one ten* into 10 *ones*. Now there are 12 *ones* and there is enough to subtract 7 *ones*. 12 *ones* take away 7 *ones* equals 5 *ones*. Finally, subtract the tens: 5 *tens* minus 3 *tens* is 2 *tens*. The difference is 25."

$$\begin{array}{ccc} & 5 & \\ & \cancel{6} & \boxed{12} \\ - & 3 & 7 \\ \hline & 2 & 5 \end{array}$$



- Teacher provides guidance on using strategies based on place value to add and subtract.
  - For example, students use strategies based on place value to solve $174 + 253$.

$$\begin{array}{cccc} & 1 & 7 & 4 \\ + & 2 & 5 & 3 \\ \hline & & & 7 \\ & 1 & 2 & 0 \\ + & 3 & 0 & 0 \\ \hline & 2 & 2 & 7 \end{array}$$

sum of ones
sum of tens
sum of hundreds

$$\begin{array}{rcccccc} 174 = & 100 & + & 70 & + & 4 & \\ 253 = & 200 & + & 50 & + & 3 & \\ \hline & 300 & + & 120 & + & 7 & = 427 \end{array}$$

Instructional Tasks

Instructional Task 1

Miranda finds 492 seashells during her vacation. She now has 1,045 seashells in her collection. How many seashells did she have in her collection before vacation?

Part A. Solve using a standard algorithm.

Part B. Indicate one step where you needed to regroup while solving and show how you did it using words or a pictorial model.

Instructional Items

Instructional Item 1

What is the sum of 1,432 and 2,981?

Instructional Item 2

What is the difference of 8,000 and 1,432?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.2.2

Benchmark

MA.3.NSO.2.2 Explore multiplication of two whole numbers with products from 0 to 144, and related division facts.

Benchmark Clarifications:

Clarification 1: Instruction includes equal groups, arrays, area models and equations.

Clarification 2: Within the benchmark, it is the expectation that one problem can be represented in multiple ways and understanding how the different representations are related to each other.

Clarification 3: Factors and divisors are limited to up to 12.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.3
- MA.3.NSO.2.4
- MA.3.AR.2.1
- MA.3.AR.2.2
- MA.3.GR.2.2
- MA.3.GR.2.4

Terms from the K-12 Glossary

- Area Model
- Commutative Property of Multiplication
- Dividend
- Divisor
- Equation
- Expression
- Factors
- Rectangular Array

Vertical Alignment

Previous Benchmarks

- MA.2.AR.3.2

Next Benchmarks

- MA.4.NSO.2.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to build conceptual understanding of what multiplication is and how it relates to division. Because the expectation of this benchmark is at the explore level, instruction should focus on building understanding of multiplication and division facts from 0 to 144 using manipulatives (e.g., counters), visual models (e.g., rectangular arrays, equal groups), discussions, estimation and drawings (e.g., rectangular arrays, equal groups) *(MTR.2.1)*.

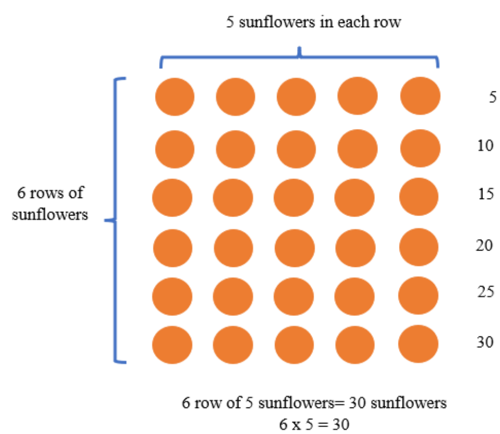
- Instruction should relate multiplication to repeated addition work that began in Grade 2. In Grade 2, students used repeated addition to find the total number of objects using rectangular arrays and equations (MA.2.AR.3.2).
- Students should explore multiplication and division through word problems, writing expressions and drawing models that match the problems' contexts *(MTR.2.1, MTR.3.1)*.
- In division, students should see examples of sharing, or partitive division (where the number of groups are given and students determine the number in each group), as well as measurement, or quotative division (where the number in each group is given and students determine the number of groups).
- Instruction should relate division facts to known multiplication facts (e.g., fact families). Fact families can be explored through arrays and equal groups *(MTR.5.1)*.

Common Misconceptions or Errors

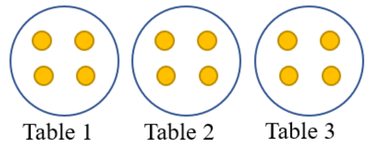
- Students may have difficulty relating word problems and real-world scenarios to models, expressions, and equations. For example, students may not differentiate the number of groups versus number in each group in multiplication, which then impacts their models, expressions, and equations.
- Students may be confused by measurement (or quotative) division, when the amount in each group is given and the number of equal-sized groups is found.

Strategies to Support Tiered Instruction

- Instruction includes demonstrating the use of counters, arrays and skip counting to model groups of objects, including the use of real-world scenarios to support students' understanding of the number of groups versus the size of each group. Students represent their models with equations to reinforce the concept of multiplication.
  - For example, a farmer is planting rows of sunflowers. He plants 6 rows with 5 sunflowers in each row. How many sunflowers does he plant?

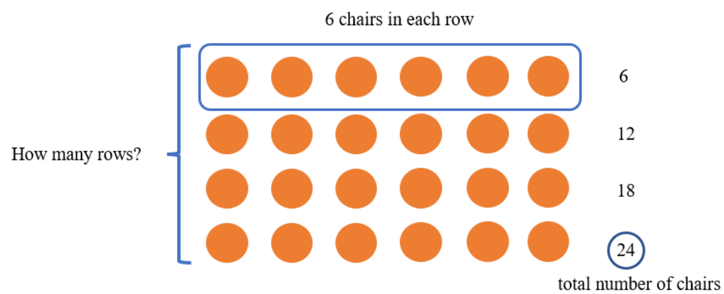


- For example, there are 3 tables in the library. There are 4 students sitting at each table. How many students are sitting at tables in the library?



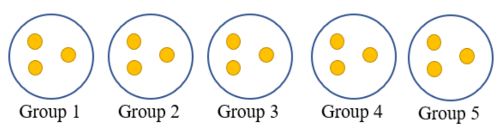
3 groups of 4 students = 12 students
$3 \times 4 = 12$

- Instruction includes demonstrating the use of counters and arrays to model division problems where the amount in each group is given and the number of equal-sized groups is found. The teacher provides real-world scenarios to represent the number of objects in each group and the number of groups Students form a group based on the context of the problem continuing to form groups of that size until the total is reached. Students can skip count to keep track of how many counters they have used, representing their models with equations to reinforce the concept of division.
  - For example, Renee is setting up chairs in the library. She is placing 24 chairs into rows. If she places 6 chairs in each row, how many rows of chairs will she have?



24 chairs divided into rows of 6
$24 \div 6 = 4$
There are 4 rows of chairs.

  - For example, there are 15 students working on an art project. The art teacher divides them into groups of 3 students to work on the project. How many groups are there?



15 students divided into groups of 3
$15 \div 3 = 5$
There are 5 groups of students.

Instructional Tasks

Instructional Task 1

Tina has 4 shelves on her bookshelf. Each row has 6 books. How many books are on Tina's bookshelf in all? Draw a model and write an equation to represent your answer.

Instructional Items

Instructional Item 1

A total of 56 chairs are in the cafeteria for an assembly. The principal arranges the chairs into 8 rows with the same number of chairs in each. Which equation shows the quotient as the number of chairs that will be in each row?

a. $56 \div 8 = 7$
b. $56 \div 8 = 48$
c. $56 \div 8 = 64$
d. $56 \div 8 = 6$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.2.3

Benchmark

MA.3.NSO.2.3 Multiply a one-digit whole number by a multiple of 10, up to 90, or a multiple of 100, up to 900, with procedural reliability.

Example: The product of 6 and 70 is 420.
Example: The product of 6 and 300 is 1,800.

Benchmark Clarifications:
Clarification 1: When multiplying one-digit numbers by multiples of 10 or 100, instruction focuses on methods that are based on place value.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2
- MA.3.NSO.2.4
- MA.3.AR.1.1
- MA.3.AR.1.2
- MA.3.GR.2.2
- MA.3.GR.2.4

Terms from the K-12 Glossary

- Expression
- Equation
- Factor
- Whole Number

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.2.2

Next Benchmarks

- MA.4.NSO.2.2
- MA.4.NSO.2.3

Purpose and Instructional Strategies

The purpose of this benchmark is for students to use place value reasoning to multiply single-digit factors (0-9) by multiples of 10 up to 90 (10, 20, 30, 40, 50, 60, 70, 80, 90) and multiples of 100 up to 900 (100, 200, 300, 400, 500, 600, 700, 800, 900). Because the expectation of this benchmark is at the procedural reliability level, students should develop accurate, reliable methods for multiplication that align with their understanding and learning style.

- Instruction should connect known facts of one-digit factors (e.g., 6×7), to then apply to products of one-digit numbers and multiples of 10 or 100 (e.g., 6×70, 60×7, 6×700, 600×7) *(MTR.5.1)*.
- Teachers should use place value representations (e.g., pictures, diagrams, base ten blocks, place value chips) to show relationships between known facts and multiplying one-digit factors by multiples of 10 or 100. For example, 3×4 can be interpreted as 3 groups of 4 ones, or 12 ones. 3×40 can be represented as 3 groups of 4 tens, or 12 tens. 12 tens is equal to **120 ones**. 3×400 can be represented as 3 groups of 4 hundreds, or 12 hundreds. 12 hundreds is equal to 120 tens or **1,200 ones** *(MTR.5.1)*.
- This standard lays the foundation for multi-digit multiplication. For benchmark 3.AR.1.1, students use the distributive property to multiply 34×8 as $(30 \times 8) + (4 \times 8)$. This benchmark (MA.3.NSO.2.3) helps students reason that $30\ x\ 8$ is the same as $3\ tens \times 8$, or $24\ tens\ (240)$.
- Instruction should not focus on "adding zeroes to the end" when multiplying one-digit factors by multiples of 10 and 100. For example, $7\ x\ 50$ should not be reduced to "7×5 with one zero at the end." This trick does not focus on place value methods, as Clarification #1 of the benchmark requires.

Common Misconceptions or Errors

- Students can quickly jump to the conclusion that they can "count zeroes" to determine the number of zeroes in the product (e.g., the product of $7\ x\ 500$ will have two zeroes because 500 has two zeroes). This can confuse students when the products of the known facts already end in zero (e.g., using $5 \times 8 = 40$ to multiply 5×80). Students who rely on this trick will often indicate that $5 \times 80 = 40$ because they see only one zero in the factors.

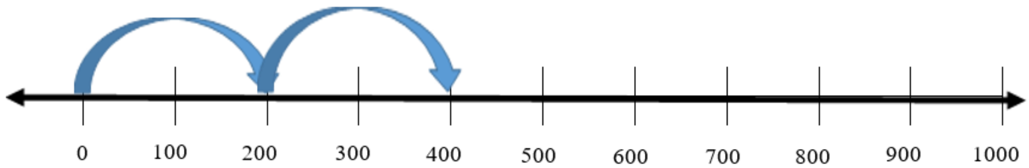
Strategies to Support Tiered Instruction

- Instruction includes opportunities to connect grouping numbers by multiples in different ways.
    - For example, students may place the following facts on the hundreds chart: 1×10, 2×10, 3×10, 4×10, 5×10, 6×10, 7×10, 8×10 and 9×10. The teacher asks students what patterns they notice.

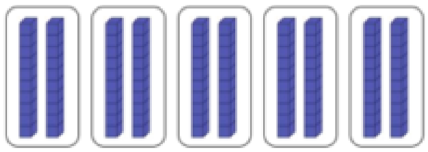
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Instruction includes opportunities to use a number line. Students skip count by multiples on the number line. This will support a conceptual understanding of what is happening with the numbers, instead of focusing on the "zero trick."

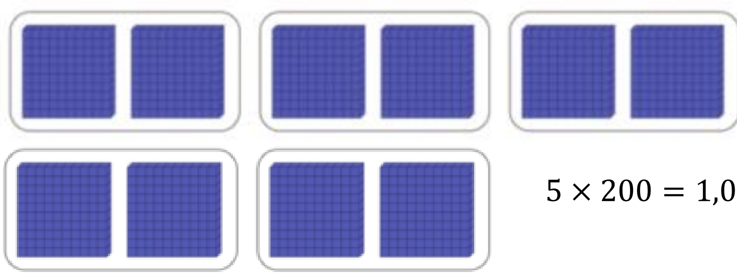
$$2 \times 200 = 400$$



- Instruction includes opportunities to connect grouping numbers by multiples.
    - For example, students use manipulatives to show that 5 groups of 20 is 100 and 5 groups of 200 is 1,000. Teacher should be explicit about the multiples and not point out the zeros trick.



$$5 \times 20 = 100$$



$$5 \times 200 = 1{,}000$$

Instructional Tasks

Instructional Task 1

The table below shows the costs for entry at the Sunnyland Amusement Park.

Type of Ticket	Cost per person
Adult (ages 13 and 54)	\$30
Child (ages 3 to 12)	\$10
Senior (ages 55 and up)	\$20
Children 2 and under	Free

a. How much does entry cost for nine adults? Write an equation to show the total cost?
b. Write an expression that shows the total cost for one senior and one 2-year-old child to attend Sunnyland Amusement Park.
c. The Suarez Family purchases 2 adult tickets, 1 senior ticket and 1 ticket for their 6-year-old daughter. Write an equation to show the total cost of entry for the family.
d. Which cost of entry is less expensive, 2 seniors or 3 children? Explain how you know using words, a picture or equations.

Instructional Items

Instructional Item 1

Write two different equations using a one-digit whole number and a multiple of 10 that show a product of 120.

$\_\_\_\_ \times \_\_\_\_ = 120$ $\qquad$ $\_\_\_\_ \times \_\_\_\_ = 120$

Instructional Item 2

Write two different equations using a one-digit whole number and a multiple of 100 that show a product of 2,400.

$\_\_\_\_ \times \_\_\_\_ = 2{,}400$ $\qquad$ $\_\_\_\_ \times \_\_\_\_ = 2{,}400$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.NSO.2.4

Benchmark

MA.3.NSO.2.4 Multiply two whole numbers from 0 to 12 and divide using related facts with procedural reliability.

Example: The product of 5 and 6 is 30.
Example: The quotient of 27 and 9 is 3.

Benchmark Clarifications:
Clarification 1: Instruction focuses on helping a student choose a method they can use reliably.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2
- MA.3.NSO.2.3
- MA.3.AR.2.2
- MA.3.AR.3.3
- MA.3.GR.2.4

Terms from the K-12 Glossary

- Expression
- Equation
- Factor
- Dividend
- Divisor
- Commutative property of multiplication
- Associative property of multiplication
- Distributive property of multiplication

Vertical Alignment

Previous Benchmarks

- MA.2.AR.3.2

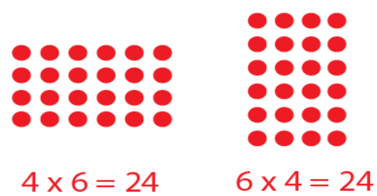
Next Benchmarks

- MA.4.NSO.2.1
- MA.4.NSO.2.2
- MA.4.NSO.2.3
- MA.4.NSO.2.4

Purpose and Instructional Strategies

The purpose of this benchmark is for students to utilize skills from the exploration stage of multiplication and division (MA.3.NSO.2.2) to develop an accurate, reliable method that aligns with the student's understanding and learning style. Procedural fluency of multiplication facts with factors up to 12 and their related division facts is not expected until Grade 4 *(MTR.2.1, MTR.3.1)*.

- This benchmark provides the opportunity for students to generalize patterns they see within the tools used during the exploration stage (e.g., rectangular arrays, equal groups) to then identify multiplication and related division facts *(MTR.4.1)*.
- Instruction that builds procedural reliability should connect multiplication understanding with the properties of multiplication (commutative, associative and distributive). The patterns students recognize help them relate facts to one another, and to use the related facts to find the products and quotients of unknown facts. In this benchmark, students should be able to explain how they know facts and how they can find products of unknown facts *(MTR.5.1)*. For example, students should recognize that $4 \; x \; 6$ and $6 \; x \; 4$ have the same product of 24 and identify this pattern as evidence of the commutative property of multiplication. This can also be discovered through arrays for multiplication using objects or drawings, where students can observe that the arrays contain the same total number of squares, but the orientation of the array has just rotated so the rows and columns are switched as shown below *(MTR.5.1)*.



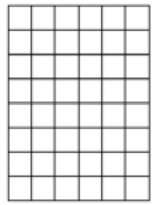
Common Misconceptions or Errors

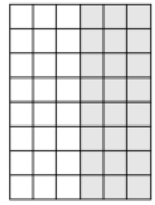
- This benchmark does not support students' memorization of multiplication and division facts. Memorization does not indicate work toward multiplication and division fact fluency. Students should be able to explain how they know multiplication and division facts, and how they can find products and quotients of unknown facts.

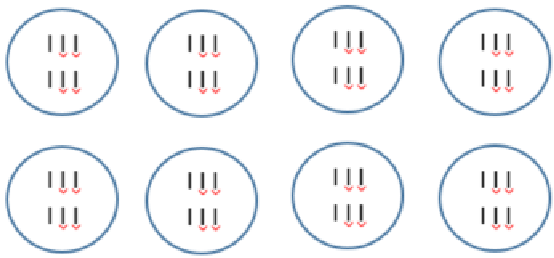
Strategies to Support Tiered Instruction

- Instruction includes opportunities to experience the properties of multiplication and division. Students use and apply properties to build procedural fluency. Students should understand that multiplication and division both involve grouping equal sets of numbers or objects.
    - For example, the teacher shows students an array of $8 \times 6 = 48$ and has them describe what they see with rows and columns. This learning can be connected to the concept of "groups of" objects, 8 groups of 6 is the same as 8 jumps of 6 on the number line.

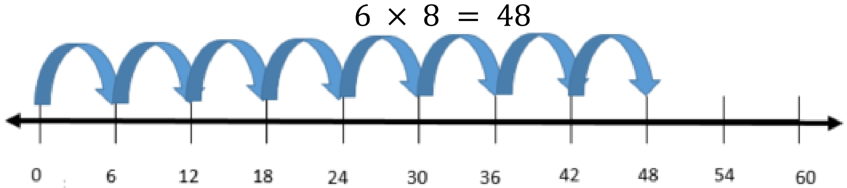
$8 \times 6 = 48$ $(8 \times 3) + (8 \times 3)$



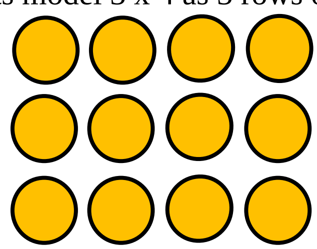




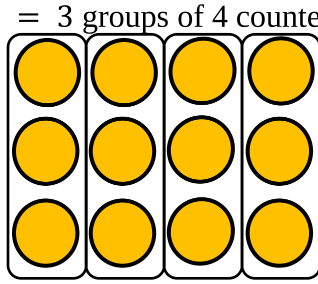
$6 \times 8 = 48$



- Teacher provides opportunities to build and manipulate what a multiplication fact looks like and then relates how it looks as division.
    - For example, students model 3 x 4 as 3 rows of 4 with counters.



The teacher then relates the multiplication model to division by separating the rows into groups. $12 = 3$ groups of 4 counters, or 12 divided by $3 = 4$.



Instructional Tasks

Instructional Task 1

Part A. Show how to find the product of 6×7 in two different ways.
Part B. Identify the related division facts from your equation in Part A.

Instructional Items

Instructional Item 1

What is the product of 11 and 4?

Instructional Item 2

Provide two division facts that have a quotient of 8.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Fractions

MA.3.FR.1 *Understand fractions as numbers and represent fractions.*

MA.3.FR.1.1

Benchmark

MA.3.FR.1.1	Represent and interpret unit fractions in the form $\frac{1}{n}$ as the quantity formed by one part when a whole is partitioned into *n* equal parts.

Example: $\frac{1}{4}$ can be represented as $\frac{1}{4}$ of a pie (parts of a shape), as 1 out of 4 trees (parts of a set) or as $\frac{1}{4}$ on the number line.

Benchmark Clarifications:
Clarification 1: This benchmark emphasizes conceptual understanding through the use of manipulatives or visual models.
Clarification 2: Instruction focuses on representing a unit fraction as part of a whole, part of a set, a point on a number line, a visual model or in fractional notation.
Clarification 3: Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.2
- MA.3.FR.1.3
- MA.3.FR.2.1
- MA.3.FR.2.2

Terms from the K-12 Glossary

- Number line

Vertical Alignment

Previous Benchmarks

- MA.2.FR.1.1
- MA.2.FR.1.2

Next Benchmarks

- MA.4.FR.2.1
- MA.4.FR.2.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to understand that unit fractions are the foundation for all fractions. Second, the purpose is for students to understand that fractions are numbers. This benchmark continues instruction of fractions from Grade 2, where students partitioned circles and rectangles into two, three or four equal-sized parts (MA.2.FR.1.1 and MA.2.FR.1.2).

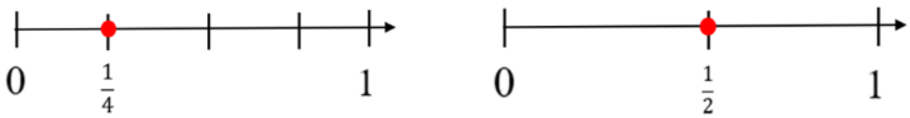
- To activate prior knowledge in Grade 3, instruction should:
  - relate how unit fractions build fractions to how whole-number units build whole numbers, and
  - show models with non-equal parts as non-examples *(MTR.2.1)*.
- Unit fractions are defined as one part when a whole is partitioned in any number of equal parts. It is in this benchmark that students conclude that the greater a unit fraction's denominator, the greater its number of parts.
- Instruction should demonstrate how to represent unit fractions using manipulatives (e.g., fraction strips, circles, relationship rods), visual area models (e.g., partitioned shapes), on a number line, and as 1 object in a set of objects *(MTR.2.1, MTR.5.1)*.
- Denominators are limited in Grade 3 to facilitate the visualizing and reasoning required while students plot, compare and identify equivalence in fractions.

Common Misconceptions or Errors

- Students can misconceive the difference between the meaning of numerators and denominators in fractions. For this reason, it is important for teachers and students to represent unit fractions in multiple ways to understand how they relate to a whole. Representations can be modeled together (e.g., fraction strips side-by-side with number lines, or relationship rods side-by-side with number lines) to help build student understanding.
- Students can misconceive that the smaller the denominator, the smaller the piece, or the larger the denominator, the larger the piece. This is due to thinking and reasoning where students worked with whole numbers (the smaller a number, the less it is, or the larger a number, the more it is). To correct this misconception, have students utilize different models, such as fraction bars and number lines, which would provide students opportunities to compare unit fractions and to reason about their sizes.
- Students can misconceive that all shapes can be partitioned the same way. To assist with this misconception, have students practice with presenting shapes other than circles, squares or rectangles to prevent students from over generalizing that all shapes can be divided the same way.

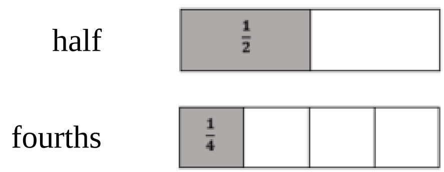
Strategies to Support Tiered Instruction

- Teacher represents unit fractions in multiple ways to show understanding of how they relate to a whole. Representations are modeled together (e.g., fraction strips side-by-side with number lines, or relationship rods side-by-side with number lines) to help build understanding.



- Instruction includes partitioning shapes into different denominators.
  - For example, students compare what they notice about partitioning a rectangle into halves verses fourths. Teacher asks students, “What do you notice about the pieces? How can we write what one piece of the rectangle is worth with a fraction?” Instruction includes the vocabulary of numerator and denominator.

$\frac{1}{n}$ = Number of equal parts



- Instruction includes shapes other than circles and rectangles. Items like pattern blocks allow students to partition shapes like hexagons and rhombi into equal sized pieces. This prevents students from over-generalizing that all shapes can be divided the same way.
- Instruction includes folding and/or cutting premade shapes into different amounts. Students benefit from beginning with halves and fourths, folding the paper in half, and then folding those halves into halves to make fourths.
  - For example, the teacher asks students, “What do you notice about the shapes? About the size? We now have 4 pieces, do we have more than we did before?” Conversation includes the size of the pieces and how that relates to the denominator.

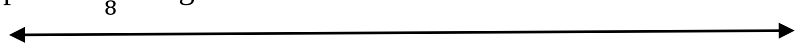
Instructional Tasks

Instructional Task 1

Terry wants to show the unit fraction $\frac{1}{8}$ using an area model, a number line, and as a set.

Part A. Into how many equal parts should Terry partition his area model? How many of those parts should be shaded? Explain in words.

Part B. Represent $\frac{1}{8}$ using the number line below.

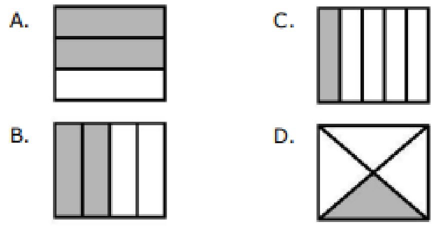


Part C. Draw a model that represents $\frac{1}{8}$ of a set of juice boxes.

Instructional Items

Instructional Item 1

Each model shown has been shaded to represent a fraction. Which model shows $\frac{1}{4}$ shaded?



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.FR.1.2

Benchmark

MA.3.FR.1.2 Represent and interpret fractions, including fractions greater than one, in the form of $\frac{m}{n}$ as the result of adding the unit fraction $\frac{1}{n}$ to itself m times.

Example: $\frac{9}{8}$ can be represented as $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$.

Benchmark Clarifications:

Clarification 1: Instruction emphasizes conceptual understanding through the use of manipulatives or visual models, including circle graphs, to represent fractions.

Clarification 2: Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.1
- MA.3.FR.1.3
- MA.3.FR.2.1
- MA.3.FR.2.2

Terms from the K-12 Glossary

- Number Line

Vertical Alignment

Previous Benchmarks

- MA.2.FR.1.1
- MA.2.FR.1.2

Next Benchmarks

- MA.4.FR.2.1
- MA.4.FR.2.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to think conceptually about fractions as they plot, compare, order and determine equivalence in Grade 3. It also allows students to develop the counting strategies and additive reasoning required to add and subtract fractions in Grade 4 *(MTR.2.1, MTR.5.1)*.

- During instruction, teachers should have students practice representing fractions using manipulatives (e.g., fraction strips, circles, relationship rods), visual area models (e.g., partitioned shapes) and on a number line. Manipulatives, visual models and number lines must extend beyond 1 so that students can represent fractions greater than *one (MTR.2.1, MTR.5.1)*.
- In instruction of MA.3.FR.1.1, students learn that unit fractions are the foundation for all fractions. MA.3.FR.1.2 builds understanding that all fractions, including fractions equal to and greater than one, decompose as the sum of unit fractions.
- In understanding fractions are numbers, students make connections about whole number operations that will allow them to perform operations with fractions in later grades. For example, understanding fractions as numbers allows students to reason that $\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$ in Grade 4 because we are adding together a total of 4 parts that are each one-third in size *(MTR.5.1)*.

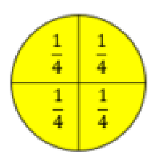
Common Misconceptions or Errors

- Students can misconceive that fractions equal to and greater than 1 can also be represented as the sum of unit fractions (e.g., $\frac{5}{2}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$). Flexible representations of models (e.g., rectangular area models that align with number lines) help students connect understanding of fractions and how they are decomposed into unit fractions.

Strategies to Support Tiered Instruction

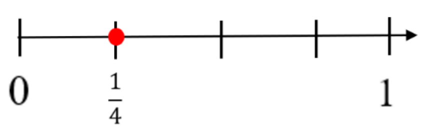
- Instruction includes modeling how fractions are decomposed. Using fraction circles, students build $\frac{4}{4}$ and then see that there are 4 pieces that make up the whole circle.
    - Example:

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}$$



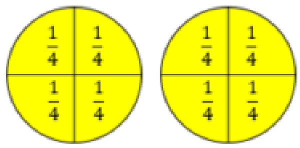
- Instruction includes more than one model so that students can experience and connect fractions in multiple ways. Flexible representations of models (e.g., rectangular area models that align with number lines) help students connect understanding of fractions and how they are decomposed into unit fractions.
    - Example:

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$



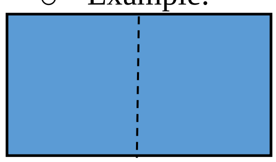
Students then apply this understanding to fractions greater than one. Using fraction circles, students build $\frac{8}{4}$ and then see that there are 8 pieces that make up two whole circles.

- Example:



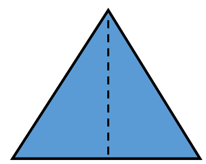
$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{8}{4}$$

- Instruction includes folding and/or cutting pre-made shapes into halves. Students physically bend the paper into halves and then label the pieces. Instruction includes relating the pieces back to the numerator and denominator and then connecting it to the equation. Using multiple shapes with the same denominators will solidify basic fraction understanding. Instruction should progress with other denominators.
    - Example:



$\frac{1}{2}$ $\frac{1}{2}$

$$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$$



$\frac{1}{2}$ $\frac{1}{2}$

$$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$$

Instructional Tasks

Instructional Task 1

Part A. How many one-fifth sized parts are added together to equal 1 whole? Prove your thinking with a visual model or number line.

Part B. How many one-fifth sized parts are added together to equal 2 wholes? Prove your thinking with a visual model or number line.

Instructional Items

Instructional Item 1

Represent the fraction $\frac{8}{3}$ as the sum of unit fractions.

Instructional Item 2

Which of the following expressions models $\frac{7}{4}$?

a. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
b. $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$
c. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
d. $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.FR.1.3

Benchmark

MA.3.FR.1.3 Read and write fractions, including fractions greater than one, using standard form, numeral-word form and word form.

Example: The fraction $\frac{4}{3}$ written in word form is four-thirds and in numeral-word form is $4\ thirds$.

Benchmark Clarifications:
Clarification 1: Instruction focuses on making connections to reading and writing numbers to develop the understanding that fractions are numbers and to support algebraic thinking in later grades.
Clarification 2: Denominators are limited to 2, 3, 4, 5, 6, 8, 10 and 12.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.1
- MA.3.FR.1.2
- MA.3.FR.2.1

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.1

Next Benchmarks

- MA.4.FR.1.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to describe fractions in different ways.

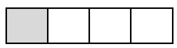
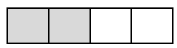
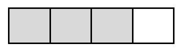
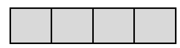
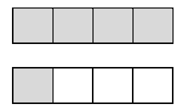
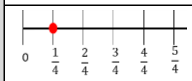
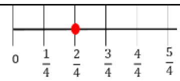
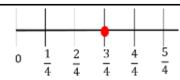
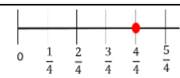
- This benchmark builds precise vocabulary for describing fractions. When students describe $\frac{4}{3}$ as 4 thirds, they build understanding that the fraction represents 4 parts that are each one-third in size *(MTR.2.1)*.
- It is also the expectation of this benchmark that students represent fractions greater than one as mixed numbers in word and numeral-word form *(MTR.2.1)*.
- During instruction, teachers should model and expect precise vocabulary from students to describe fractions *(MTR.4.1)*.

Common Misconceptions or Errors

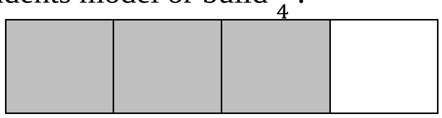
- Students can misinterpret fractions as two numbers that are being compared (e.g., reading “1 over 2” instead of one-half). The use of precise vocabulary helps them understand that a fraction is a representation of one number.
- Students can misinterpret that a fraction always models part of one whole. Exceptions to this misconception are fractions greater than one or fractions representing on number lines and in sets of objects.

Strategies to Support Tiered Instruction

- Instruction includes opportunities for practice in naming fractions correctly in multiple ways. Students use a chart to correctly name fractions. To increase appropriate terminology for naming fractions, students use visual representations with the naming of the fractional parts, as well as build fractions with models as well as number lines.

	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$
	$1\ fourth$	$2\ fourths$	$3\ fourths$	$4\ fourths$	$5\ fourths$
	One-fourth	Two-fourths	Three-fourths	Four-fourths	Five-fourths
					
					

  - For example, students model or build $\frac{3}{4}$.



    - Teacher asks, “How can we describe this fraction model?” while guiding students to the understanding that $\frac{3}{4}$ is $3\ fourths$ or 3 of the $\frac{1}{4}$ pieces. The use of precise vocabulary helps them understand that the same number can be represented by different visual models and different verbal expressions.
- Instruction includes opportunities to practice naming fractions correctly in multiple ways with concrete materials and models.
  - For example, students partition a shape or paper into halves. The teacher asks “What do you notice about the pieces? What do we call each piece? How can we write what one piece of the shape is worth with a fraction?” Instruction involves the vocabulary of numerator and denominator. Students are prompted to use the language of *one half* and then connect that to the standard form. The use of precise vocabulary helps students understand that a fraction is a representation of one number.

Instructional Tasks

Instructional Task 1

Part A. Reynaldo says that the fraction $\frac{8}{7}$ is written as $8\ sevenths$. Jonathon says that the fraction $\frac{8}{7}$ is written as $7\ eighths$. Who is correct?

Part B. What is another way to represent $\frac{8}{7}$? Draw a model or write an equation.

Instructional Items

Instructional Item 1

Select all the ways to represent $\frac{8}{3}$.

a. $Eight\ thirds$
b. $8\ thirds$
c. $3\ eighths$
d. $Two\ and\ two\ thirds$
e. $Three\ and\ two\ thirds$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.FR.2 *Order and compare fractions and identify equivalent fractions.*

MA.3.FR.2.1

Benchmark

MA.3.FR.2.1	Plot, order and compare fractional numbers with the same numerator or the same denominator.

Example: The fraction $\frac{3}{2}$ is to the right of the fraction $\frac{3}{3}$ on a number line so $\frac{3}{2}$ is greater than $\frac{3}{3}$.

Benchmark Clarifications:
Clarification 1: Instruction includes making connections between using a ruler and plotting and ordering fractions on a number line.
Clarification 2: When comparing fractions, instruction includes an appropriately scaled number line and using reasoning about their size.
Clarification 3: Fractions include fractions greater than one, including mixed numbers, with denominators limited to 2, 3, 4, 5, 6, 8, 10 and 12.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.2.2
- MA.3.NSO.1.3

Terms from the K-12 Glossary

- Number Line

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.3

Next Benchmarks

- MA.4.FR.1.4
- MA.4.FR.1.3

Purpose and Instructional Strategies

The purpose of this benchmark is for students to plot and order fractions with the same numerator (e.g., $\frac{3}{4}, \frac{3}{2}, \frac{3}{8}$) or fractions with the same denominator (e.g., $\frac{3}{5}, \frac{10}{5}, \frac{7}{5}$) to compare them by their location on a number line.

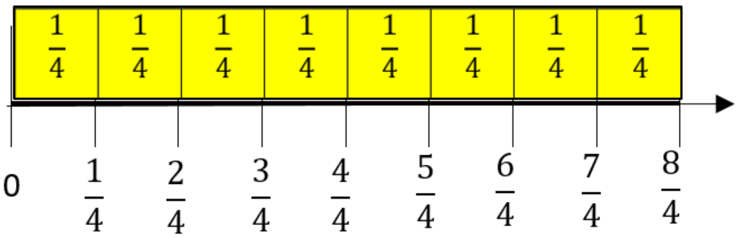
- During instruction, teachers should provide students opportunities to practice using the number line, which will assist students with understanding the difference in size when fractions have the same numerator (the size of the parts) and with comparing fractions with the same denominator (number of parts) *(MTR.2.1)*.
- Through making connections to rulers, students see that appropriately scaled number lines allow for comparisons of fraction size. Students should also utilize open number lines as to practice creating their own appropriately scaled number lines *(MTR.2.1)*.
- Instruction should model that fractional units on a number line represent intervals that are its unit fraction in size. For example, $\frac{5}{3}$ on a number line is represented by 5 units from 0 that are each one-third in length. Second, number lines help students see comparisons of fractions to the same whole and will continue as students compare fractions with different numerators and denominators in Grade 4. Finally, number lines reinforce Clarification 3 for MA.3.FR.1.3, that fractions are numbers *(MTR.2.1, MTR.5.1)*.

Common Misconceptions or Errors

- Students can confuse that when numerators are the same in fractions, larger denominators represent smaller pieces, and smaller denominators represent larger pieces.
- When fraction comparisons are made using area models, students may confuse that the size of the whole for each model must be the same size.

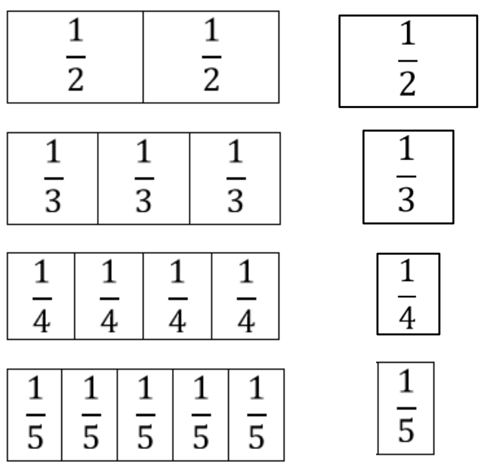
Strategies to Support Tiered Instruction

- Instruction includes opportunities to use concrete models and drawing of number lines to connect learning with fraction understanding.
    - For example, students plot fourths on the number line. Utilizing fraction strips or tiles, students can connect fractional parts to the measurement on a number line.



Conversation includes what students notice about the fraction on the number line. "How many fourths are in three-fourths? What do we notice about the size of $\frac{1}{4}$ compared to $\frac{3}{4}$?" Students have opportunities to describe the distance from the 0 as well as the distance from other benchmark fractions.

- Instruction includes opportunities to use fraction manipulatives, concrete models and drawings. The teacher begins instruction by modeling fractional pieces with their fraction name. It is important that students see that the fractions that they are building and comparing refer to the same size whole.
    - For example, students build fractions tiles or models to equal the same size one whole like below.



Students pull out the unit fraction of each of the fraction models. Conversations include what students notice about the size of each piece and what students notice about the size of the piece compared to the denominators. "Why is $\frac{1}{2}$ larger than $\frac{1}{5}$?"

Instructional Tasks

Instructional Task 1

Clara says that $\frac{5}{4}$ is greater than $\frac{5}{2}$ because 4 is greater than 2. Prove why she is incorrect using the number line below.

Instructional Items

Instructional Item 1

Order the fractions below from least to greatest.

$$\frac{8}{5}, \frac{8}{3}, \frac{8}{10}, \frac{8}{1}$$

Instructional Item 2

Compare 7 fourths and 3 fourths using $<, =, or\ >$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.FR.2.2

Benchmark

MA.3.FR.2.2 Identify equivalent fractions and explain why they are equivalent.

Example: The fractions $\frac{1}{1}$ and $\frac{3}{3}$ can be identified as equivalent using number lines.

Example: The fractions $\frac{2}{4}$ and $\frac{2}{6}$ can be identified as not equivalent using a visual model.

Benchmark Clarifications:

Clarification 1: Instruction includes identifying equivalent fractions and explaining why they are equivalent using manipulatives, drawings, and number lines.

Clarification 2: Within this benchmark, the expectation is not to generate equivalent fractions.

Clarification 3: Fractions are limited to fractions less than or equal to one with denominators of 2, 3, 4, 5, 6, 8, 10 and 12. Number lines must be given and scaled appropriately.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.2.1

Terms from the K-12 Glossary

- Number Line

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.2

Next Benchmarks

- MA.4.FR.1.3
- MA.4.FR.1.4

Purpose and Instructional Strategies

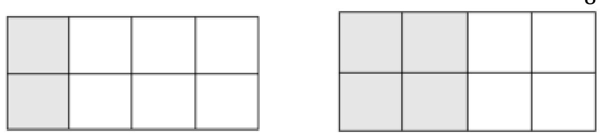
- The purpose of this benchmark is for students to identify equivalent fractions both on appropriately scaled number lines and on area models, and to justify how they know *(MTR.2.1, MTR.4.1)*.
- Instruction should prioritize tasks that allow for students to reason why fractions are equivalent using the models instead of the algorithm. Students are not expected to generate equivalent fractions until Grade 4 *(MTR.2.1)*.

Common Misconceptions or Errors

- Students can confuse that when numerators are the same in fractions, larger denominators represent smaller pieces, and smaller denominators represent larger pieces.

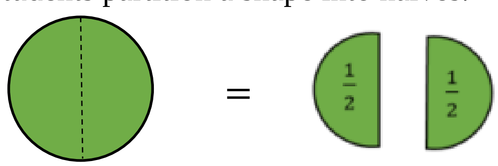
Strategies to Support Tiered Instruction

- Instruction includes opportunities to use concrete models and drawings to solidify understanding of fraction equivalence. Students use models to describe why fractions are equivalent or not equivalent when referring to the same size whole.
    - For example, when looking at $\frac{2}{8}$ and $\frac{4}{8}$, conversation includes that both fraction models are the same size, so when comparing them we are comparing the same size whole. Students can see that 2 out of the 8 are shaded in the first model and 4 out of the 8 are shaded in the second model, making the $\frac{4}{8}$ greater than $\frac{2}{8}$.



$\frac{2}{8}$ $\frac{4}{8}$

- Instruction includes opportunities to use concrete models and drawings to solidify understanding of fraction equivalence. Students use models to describe why fractions are equivalent or not equivalent when referring to the same size whole. Instruction includes partitioning shapes with halves, thirds and fourths and then comparing the pieces used.
    - For example, students partition a shape into halves.

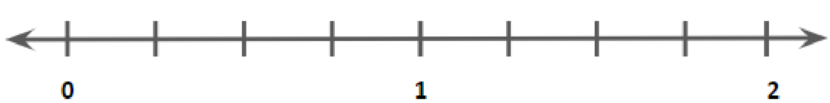


Conversation includes observations about the shape partitioned into two equal pieces. The teacher models writing the fractional parts of $\frac{1}{2}$ so that students can make the connection of the denominator representing the number of pieces. Students then practice partitioning shapes into thirds and fourths for this same understanding.

Instructional Tasks

Instructional Task 1

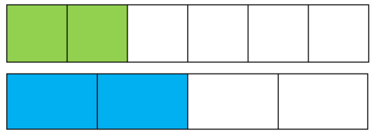
Plot the fractions $\frac{6}{4}$ and $\frac{3}{2}$. Use your number line to determine whether the fractions are equivalent. Justify your argument in words.



Instructional Items

Instructional Item 1

Use the area models below to determine whether the fractions they represent are equivalent.



a. The model shows that 2 sixths and 2 fourths are equivalent because the area models each have 2 shaded parts.
b. The model show that 2 sixths and 2 fourths are equivalent because the area models show the size of the shaded parts are equal when the size of each whole is the same.
c. The model shows that 2 sixths and 2 fourths are not equivalent because 2 sixths is greater than fourths when the size of each whole is the same.
d. The model show that 2 sixths and 2 fourths are not equivalent because the area models show the size of the shaded parts are not equal when the size of each whole is the same.

Instructional Item 2

Select all the fractions that are equivalent to a whole number.

a. $\frac{3}{3}$
b. $\frac{5}{10}$
c. $\frac{8}{2}$
d. $\frac{15}{7}$
e. $\frac{1}{6}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Algebraic Reasoning

MA.3.AR.1 *Solve multiplication and division problems.*

MA.3.AR.1.1

Benchmark

MA.3.AR.1.1	Apply the distributive property to multiply a one-digit number and two-digit number. Apply properties of multiplication to find a product of one-digit whole numbers.

Example: The product 4×72 can be found by rewriting the expression as $4 \times (70 + 2)$ and then using the distributive property to obtain $(4 \times 70) + (4 \times 2)$ which is equivalent to 288.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is to apply the associative and commutative properties of multiplication, the distributive property and name the properties. Refer to K-12 Glossary (Appendix C).
Clarification 2: Within the benchmark, the expectation is to utilize parentheses.
Clarification 3: Multiplication for products of three or more numbers is limited to factors within 12. Refer to Properties of Operations, Equality and Inequality (Appendix D).

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.3
- MA.3.NSO.2.4

Terms from the K-12 Glossary

- Expression
- Equation
- Distributive property
- Factors

Vertical Alignment

Previous Benchmarks

- MA.2.NSO.1.2

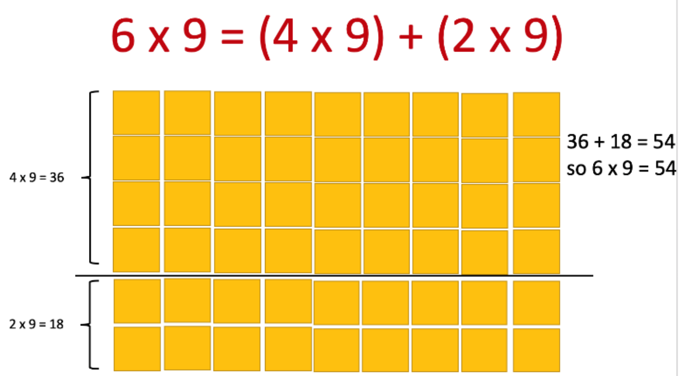
Next Benchmarks

- MA.4.NSO.2.2
- MA.4.NSO.2.3

Purpose and Instructional Strategies

The purpose of this benchmark is for students to apply what they have learned about multiplication of one-digit numbers and multiples of ten to then multiply a one-digit number and a two-digit number (MA.3.NSO.2.3).

- Students are introduced to the distributive property of multiplication over addition as a strategy for using products that they know in order to solve products that they do not know. For example, if students are asked to find the product of $6 x 9$, they might decompose 6 into 4 and 2 and then multiply $4 x 9$ and $2 x 9$ to arrive at $36 + 18$, which equals 54. Because of the distributive property, students use parentheses to show how to decompose two-digit numbers by the value of its tens and ones, and then multiply the one-digit number by both the values of the two-digit number's tens and ones values and find the sum of those products. The application of the commutative and associative properties of multiplication allow for two-digit numbers to be decomposed and multiplication expressions reorganized so that the distributive property can work *(MTR.2.1)*.



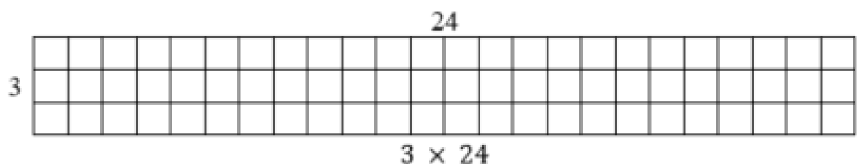
- During instruction, teachers should model where the properties are applied while multiplying and expect students to explain how they work during explanations of their strategies and solutions. Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty *(MTR.2.1, MTR.4.1)*.
- Building understanding of the distributive property in Grade 3 will help students decompose larger numbers as they continue to multiply multi-digit numbers with procedural reliability and procedural fluency in Grade 4. Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty.

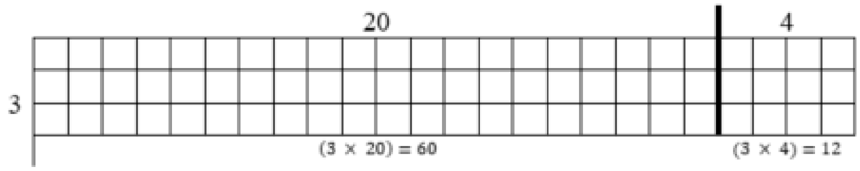
Common Misconceptions or Errors

- Students can be confused about how to write expressions using the distributive property. One common mistake that students make is writing an expression 4×72 as $(4 \times 70) \times (4 \times 2)$ instead of $(4 \times 70) + (4 \times 2)$. Instruction should show concrete models (e.g., base ten drawings) along with equations so students can understand the relationship between multiplication and addition while applying the property and writing expressions.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to use concrete models and drawings along with equations to increase understanding of the relationship between multiplication and addition when applying the distributive property and writing equations. The teacher begins by modeling a one-digit number multiplied by a one-digit number, guiding students to decompose one of the factors and use models or drawings to demonstrate the reorganization of the multiplication expression using parentheses. Next, the teacher models multiplication of a one-digit number by a two-digit number, guiding students to decompose the two-digit number into the value of the tens and the ones using models or drawings. The teacher clarifies that the decomposed factor can be represented in expanded form by adding the tens and the ones, repeating with additional one-digit by two-digit multiplication equations.
  - For example, the teacher uses a model or drawing to use the distributive property to solve $3 \; x \; 24$.

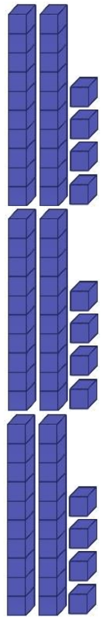




$3 \times (20 + 4)$
$(3 \times 20) + (3 \times 4)$
$(3 \times 20) + (3 \times 4) = 60 + 12$
$(3 \times 20) + (3 \times 4) = 72$
so $3 \times 24 = 72$

- Teacher provides opportunities to apply the distributive property to solve one-digit by two-digit multiplication equations using base-ten blocks or place value disks. The teacher provides the equation and guides students to decompose the two-digit number into the value of the tens and the ones using manipulatives. If needed, the teacher prompts students to count by 10s and 1s using the base-ten blocks or place value disks.

- For example, the teacher uses base-ten blocks to solve 3×24 while asking guiding questions such as "How many tens are in 24?" "How many ones are in 24?" "How would we write 24 in expanded form?"



3×24
$3 \times (20 + 4)$
$(3 \times 20) + (3 \times 4)$
$(3 \times 20) + (3 \times 4) = 60 + 12$
$(3 \times 20) + (3 \times 4) = 72$
so $3 \times 24 = 72$

$(3 \times 4) = 12$

Instructional Tasks

Instructional Task 1

In each equation, find the missing value, *n*.

Part A. $4 \times 52 = (4 \times 50) + (4 \times n)$
Part B. $n \times 3 = (20 \times 3) + (9 \times 3)$
Part C. $8 \times 36 = (n \times 30) + (n \times 6)$
Part D. $48 \times 6 = n$

Instructional Task 2

Tory tried to use the associative and commutative properties to create the following equations. Using pictures and/or words, explain why Tory is incorrect.

4 x (11 + 6) = (4 x 11) + 6
4 x (11 + 6) = 11 x (4 + 6)

Instructional Items

Instructional Item 1

Which of the following correctly uses the distributive property to multiply 8×39 ?

a. $(8 \times 30) \times (8 \times 9) = 24 + 72 = 96$
b. $(8 \times 30) + (8 \times 9) = 240 + 56 = 296$
c. $(8 \times 30) + (8 \times 9) = 38 + 17 = 45$
d. $(8 \times 30) + (8 \times 9) = 240 + 72 = 312$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.1.2

Benchmark

MA.3.AR.1.2 Solve one- and two-step real-world problems involving any of four operations with whole numbers.

Example: A group of students are playing soccer during lunch. How many students are needed to form four teams with eleven players each and to have two referees?

Benchmark Clarifications:
Clarification 1: Instruction includes understanding the context of the problem, as well as the quantities within the problem.
Clarification 2: Multiplication is limited to factors within 12 and related division facts. Refer to Situations Involving Operations with Numbers (Appendix A).

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.1/2.2/2.3/2.4
- MA.3.AR.2.1/2.2/2.3

Terms from the K-12 Glossary

- Expression
- Equation

Vertical Alignment

Previous Benchmarks

- MA.2.AR.1.1

Next Benchmarks

- MA.4.AR.1.1/1.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to apply all four operations to solve one- and two-step real-world problems. This benchmark continues the work done in grade 2 solving real-world problems using addition and subtraction (MA.2.AR.1.1).

- Instruction should facilitate students' understanding of contexts and quantities within word problems.
- The emphasis on teaching problem-solving strategies should focus on the comprehension of problem contexts and what quantities represent in them. Examples of questions that help students comprehend word problems are:
  - What is happening in the real-world problem?
  - What do you need to find out?
  - What do the quantities represent in the problem?
  - What will the solution represent in the problem? *(MTR.1.1, MTR.4.1, MTR.6.1)*
- Teachers should model answering these questions through rectangular arrays, base-ten blocks, counters and think-alouds. In addition, teachers should help students explore estimation strategies to determine reasonable ranges for solutions (e.g., rounding, finding low and high estimates) and teach problem-solving strategies that build comprehension (e.g., Three Reads) *(MTR.4.1, MTR.5.1, MTR.6.1)*.

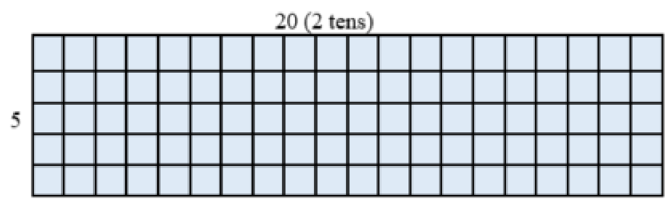
Common Misconceptions or Errors

- Students may have difficulty creating effective models (e.g., drawings, equations) that will help them solve real-world problems. To assist students, provide opportunities for them to estimate solutions and try different models before solving. Beginning instruction by showing problems without their quantities is a strategy for helping students determine what steps and operations will be used to solve.
- Students may also have difficulty identifying when real-world problems require two steps to solve and will complete only one of the steps. Focusing on comprehension of real-world problems helps students determine what step(s) are required to solve.

Strategies to Support Tiered Instruction

- Instruction provides opportunities for students to estimate solutions and try different models before solving.
- Instruction includes opportunities to create models (e.g., equations, drawings, manipulatives) to help solve real-world problems. The teacher uses guided questioning to support comprehension, considering levels of reading proficiency for students who may struggle with word problems—some students may need to hear the problems read aloud. The teacher provides opportunities to estimate solutions and try different models before solving, beginning instruction by showing problems without their quantities is a strategy to help students determine what steps and operations will be used to solve.
    - For example, the teacher reads aloud the following problem: Keisha and Diego are selling pies for a fundraiser. Each pie costs five dollars. If Keisha sells 15 pies and Diego sells 5 pies, how much money did they earn for the fundraiser?
        - The teacher uses questioning to ensure comprehension (e.g., "What do you need to find out?" "What do the quantities represent in the problem?" "What will the solution represent in the problem?").
        - The teacher models how to represent this problem using an equation and a drawing:

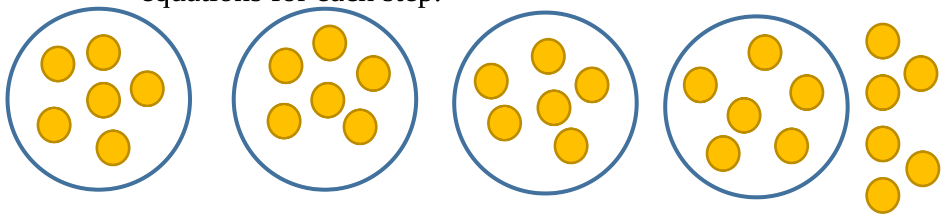
$$(15 + 5) \times 5$$
$$20 \times 5$$



Total amount earned: $20 \times 5 = 100$ dollars

        - The teacher repeats with additional two-step problems, guiding students to create appropriate models to support problem-solving.
    - For example, the teacher reads aloud the following problem: Antwan is helping the art teacher get ready for art club. There are a total of 30 paintbrushes. The art teacher asked Antwan to put 6 paintbrushes on each of the 4 tables in the room and then put the rest on the counter. How many paint brushes will he put on the counter?
        - The teacher uses questioning to ensure comprehension (e.g., "What do you need to find out?" "What do the quantities represent in the problem?" "What will the solution represent in the problem?").

- The teacher models the problem using counters, prompting the students to demonstrate each step of the problem while writing the corresponding equations for each step.



$$6 \times 4 = 24$$
$$30 - 24 = 6$$

- The teacher repeats with additional two-step problems, guiding students to create appropriate models using manipulatives to support problem-solving. Some students may benefit from "acting out" the story in the problem to support the problem-solving process.

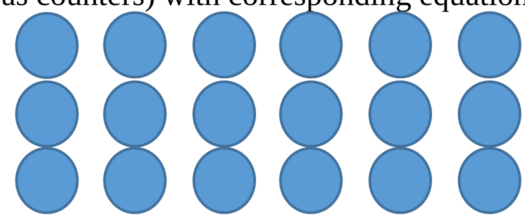
- Instruction includes guided practice identifying and completing two steps in a real-world problem. The teacher uses guided questioning to support comprehension considering levels of reading proficiency for students who may struggle with word problems—some students may need to hear the problems read aloud. The teacher uses explicit prompts for each step.
  - For example, the teacher reads aloud the following problem: Suni is taking piano lessons. Her piano teacher told her to practice for 90 minutes this week. On Monday, she practiced 15 minutes. She practiced 20 minutes on Tuesday and 25 minutes on Wednesday. How much more time does she still need to practice this week?
    - The teacher uses guided questioning and prompts to help students to identify the steps (e.g., "What do you already know?" "What do you need to find out?" "What do we need to do before we can find out the remaining time she has left to practice?"). Through questioning, the teacher guides students to identify the first step: adding the amount of time Suni has already practiced.
    - The teacher uses a model to represent the problem and an equation to represent the first step.
      $15 + 20 + 25 = 60$
    - After students complete the first step, the teacher uses questioning to prompt next step (e.g., "What does the sum we just found show us?" "What do you need to find out to solve this problem?" "What should we do next?"). The problem may need to be reread aloud.

$$15 + 20 + 25 = 60$$
$$60 + \_\_ = 90$$
$$90 - 60 = 30 \text{ minutes}$$

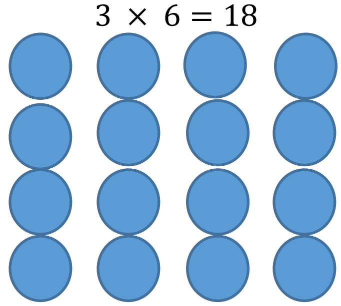
Monday	Tuesday	Wednesday	Remaining Time
15	20	25	?
Total = 90 minutes			

- The teacher repeats with additional two-step problems, guiding students to identify and solve each step.

- For example, the teacher reads aloud the following problem: Rahim is learning about instruments in music class. He learns that guitars have six strings and mandolins have four strings. If there are three guitars and four mandolins in the classroom, how
  many strings are there altogether on the guitars and mandolins?
  - The teacher uses guided questioning and prompts to help students to identify the steps (e.g., “What do you already know?” “What do you need to find out?” “What do we need to do before we can find out the remaining time she has left to practice?”). Through questioning, the teacher guides students to identify the first step: Multiplying the number of strings by the number of each instrument.
  - The teacher guides the students to create a model (using manipulatives such as counters) with corresponding equations.



$3 \times 6 = 18$



$4 \times 4 = 16$

  - After students complete the first step, the teacher uses questioning to prompt next step (e.g., “What have we learned about the numbers of strings?” “What do you need to find out to solve this problem?” “What should we do next?”). The problem may need to be reread aloud.
    $18 + 16 = 34$ total strings
  - The teacher repeats with additional two-step problems, guiding students to identify and solve each step using manipulatives. Some students may benefit from “acting out” the story in the problem to support the problem-solving process.

Instructional Tasks

Instructional Task 1

Solve the problem. Oak Hill Elementary third grade students are taking a field trip to the zoo. There are 71 students who paid to attend the field trip. Of those that paid, 8 students cannot go on the day of the trip. There needs to be 7 groups at the zoo and each group must have an equal number of students. How many students will be in each group on the field trip?

Instructional Items

Instructional Item 1

For a school food drive, three students bring in cases of canned goods to donate. Uriel brings 4 cases, Paola brings 6 cases, and Mika brings 5 cases. Each case contains 8 canned goods. How many canned goods in all does the school collect?

Instructional Item 2

A bookstore has 8 boxes of books. Each box contains 10 books. On Monday, the bookstore sold 16 books. How many books remain to be sold?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.2 *Develop an understanding of equality and multiplication and division.*

MA.3.AR.2.1

Benchmark

MA.3.AR.2.1 Restate a division problem as a missing factor problem using the relationship between multiplication and division.

Example: The equation $56 \div 7 = ?$ can be restated as $7 \times ? = 56$ to determine the quotient is 8.

Benchmark Clarifications:
Clarification 1: Multiplication is limited to factors within 12 and related division facts.
Clarification 2: Within this benchmark, the symbolic representation of the missing factor uses any symbol or a letter.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4
- MA. 3.AR.1.2

Terms from the K-12 Glossary

- Equation
- Factor

Vertical Alignment

Previous Benchmarks

- MA.1.AR.2.1
- MA.2.AR.2.1

Next Benchmarks

- MA. 4.AR.2.1

Purpose and Instructional Strategies

The purpose of this benchmark is to build students' fluency with division facts by relating them to known multiplication facts. Division is often more challenging for students than multiplication, so relating division to multiplication helps to determine quotients. Students learned a similar strategy when relating subtraction to addition in Grade 1 (MA.1.AR.2.1).

- Instruction should have students build and use fact families to relate division and multiplication equations. It is important for students to understand that multiplication and division are inverse operations. During instruction, students should have practice with solving and explaining division problems that can also be represented as an unknown factor in multiplication problems *(MTR.3.1, MTR.5.1)*.
- To help students understand the relationships between division problems and unknown factor problems conceptually (and to build understanding about fact families), teachers should utilize arrays that show 4 related multiplication and division facts. In addition to arrays, instruction of this standard pairs well with MA.3.AR.1.2 while students solve one- and two-step real-world problems. When students translate problem contexts to division equations, this benchmark helps students find solutions *(MTR.3.1, MTR.5.1)*.

Common Misconceptions or Errors

- Students may have difficulty understanding that the quotient of a division equation will become a factor in a multiplication equation. Allowing students to use an array model and/or reinforcing fact families may help to clarify the relationship.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to use array models to practice relating multiplication and division as inverse operations. The teacher shows an array model and guides students to identify the factors and the product, having them assist in writing the corresponding equation. The teacher guides students to complete the fact family using prompts as needed, reminding them that multiplication and division are inverse operations. After practicing with several examples, students practice completing fact families without arrays, solving for an unknown factor.
  - For example, students draw an array model to show 3×7.

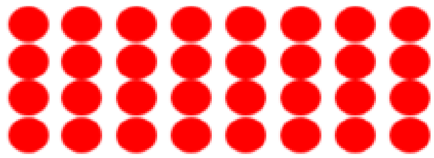
    ▲▲▲▲▲▲▲
    ▲▲▲▲▲▲▲
    ▲▲▲▲▲▲▲

    $3 \times 7 = 21$
    $7 \times 3 = 21$
    $21 \div 7 = 3$
    $21 \div 3 = 7$
  - For example, the students write the fact family and solve for $42 \div 6$.

$42 \div 6 = \_$	$42 \div 6 = 7$
$42 \div \_ = 6$	$42 \div 7 = 6$
$6 \times \_ = 42$	$6 \times 7 = 42$
$\_ \times 6 = 42$	$7 \times 6 = 42$
- Teacher provides opportunities to use manipulatives to practice relating multiplication and division as inverse operations. The teacher guides students to develop a model using

manipulatives (e.g., counters or base-ten blocks) and uses explicit instruction and questioning to help students to identify the related equation. Additionally, the teacher guides students to complete the fact family using explicit instruction, verbal prompts and nonverbal cues while reminding students that multiplication and division are inverse operations. After practicing with several examples, students practice completing fact families without arrays, solving for an unknown factor with the support of number cards.

- For example, the teacher uses counters to show an array to represent 4×8 and asks guiding questions to help students build the array. With prompting, the teacher guides students to identify the product and write the complete fact family.



$$4 \times 8 = 32$$
$$8 \times 4 = 32$$
$$32 \div 8 = 4$$
$$32 \div 4 = 8$$

- For example, students use number cards to rearrange equations to create all four parts to the fact family and solve for a missing factor. Students may also write on notecards for this activity. One card should have the multiplication symbol on one side and the division symbol on the other. The teacher uses a blank card for the missing factor until the students solve. Students move each card to a different location to build the entire fact family and record each equation on a sheet of paper or mini white board as they manipulate the cards.

Instructional Tasks

Instructional Task 1

Part A. Write a multiplication equation that can be used to find the quotient $48 \div 12$. Use *n* to represent the unknown factor.

Part B. What is the quotient?

Instructional Items

Instructional Item 1

Which of the following equations can be used to find the quotient $72 \div 8$?

a. $8 \times ? = 72$
b. $72 \times 8 = ?$
c. $72 - 8 = ?$
d. $? + 8 = 72$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.2.2

Benchmark

MA.3.AR.2.2 Determine and explain whether an equation involving multiplication or division is true or false.

Example: Given the equation $27 \div 3 = 3 \times 3$, it can be determined to be a true equation by dividing the numbers on the left side of the equal sign and multiplying the numbers on the right of the equal sign to see that both sides are equivalent to 9.

Benchmark Clarifications:
Clarification 1: Instruction extends the understanding of the meaning of the equal sign to multiplication and division.
Clarification 2: Problem types are limited to an equation with three or four terms. The product or quotient can be on either side of the equal sign.
Clarification 3: Multiplication is limited to factors within 12 and related division facts.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.1.3
- MA.3.NSO.2.2/2.4
- MA.3.AR.1.2

Terms from the K-12 Glossary

- Equation
- Expression
- Equal sign

Vertical Alignment

Previous Benchmarks

- MA.2.AR.2.1

Next Benchmarks

- MA.4.AR.2.1

Purpose and Instructional Strategies

The purpose of this benchmark is to extend the understanding of the meaning of the equal sign in multiplication and division situations. In Grades 1 and 2, students determined and explained when addition and subtraction equations were true or false (MA.1.AR.2.2, MA.2.AR.2.1).

- Instruction should emphasize that the equal sign can be read as “the same as” to show the balance of two multiplication and/or division expressions. When those expressions are evaluated as the same product or quotient, the equation is balanced, or true. If those expressions evaluate differently, then the equation is not balanced, or false *(MTR.2.1, MTR.5.1).*
- When students explain whether an equation is true or false, they should justify by explaining the equivalence of its expressions. (Note: The expectation of this benchmark is not to compare the expressions of a false equation using symbols of inequality, $<$ or $>$.) *(MTR.4.1, MTR.6.1)*

Common Misconceptions or Errors

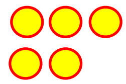
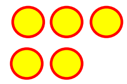
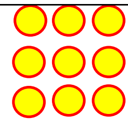
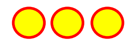
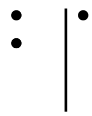
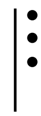
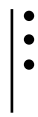
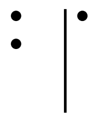
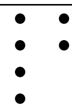
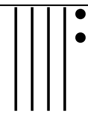
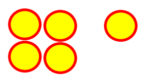
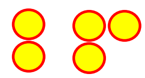
- By Grade 3, students may grow to expect equation solutions to be represented as the expressions on the right side of the equal sign. When having students evaluate true or false equations with only three terms (e.g., $18 = 3 \times 6$), teachers should give examples showing products and quotients on the left side.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to explore the meaning of the equal sign. The teacher provides clarification that the equal sign means "the same as" rather than "the answer is." Multiple examples are provided to evaluate equations as true or false using the four operations with the answers on both the left and right side of the equation, beginning by using single numbers on either side of the equal sign to build understanding. The same equations are written in different ways to reinforce the concept.
  - For example, the teacher shows the following equations, asking students if they are true or false statements. Students explain why each equation is true or false, repeating with additional true and false equations using the four operations.

Example	True/False	Sample Student Rationale
$5 = 5$	True	They are both the same number; five is the same as five.
$9 = 3$	False	Nine and three have different values; they are not the same.
$2 + 11 = 13$	True	When you add two and eleven, the total has a value of thirteen.
$13 = 2 + 11$	True	The value of thirteen is the same as the value of two and eleven combined.
$4 + 2 = 42$	False	The sum of four and two is six, not forty-two.
$25 - 5 = 20$	True	When you take five away from twenty-five, the difference is twenty.
$20 = 25 - 5$	True	The value of twenty is the same as the difference between twenty-five and five.
$20 = 25 + 5$	False	The value of twenty-five plus five is thirty, not twenty.
$4 + 1 = 2 + 3$	True	Four plus one has a value of five. Two plus three also has a value of five.
$2 \times 3 = 8 - 2$	True	Two times three has a value of six. Eight minus two also has a value of six.

- Teacher provides opportunities to explore the meaning of the equal sign using visual representations (e.g., counters, drawings, base-ten blocks) on a t-chart to represent equations. The teacher provides clarification that the equal sign means "the same as" rather than "the answer is." Multiple examples are provided for students to evaluate equations as true or false using the four operations with the answers on both the left and right side of the equation, beginning by using single numbers on either side of the equal sign to build understanding. The same equations are written in different ways to reinforce the concept.
  - For example, the teacher shows the following equations. Students use counters, drawings, or base-ten blocks on a t-chart to represent the equation. The teacher asks students if they are true or false statements and has them explain why each equation is true or false, repeating with additional true and false equations using the four operations.

$5 = 5$	True			They are both the same number; the same amount is on both sides.
$9 = 3$	False			Nine and three have different values; there is a different number on each side.
$2 + 11 = 13$	True			When you add two and eleven, the total has a value of thirteen. Each side has the same amount.
$13 = 2 + 11$	True			The value of thirteen is the same as the value of two and eleven combined. Each side has the same amount.
$4 + 2 = 42$	False			The sum of four and two is six, not forty-two. The value on each side is different.
$4 + 1 = 2 + 3$	True			Four plus one has a value of five. Two plus three also has a value of five. Each side has the same number of counters.

Instructional Tasks

Instructional Task 1

Two equations are below. One equation is true, and the other equation is false. Choose one of the equations and explain why it is true or false.

$$2 \times 3 = 4 \times 6 \qquad\qquad 2 \times 12 = 4 \times 6$$

Instructional Items

Instructional Item 1

Which of the following describes the equation $16 \div 2 = 36 \div 9$?

a) This equation is true because the expressions on each side have a quotient of 8.
b) The equation is true because the expressions on each side have a quotient of 4.
c) This equation is false because the expressions on each side have a quotient of 8.
d) This equation is false because the quotient on the left is 8 and the quotient on the right is 4.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.2.3

Benchmark

MA.3.AR.2.3 Determine the unknown whole number in a multiplication or division equation, relating three whole numbers, with the unknown in any position.

Benchmark Clarifications:
Clarification 1: Instruction extends the development of algebraic thinking skills where the symbolic representation of the unknown uses any symbol or a letter.
Clarification 2: Problems include the unknown on either side of the equal sign.
Clarification 3: Multiplication is limited to factors within 12 and related division facts. Refer to Situations Involving Operations with Numbers (Appendix A).

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4
- MA.3.AR.1.2

Terms from the K-12 Glossary

- Equation
- Expression

Vertical Alignment

Previous Benchmarks

- MA.2.AR.2.2

Next Benchmarks

- MA.4.AR.2.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to find an unknown value represented by a symbol or letter in a multiplication or division equation, continuing the work from Grade 2, where students found unknown values in addition and subtraction equations.

- Instruction that emphasizes the relationship between related facts in a fact family helps students use known values to solve for unknown values. For example, a fact family could be used to help students determine the unknown value in the equation $72 \div ? = 9$ *(MTR.5.1)*.
$$72 \div ? = 9$$
$$72 \div 9 = ?$$
$$9 \times ? = 72$$
$$? \times 9 = 72$$
Students can use any of these related facts to determine that the unknown value is 8. Teachers should encourage students to use such equations to justify their solutions *(MTR.6.1)*.
- In the primary grades, students used fact families to find missing addends and understand the relationship between addition and subtraction.
- Understanding and using related facts to solve for unknown values is an important algebraic understanding for using inverse operations to solve equations in future mathematics courses *(MTR.5.1)*.

Common Misconceptions or Errors

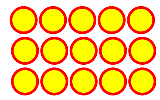
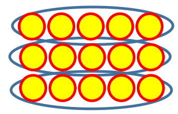
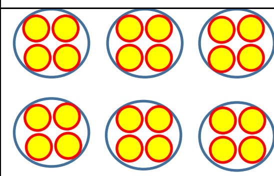
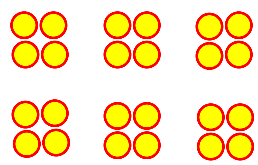
- By Grade 3, many students expect the solutions of equations to be an expression on the right side of the equal sign. When students determine unknown values in multiplication or division equations, give examples with the product or quotient on the left side.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to explore the meaning of the equal sign within the context of multiplication and division. The teacher provides clarification that the equal sign means "the same as" rather than "the answer is," supporting the understanding that the product and quotient can be on either the left or the right side of the equal sign. Multiple examples are provided for students to solve for the unknown with the product or quotient on both the left and right side of the equation. The teacher uses the same equations written in different ways to reinforce the concept.
    - For example, the teacher shows the following equations, asking students to solve for the unknown. Students explain why each equation is true after solving, repeating with additional examples.

Example	Unknown	Sample Student Rationale
$3 \times 5 = ?$	15	Three groups of five is the same as fifteen. If I count by fives three times, I get fifteen.
$15 = \_ \times 5$	3	When I count by fives, I count three times before I get to fifteen.
$4 = 24 \div \_$	6	If I sort twenty-four into four equal groups, there are six in each group.
$\_ \div 4 = 6$	24	Four groups of six in each group is twenty-four. If I count by fours, six times, I get twenty-four and multiplication and division are inverse operations.

- Teacher provides opportunities to explore the meaning of the equal sign within the context of multiplication and division using visual representations (e.g., counters, drawings, base-ten blocks) to represent the equations. The teacher provides clarification that the equal sign means "the same as" rather than "the answer is," supporting the understanding that the product and quotient can be on either the left or the right side of the equal sign. Multiple examples are provided for students to solve for the unknown with the product or quotient on both the left and right side of the equation, using the same equations written in different ways to reinforce the concept.
    - For example, the teacher shows the following equations, asking students to solve for the unknown and explain why each equation is true after solving. Students use counters, drawings, or base-ten blocks to represent the equation, repeating with additional operations.

$3 \times 5 = \_$	15		Three groups of five is the same as fifteen. If I make three rows of five counters, I have a total of fifteen.
$15 = \_ \times 5$	3		When I count by fives, I count three times before I get to fifteen. If I have fifteen counters and put them in rows of five, I have three rows.
$4 = 24 \div \_$	6		If I sort twenty-four into equal groups of four, there are six equal groups.
$\_ \div 4 = 6$	24		Six groups of four in each group is twenty-four. If I make six groups with four counters in each group, I have a total of twenty-four counters and multiplication and division are inverse operations.

Instructional Tasks

Instructional Task 1

Sam is having trouble deciding whether the value of n that makes the equation below true is 4 or 36. Which number is correct? Show your thinking using an equation or array.

$$3 = n \div 12$$

Instructional Items

Instructional Item 1

What value of n makes the equation below true?

$$n \div 6 = 5$$

Instructional Item 1

What is the value of the unknown number in the equation $7 \times n = 56$?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.3 *Identify numerical patterns, including multiplicative patterns.*

MA.3.AR.3.1

Benchmark

MA.3.AR.3.1	Determine and explain whether a whole number from 1 to 1,000 is even or odd.

Benchmark Clarifications:
Clarification 1: Instruction includes determining and explaining using place value and recognizing patterns.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks
- MA.2.AR.3.1

Next Benchmarks
- MA.4.AR.3.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to relate odd and even numbers to factors and multiples. In Grade 2, students learn to represent an even number using two equal groups or two equal addends and as odd number as two equal groups or two equal addends with one left over (MA.2.AR.3.1). In Grade 3, instruction extends to use patterns to generalize whether any number is odd or even *(MTR.2.1, MTR.5.1)*.

- Instruction should connect multiples of 2 to the patterns that the ones digit in any even number is 0, 2, 4, 6, or 8. By teaching this benchmark with MA.3.AR.3.2, students can see that multiples of 2 can form any even number. If a number is not a multiple of 2, then the number is odd *(MTR.5.1)*.
- These beginning understandings about multiples will help students explore factors and divisibility with prime and composite numbers in Grade 4.

Common Misconceptions or Errors

- Students may confuse that in an even number, the ones digit indicates whether it is a multiple of 2. For example, students may look at the number 883 is even because the digit 8 in the hundreds and tens places are even.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to practice identifying if multi-digit numbers are even or odd by using a place-value chart. The teacher explains that even numbers can be represented by two equal groups or two equal addends and that odd numbers can be represented by two equal groups or two equal addends with one left over while modeling using visual representations with several examples (e.g., drawings, tally marks).
  - For example, the teacher uses visual representations to identify numbers are even or odd by sorting into two equal groups using drawings or tally marks and enters numbers into a place value chart while asking "What do you notice about the

digits in the ones place?" Students should explain that even numbers all have digits in the ones place that are multiples of 2 (0, 2, 4, 6, or 8). Additional examples are used in the place-value chart to practice identifying if numbers are even or odd by looking at the digits in the ones place.

Hundreds	Tens	Ones	Tallies/Drawings		Even or Odd?
	1	3			Odd
	2	8			Even
		7			Odd
	3	0			Even
	1	6			Even
2	3	2			Even
8	8	1			Odd
4	9	4			Even

- For example, students use counters to identify if numbers are even or odd by sorting into two equal groups and enter numbers into a place value chart.

Hundreds	Tens	Ones	Multiples of 2: 0, 2, 4, 6, 8	
			Visual Representation	Even or Odd?
	1	3		Odd
	2	8		Even
		7		Odd
	3	0		Even
	1	6		Even

- As in the previous example, the students use counters by sorting into two equal groups. The teacher asks, "How many total counters are there, and what is the digit in the ones place?" Students should explain that even numbers all have digits in the ones place that are multiples of 2 (0, 2, 4, 6, or 8). Additional examples are used in the place-value chart to practice identifying if numbers are even or odd by looking at the digits in the ones place.

Instructional Tasks

Instructional Task 1

Is the number 461 even or odd? Explain how you know.

Instructional Items

Instructional Item 1

Determine whether the numbers are even or odd in the table below.

	Even	Odd
883	☐	☐
19	☐	☐
538	☐	☐
1,000	☐	☐
727	☐	☐

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.3.2

Benchmark

MA.3.AR.3.2	Determine whether a whole number from 1 to 144 is a multiple of a given one-digit number.

Benchmark Clarifications:

Clarification 1: Instruction includes determining if a number is a multiple of a given number by using multiplication or division.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.2.AR.3.2

Next Benchmarks

- MA.4.NSO.2.1
- MA.4.AR.3.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to determine whether a whole number is a multiple of a given one-digit number (e.g., Is 45 a multiple of 5?). Understanding of multiples extends what students learned in Grade 2 about skip-counting (e.g., skip-counting by 2s results in multiples of 2). Building a strong foundational understanding of multiples prepares students for relating multiples and factors to prime and composite numbers in Grade 4 (MA.4.AR.3.1).

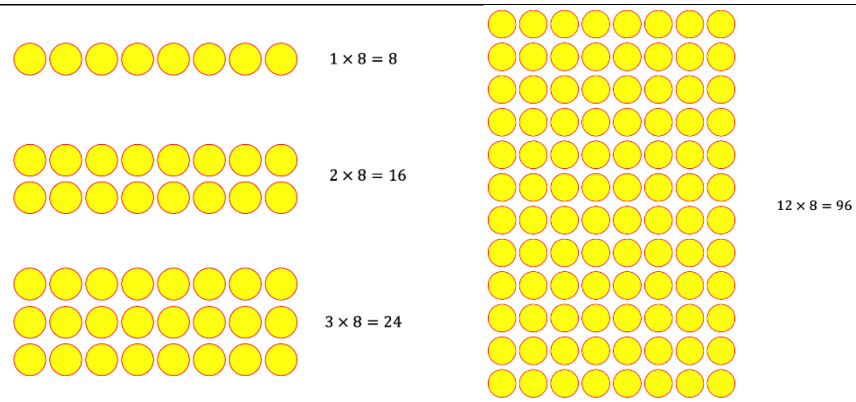
- Understanding of multiples extends from multiplication by expecting students to understand that the products of the given one-digit number and other factors create multiples of that one-digit number. For example, the products of $5 \, x \, 1$, $5 \, x \, 2$, $5 \, x \, 3, \ldots$ are multiples of 5 (5, 10, 15,…). Understanding of multiples extends from division by expected students to understand if a given whole number from 1 to 144 is divisible by a given-one-digit number, then that dividend is a multiple of it (e.g., 45 is divisible by 5, so 45 is a multiple of 5) *(MTR.5.1)*.
- The focus of instruction should be on the vocabulary of multiples as it relates to multiplication and division. Students should first have a strong understanding of how multiplication and division work before developing their knowledge of multiples. Instruction can include real-world applications (e.g., Can 45 cookies be placed into 5 bags with an equal number in each bag?) *(MTR.4.1, MTR.5.1)*.

Common Misconceptions or Errors

- When listing multiples of numbers, students may not list the number itself. It is important to emphasize that the smallest multiple is the number itself. Having students write multiples of a number by consecutive factors beginning with one.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to write multiples of a number by consecutive factors beginning with the factor 1.
- Instruction includes opportunities to connect finding multiples to skip counting.
    - For example, to find the multiples of 8, students can generate lists of multiples beginning with 1×8. Their generated list should include each of the counting numbers through 12×8. Students model generating multiples with counters. The teacher asks students to make one group of 8, having them record how many counters there are in an equation ($1 \times 8 = 8$). Next, students add another group of 8, recording the number of counters in an equation ($2 \times 8 = 16$). Students add more groups of 8 while recording the number of counters they have in an equation. Students should make all multiples of 8 through $12 \times 8 = 96$. When students have created their multiples, they record the products in a horizontal list in order from $1 \times 8 = 8$ to $12 \times 8 = 96$ and explain the connection between the products in their equations and the multiples in their list.



Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96

Instructional Tasks

Instructional Task 1

Use a visual model or write an equation to show whether 27 is a multiple of 3.

Instructional Task 2

Use a visual model or write an equation to show whether 36 is a multiple of 8.

Instructional Items

Instructional Item 1

Select all the numbers below that are multiples of 8.

a. 28
b. 56
c. 18
d. 24
e. 30

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.AR.3.3

Benchmark

MA.3.AR.3.3 Identify, create and extend numerical patterns.

Example: Bailey collects 6 baseball cards every day. This generates the pattern 6, 12, 18, ... How many baseball cards will Bailey have at the end of the sixth day?

Benchmark Clarifications:
Clarification 1: The expectation is to use ordinal numbers (1^{st}, 2^{nd}, 3^{rd}, …) to describe the position of a number within a sequence.
Clarification 2: Problem types include patterns involving addition, subtraction, multiplication or division of whole numbers.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.K.NSO.1.3

Next Benchmarks

- MA.4.AR.3.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to identify, create and extend numerical patterns using all four operations. Understanding of ordinal numbers from Kindergarten is the foundation for describing the sequence of numbers in a pattern.

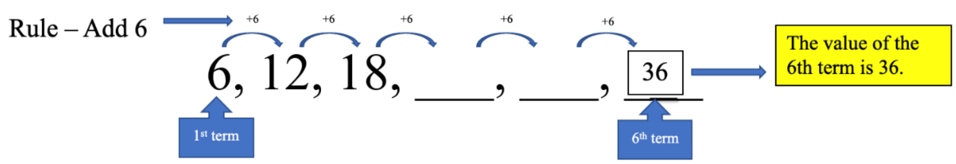
- "Identifying" a numerical pattern requires students to determine when a pattern exists in a sequence of numbers, and to potentially determine a rule that can be used to find each term in the sequence. For example, students may be asked whether a pattern exists in the numbers 20, 17, 14, 11,... and to discuss possible rules used to determine the next term.
- "Creating" a numerical pattern requires students to write a pattern given a rule and starting value. For example, students may be asked to write the first five terms of a sequence that begins with 500 and then create each successive term by subtracting 35 from the previous term.
- Finally, "extending" asks students to identify a future term in a sequence when provided with a rule. For example, students may be asked to find the next three terms in which each term is multiplied by 2 to get the next term 2: 1, 2, __, __, __ *(MTR.2.1, MTR.5.1)*.
- Instruction of this standard can begin by relating patterns to skip-counting to explore patterns in sequences of numbers and look for relationships in the patterns and be able to describe and make generalizations. When exploring patterns, teachers should allow for students to describe pattern rules flexibly. For example, in the pattern 6, 12, 18,…, one student may describe the pattern's rule as "add 6." Another student may describe the rule as, "add 7, then subtract 1" or "list the multiples of 6." Classroom discussion could compare these rules *(MTR.2.1, MTR.4.1)*.
- Instruction should be limited to whole numbers and operations that are appropriate for Grade 3.
- This foundation for identifying and using patterns extends into Grades 4 and 5 to build algebraic thinking for functions in middle and high school.

Common Misconceptions or Errors

- Students can confuse a term's number and its value in the sequence. For example, in the pattern 6, 12, 18,…, students can struggle to understand that even though 12 is the 2^{nd} term, 6 is being added to it to find the value of the 3^{rd} term (18). Encourage students to use precise vocabulary while describing patterns to address this confusion.

Strategies to Support Tiered Instruction

- Instruction includes explicit vocabulary instruction regarding patterns (first term, second term, third term…, rule, value, etc.). Instruction also includes relating the pattern to skip counting where appropriate.
    - Example:



    - For example, a 100 chart may be a referent that can be used for arithmetic patterns. The teacher makes connections between the rule and counting on the 100s chart.

1	**2**	**3**		**5**	**6**	**7**	**8**	**9**	
11	**12**	**13**	**14**	**15**		**17**	**18**	**19**	**20**
21		**23**	**24**	**25**	**26**	**27**		**29**	**30**
31	**32**	**33**		**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**

Rule – Add 6

Value of the 6^{th} Term – 34

Instructional Tasks

Instructional Task 1

Part A. Write a pattern that shows the first 10 multiples of 6.

Part B. What do you notice about the ones digits of the pattern's numbers?

Part C. What would you expect the ones digit of the 12th multiple to be? Explain how you know using the pattern you observed.

Instructional Items

Instructional Item 1

What are the fourth and fifth terms of the sequence below that follows the rule "subtract 4"?

$$34, 30, 26, \_\_\_, \_\_\_$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.M.1 *Measure attributes of objects and solve problems involving measurement.*

MA.3.M.1.1

Benchmark

MA.3.M.1.1	Select and use appropriate tools to measure the length of an object, the volume of liquid within a beaker and temperature.

Benchmark Clarifications:
Clarification 1: Instruction focuses on identifying measurement on a linear scale, making the connection to the number line.
Clarification 2: When measuring the length, limited to the nearest centimeter and half or quarter inch.
Clarification 3: When measuring the temperature, limited to the nearest degree.
Clarification 4: When measuring the volume of liquid, limited to nearest milliliter and half or quarter cup.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.1/1.2
- MA.3.FR.2.1/2.2
- MA.3.GR.1.2
- MA.3.GR.2.3/2.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.2.M.1.1/1.2

Next Benchmarks

- MA.4.M.1.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to choose appropriate tools to measure length, liquid volume, and temperature. In Grade 3, students continue to build their understanding of measuring lengths from Grades 1 and 2. In Grade 3, they also measure liquid volume and temperature.

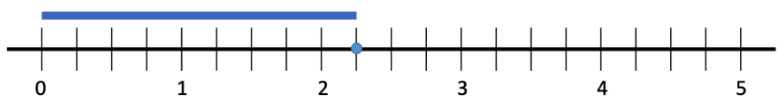
- Instruction should connect students' understandings about number lines and rulers to tools that measure liquid volume and temperature. This will help students make sense of measuring units (including half and quarter) with different tools *(MTR.1.1, MTR.2.1)*.
- To make instruction meaningful for students, this benchmark should be taught with MA.3.M.1.2 so students can choose appropriate tools when given problems in real-world contexts.
- Instruction should model and allow students to interact with hands-on activities to choose tools and measure appropriately.

Common Misconceptions or Errors

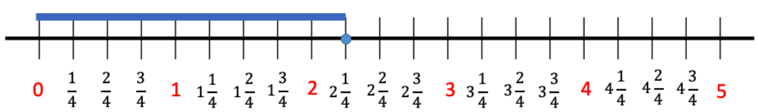
- Students who struggle to identify benchmarks on number lines can also struggle to measure units of length, liquid volume, and temperature. To assist students, teachers should allow students to measure often and provide feedback. Students can also complete error and reasoning analysis activities to identify this common measurement difficulty.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to measure often and provide feedback. Use error and reasoning analysis activities to address common measurement difficulties.
- Instruction includes opportunities to find the locations of points on number lines. Number lines should be represented vertically and horizontally. Instruction includes whole number values and fractions, including fractions greater than one.
  - For example, number lines should be included with benchmarks instead of every number in the sequence included. The blue line below extends from the 0 mark on the number line to the first hashmark beyond 2. The dot plotted on the number line identifies the end of the blue line. Since each whole number interval is partitioned into four equal parts, the first hashmark beyond 2 is represented as $2\frac{1}{4}$.



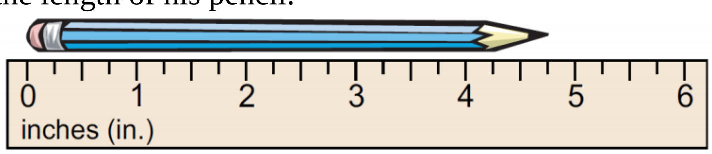
  - For example, number lines can also have all numbers included to represent the values between the benchmarks.



Instructional Tasks

Instructional Task 1

Jonah measures the length of his pencil.



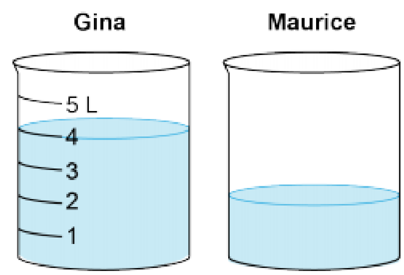
Part A. What is the length of his pencil, in inches?

Part B. Why is a ruler an appropriate tool for Jonah to measure the pencil's length?

Instructional Items

Instructional Item 1

Gina and Maurice have same-sized containers filled with different amounts of water, as shown. Gina's container has 4 liters (L) of water. About how much water, in liters (L), does Maurice's container have?



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.M.1.2

Benchmark

MA.3.M.1.2 Solve real-world problems involving any of the four operations with whole-number lengths, masses, weights, temperatures or liquid volumes.

Example: Ms. Johnson's class is having a party. Eight students each brought in a 2-liter bottle of soda for the party. How many liters of soda did the class have for the party?

Benchmark Clarifications:

Clarification 1: Within this benchmark, it is the expectation that responses include appropriate units.

Clarification 2: Problem types are not expected to include measurement conversions.

Clarification 3: Instruction includes the comparison of attributes measured in the same units.

Clarification 4: Units are limited to yards, feet, inches; meters, centimeters; pounds, ounces; kilograms, grams; degrees Fahrenheit, degrees Celsius; gallons, quarts, pints, cups; and liters, milliliters.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.1/1.2
- MA.3.FR.2.1/2.2
- MA.3.AR.1.2
- MA.3.GR.1.2
- MA.3.GR.2.3/2.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.2.M.1.3

Next Benchmarks

- MA.4.M.1.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to apply what they have learned about measurement to solve real-world problems.

- When solving real-world problems, instruction should facilitate students' understandings of contexts and quantities *(MTR.4.1, MTR.5.1, MTR.7.1)*.
- Recommendations for helping students comprehend and solve real-world problems can be found in this document for benchmark MA.3.AR.1.2.

Common Misconceptions or Errors

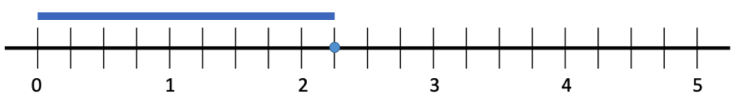
- Students who struggle to identify benchmarks on number lines can also struggle to measure units of length, liquid volume, and temperature. Allow students to measure often and receive feedback. Students can also use error and reasoning analysis activities to identify this common measurement difficulty.
- Students may have difficulty creating effective models (e.g., drawings, equations) that will help them solve real-world problems. To assist students, provide opportunities for them to estimate solutions and try different models before solving. Beginning instruction by showing problems without their quantities is a strategy for helping students determine what steps and operations will be used to solve.
- Students can struggle to identify when real-world problems require two steps to solve and will complete only one of the steps. Focusing on comprehension of real-world problems helps students determine what step(s) are required to solve.

Strategies to Support Tiered Instruction

- Instruction includes providing opportunities to estimate solutions and try different models before solving. Instruction begins by showing problems without their quantities to determine what steps and operations will be used to solve. Teaching problem-solving strategies should focus on the comprehension of problem contexts and what quantities represent in them.
  - For example, "For a science experiment in Mr. Thomas's 3rd grade class, each student needs ***some*** milliliters of water. If there are ***some*** students in Mr. Thomas's class, how many milliliters will be needed in all?" Students will notice that the quantities have been removed from the problem. This will help them to determine what the quantities represent and which operation to choose to solve the problem. The numberless word problem may also be written as ______ students × _______ milliliters of water = ______ milliliters needed in all.
- Teacher encourages exploration of estimation strategies to determine reasonable ranges for solutions (e.g., rounding, finding low and high estimates) and teach problem-solving strategies that build comprehension.
  - For example, the 3-Reads Protocol is a close reading strategy for solving problems that focuses on comprehension of the word problem.
    - The problem is read 3 times, each for a different purpose.
      - What is the problem, context, or story about?
      - What are we trying to find out?
      - What information is important in the problem?

What is the problem about?	What are we trying to find out?	What information is important to the problem?
• Students need water for a science experiment.	• How much water is needed for the whole class?	• Amount of students • Amount of water each student needs

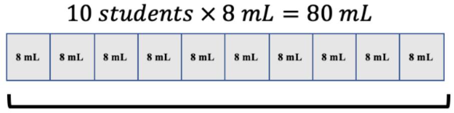
- Instruction includes opportunities to measure often and provide feedback. Use error and reasoning analysis activities to address common measurement difficulties.
- Instruction includes opportunities to find the locations of points on number lines. Number lines should be represented vertically and horizontally. Instruction includes whole number values and fractions, including fractions greater than one.
    - For example, number lines should be included with benchmarks instead of every number in the sequence included. The blue line below extends from the 0 mark on the number line to the first hashmark beyond 2. The dot plotted on the number line identifies the end of the blue line. Since each whole number interval is partitioned into four equal parts, the first hashmark beyond 2 is represented as $2\frac{1}{4}$.



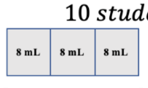
    - For example, number lines can also have all numbers included to represent the values between the benchmarks.
    - For example, teaching problem-solving strategies should focus on the comprehension of problem contexts and what quantities represent in them.
- Instruction includes an emphasis on teaching problem-solving strategies, focusing on the comprehension of problem contexts and what quantities represent in them.
    - For example, questions that help students comprehend word problems are:
        - What is happening in the real-world problem?
        - What do you need to find out?
        - What do the quantities represent in the problem?
        - What will the solution represent in the problem?
    - For example, "For a science experiment in Mr. Thomas's 3rd grade class, each student needs 8 milliliters of water. If there are 23 students in Mr. Thomas's class, how many milliliters will be needed in all?"

$80 mL + 80\ mL + 24\ mL = 184\ mL$

$10\ students \times 8\ mL = 80\ mL$



$10\ students \times 8\ mL = 80\ mL$



$3\ students \times 8\ mL = 24\ mL$

- Teacher guides exploration in estimation strategies to determine reasonable ranges for solutions (e.g., rounding, finding low and high estimates) and teaches problem-solving strategies that build comprehension (e.g., Three Reads).
    - For example, the 3-Reads Protocol is a close reading strategy for solving problems that focuses on comprehension of the word problem.
        - The problem is read 3 times, each for a different purpose.
            - What is the problem, context, or story about?
            - What are we trying to find out?
            - What information is important in the problem?

What is the problem about?	What are we trying to find out?	What information is important to the problem?
• Students need water for a science experiment.	• How much water is needed for the whole class?	• 23 students • Each student needs 8 mL

Instructional Tasks

Instructional Task 1

Each year, the Tallahassee Pumpkin Festival hosts a contest to find the largest pumpkin grown that season. The winner of the competition has the greatest mass, in grams. The masses of the contest entries are in the table below.

A	8,164
B	7,322
C	9,002
D	6,488
E	7,450
F	8,098
G	6,341

Part A. Which pumpkin won the contest?

Part B. What is the difference of the mass, in grams, between the first and second place winning pumpkins?

Instructional Items

Instructional Item 1

For a science experiment in Mr. Thomas's 3rd grade class, each student needs 8 milliliters of water. If there are 23 students in Mr. Thomas's class, how many milliliters will be needed in all?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.M.2 *Tell and write time and solve problems involving time.*

MA.3.M.2.1

Benchmark

MA.3.M.2.1	Using analog and digital clocks tell and write time to the nearest minute using a.m. and p.m. appropriately.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is not to understand military time.

Connecting Benchmarks/Horizontal Alignment

- MA.3.M.1.1

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks
- MA.2.M.2.1

Next Benchmarks
- MA.4.M.2.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to tell time to the nearest minute, using a.m. and p.m. appropriately. In Grade 2, students tell and write time on analog and digital clocks to the nearest five minutes, including using language that expressions fractional parts of an hour (e.g., “half of,” “half past,” “quarter of,” “quarter after,” and “quarter til”). Students also bring understanding about a.m. and p.m. from Grade 2, and they also related partitioned circles to number lines with the purpose of helping them count by 5s.

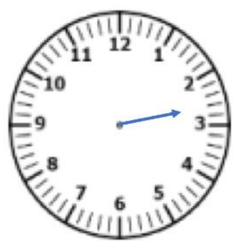
- Instruction should connect how students can count by fives and ones to identify the exact time on an analog clock. For example, if the time on an analog clock shows 3:19, students should know that they can use the minute hand to count by 5s to land at the 3 on the clock (15 minutes after the hour), and then count ahead 4 more minutes to represent 19 minutes. Students could also count by 5s to get to the 4 on the clock (20 minutes after the hour), and then count back one to get to 3:19. During instruction, allowing students opportunities to use flexible strategies for telling time will build understanding and continue to connect telling time to using number lines *(MTR.4.1, MTR.5.1)*.
- Manipulatives that help students tell and write time are Judy clocks, virtual clocks, and number lines (that can be folded as a circle around a clock and unfolded to show a linear representation) *(MTR.2.1)*. It is important to note that when using number lines during instruction, students should be given the opportunities to determine the intervals and size of jumps on their number line. This approach also connects to measuring lengths (MA.3.M.1.1).

Common Misconceptions or Errors

- Students can misrepresent the location of the hour hand when expressing a given time on an analog clock. For example, when representing the hour hand for 3:19, students can be unsure where the hour hand is located between the 3 and 4. Model reasoning with students that the hour hand should be less than half way between 3 and 4 because 3:19 is before 3:30 when the hour hand would be in the middle. Allow for classroom discussions that encourage students to justify the location of hour hands between benchmarks when representing analog time.

Strategies to Support Tiered Instruction

- Instruction includes classroom discussions that encourage students to justify the location of hour hands between benchmarks when representing analog time.
- Instruction includes how the hour hand moves around the clock. Instruction includes using a one-handed (hour hand only) clock. As students receive given times from the teacher, they should reason the location of the hour hand for that given time.
  - For example, the teacher models where the hour hand of the clock should be if the time is 2:37, reasoning for the students so they understand that they should point the hour hand slightly more than halfway between the 2 and the 3 on the clock because 2:37 is just past 2:30.

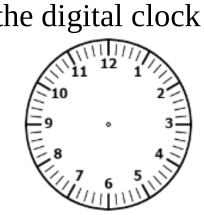


- Instruction includes understanding that the hour hand moves around the clock. Instruction includes using a geared manipulative clock. This clock will demonstrate the relationship between the minute hand and hour hand moving around the clock.
  - For example, the teacher moves the hands on the clock so the hour hand is slightly more than half-way between the 2 and the 3 asking, "What time do you think it is on the clock?" (The clock reads approximately 2:37.) The teacher allows for classroom discussions that encourage students to justify the location of hour hands between benchmarks when representing analog time.

Instructional Tasks

Instructional Task 1

Show the same time represented on the digital clock on the analog clock below.



Instructional Items

Instructional Item 1

Alex goes to the grocery store in the morning at the time shown.

What time does Alex go to the grocery store? Write the time on the line and circle a.m. or p.m.

_________ a.m./p.m.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.M.2.2

Benchmark

MA.3.M.2.2 Solve one- and two-step real-world problems involving elapsed time.

Example: A bus picks up Kimberly at 6:45 a.m. and arrives at school at 8:15 a.m. How long was her bus ride?

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is not to include crossing between a.m. and p.m.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2
- MA.3.AR.1.2

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks
- MA.2.M.2.1

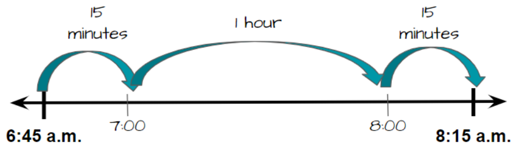
Next Benchmarks
- MA.4.M.2.1

Purpose and Instructional Strategies

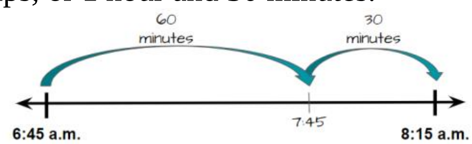
The purpose of this benchmark is for students to apply their understanding of telling and writing time to solve one- and two-step real-world problems involving elapsed time. Elapsed time can be represented within a single hour (e.g., determining when a half-hour gym class would end if it began at 8:10 a.m.) or crossing into the next hour (e.g., determining when a half-hour gym class will end if it began at 8:45 a.m.). Elapsed time should not include crossing between a.m. and p.m. This is the first grade where students will be expected to determine elapsed time.

- When solving problems with elapsed time, students may see different problem types. Students may see result unknown problems (e.g., determining when an activity ends, given the starting time and length of activity), change unknown problems (e.g., determining the length of an activity, given the starting and end times), or start unknown problems (e.g., determining the starting time, given the length of the activity and ending time) *(MTR.2.1, MTR.7.1)*.

- A great way for students to work with elapsed time problems is to use number lines. It is important to note that when using number lines during instruction, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-marked number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students). Open number lines encourage students to jump from one point on the line to another any way they choose, allowing them to calculate flexibly. Students should compare their open number line strategies with one another, and then make connections between them during classroom discussions.
- In real-world elapsed time problems, students use open number lines to represent solutions in many ways. Two open number lines that represent the benchmark's example are below.



  - In this example, the student counted up to benchmark hours, then an addition 15 minutes to jump to 8:15 a.m. The student would reason that the elapsed time is the sum of the jumps, or 1 hour and 30 minutes.



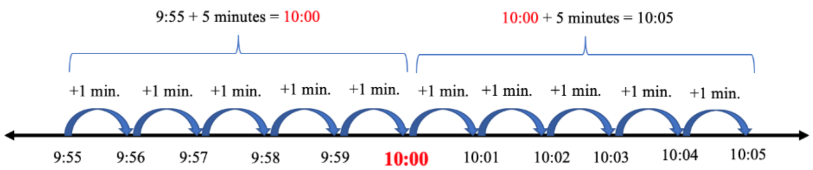
  - In this example, the student jumped 60 minutes to 7:45 a.m., and then another 30 minutes to 8:15 a.m. In this example, the student would represent the answer as 60 minutes + 30 minutes, or 90 minutes.
  - Notice that both the answers of 1 hour and 30 minutes and 90 minutes are acceptable. Students' solutions may be expressed as hours and minutes or minutes only. Conversion from minutes to hours or hours to minutes is not expected in Grade 3, so students should see both as correct *(MTR.2.1, MTR.5.1)*.
- In addition to number lines, Judy clocks provide a great visual to help students identify elapsed time and can be used to help students solve real-world problems *(MTR.2.1)*.
- Elapsed time problems can involve multiplication and division. For example, if Petra starts running laps at 9:55 a.m. and runs 6 laps at 2 minutes per lap, what time does she finish?

Common Misconceptions or Errors

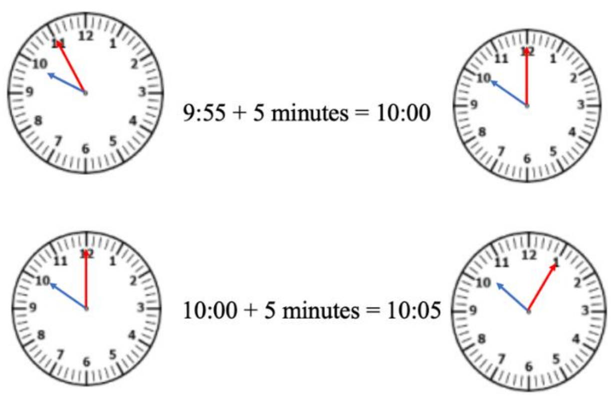
- Students can confuse when time crosses the hour because it does not follow the familiar base ten pattern. For example, students can misinterpret that the elapsed time between 9:55 a.m. and 10:05 a.m. and state that the elapsed time is 50 minutes because they have found the difference from 55 to 105. The use of number lines and clocks side-by-side help students build understanding about how elapsed time is calculated.

Strategies to Support Tiered Instruction

- Instruction includes the use of number lines and clocks side-by-side to help students build understanding about how elapsed time is calculated.
- Instruction includes using a number line and counting by ones to demonstrate what happens when time crosses the hour because it does not follow the familiar base ten pattern.
  - For example, use a number line to find the elapsed time between 9:55 a.m. and 10:05 a.m. and explain what happens when time crosses the hour at 10:00 a.m.



- Instruction includes using a geared manipulative clock.
  - For example, the teacher uses a geared manipulative clock model how to find the elapsed time between 9:55 a.m. and 10:05 a.m. Students should move the minute of the hand one minute at a time from 9:55 to 10:00. After each minute, the teacher asks students to record what time it is. The teacher should have students pay special attention to what happens when the minute hand moves from 9:59 to the next minute.



Instructional Tasks

Instructional Task 1

Recess began at the time shown on Clock A. Recess ended at the time shown on Clock B.

Clock A

Clock B

How many minutes were spent at recess?

Instructional Task 2

Anthony began reading at the time shown on Clock A. He stopped at the time shown on Clock B.

Clock A

Clock B

How many minutes did Anthony spend reading?

Instructional Items

Instructional Item 1

Each week, Victor attends violin lessons that last 55 minutes. If the lesson begins at 4:30 p.m., what time will it end?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.GR.1 *Describe and identify relationships between lines and classify quadrilaterals.*

MA.3.GR.1.1

Benchmark

MA.3.GR.1.1	Describe and draw points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines. Identify these in two-dimensional figures.

Benchmark Clarifications:

Clarification 1: Instruction includes mathematical and real-world context for identifying points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines.

Clarification 2: When working with perpendicular lines, right angles can be called square angles or square corners.

Connecting Benchmarks/Horizontal Alignment

- MA.3.GR.2.4
- MA.3.DP.1.1

Terms from the K-12 Glossary

- Line
- Parallelogram
- Rectangle
- Square
- Triangle

Vertical Alignment

Previous Benchmarks

- MA.2.GR.1.1

Next Benchmarks

- MA.4.GR.1.1/1.2

Purpose and Instructional Strategies

The purpose of this benchmark is to build important vocabulary that allows students to describe and draw points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines, and to identify examples in two-dimensional figures represented in mathematical and real-world contexts. In Grade 2, students were expected to identify and draw two-dimensional figures based on their defining attributes. In Grade 3, students can describe and draw geometric figures using more formal vocabulary developed in this benchmark. Therefore, instruction of this benchmark relies heavily on vocabulary development for students to internalize definitions and make connections between the concepts. In Grade 4, students will explore and draw angles beyond square angles.

- In mathematical contexts, students find evidence of points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines in models of two-dimensional figures (e.g., quadrilaterals, triangles). In real-world contexts, students identify evidence of these geometric attributes in real-life images (e.g., aerial views of city maps, photos of objects) *(MTR.4.1, MTR.7.1)*.
- This vocabulary development will be necessary as students identify and draw quadrilaterals based on their defining attributes (MA.3.GR.1.2). It will also be beneficial in other areas as students begin to read, draw and understand graphs and diagrams.
- Instruction should also consider activities that encourage student discussions rich in mathematical reasoning opportunities. Mathematical discussions and reasoning activities give students the practice necessary to use the vocabulary and internalize it in meaningful ways. An example of a mathematical reasoning activity that builds vocabulary understanding is in the instructional task below *(MTR.4.1)*.
- Two additional notes about instruction of this benchmark:
    - Images of figures used in instruction should not include hatch marks.
    - Because formal instruction of angle measurements does not begin until Grade 4, students can refer to right angles in perpendicular lines as “square angles” or “square corners.”

Common Misconceptions or Errors

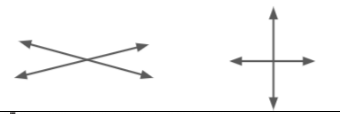
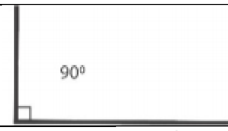
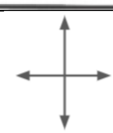
- Students can confuse some pairs of intersecting lines as perpendicular. Encourage students to justify their thinking whenever they reason about geometric concepts. For example, students can use the corners of a standard sheet of paper as a comparison to determine whether a pair of intersecting lines is perpendicular.
- Students may struggle to identify examples of points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines in real-world examples. Classroom instruction should include many examples for students to explore.

Strategies to Support Tiered Instruction

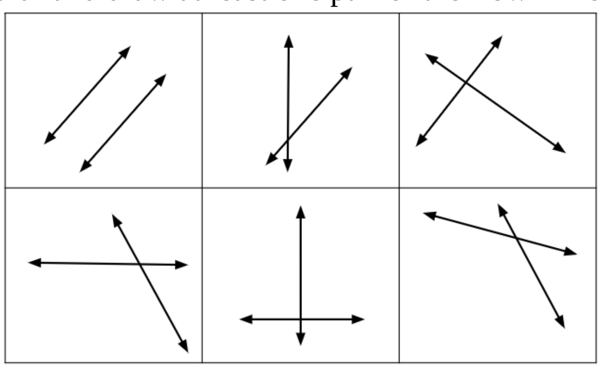
- Instruction provides opportunities to justify thinking when reasoning about geometric concepts.
    - For example, the teacher demonstrates how to use the corners of a standard sheet of paper as a comparison to determine whether a pair of intersecting lines is perpendicular.
- Instruction includes the use of key vocabulary, referencing definitions for terms such as intersecting lines, right angle, and perpendicular lines. The teacher draws examples of

intersecting lines that are both perpendicular and not perpendicular and has students explain which they are and justify their reasoning.

  - For example, teachers provide key vocabulary as shown below for students to refer to. The teacher will then draw sets of lines, some that do not intersect, some that intersect but do not create right angles, and other sets that do create right angles. Students determine which pairs of intersecting lines can be classified as perpendicular and explain why.

	Definition	**Example**
Intersecting Lines	Lines that cross.	
Right Angle	An angle that measures exactly 90 degree or forms a square corner.	
Perpendicular Lines	Lines that cross and form right angles.	

- The teacher provides a tool such as a square tile or the corner of a piece of paper to identify intersecting lines that create right angles and classify those as perpendicular lines and those that do not form right angles as intersecting but not perpendicular. Students use the tool to draw some of their own intersecting lines that would be examples of both.
  - For example, the teacher may provide students with a graphic organizer like the one shown below and a set of cards with pairs of lines (examples shown below). The students use the tool to sort the cards into perpendicular and not perpendicular and draw at least one pair of their own lines for each category.



Intersecting Lines	
Not Perpendicular	Perpendicular

- Instruction includes real-world examples of points, lines, line segments, rays, intersecting lines, perpendicular lines, and parallel lines. The teacher provides images of real-world

examples that include geometric figures. Students identify the geometric figure in the example.
    - For example, the teacher provides an image of railroad tracks to represent parallel lines, a speed sign to represent perpendicular lines, a balance beam to represent a line segment, and other common images.
- Instruction includes real-world examples of points, lines, line segments, rays, intersecting lines, perpendicular lines, and parallel lines. The teacher points out items in the classroom that are examples of the geometric terms listed above and has students identify which term it is an example of.
    - For example, if the teacher points out a poster with the number one or the letter l on it, students will say it represents a line segment. If the teacher points out the window, students will say the top and bottom of the window shows parallel lines, while the corners of the window show perpendicular lines.
    - For example, students to find their own examples within in the classroom and explain which geometric term they notice in the figure.

Instructional Tasks

Instructional Task 1

Are intersecting lines always, sometimes or never parallel? Show your thinking.

Instructional Task 2

Are intersecting lines always, sometimes or never perpendicular? Show your thinking.

Instructional Task 3

Draw a geometric figure with parallel and perpendicular sides.

Instructional Items

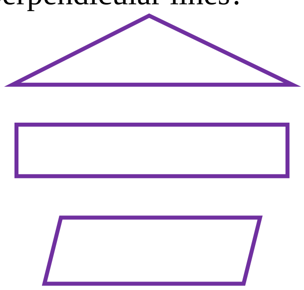
Instructional Item 1

Which of the following figures show perpendicular lines?

I.

II.

III.



a. I only
b. II only
c. II and III
d. I, II and III

*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

MA.3.GR.1.2

Benchmark

MA.3.GR.1.2	Identify and draw quadrilaterals based on their defining attributes. Quadrilaterals include parallelograms, rhombi, rectangles, squares and trapezoids.

Benchmark Clarifications:
Clarification 1: Instruction includes a variety of quadrilaterals and a variety of non-examples that lack one or more defining attributes when identifying quadrilaterals.
Clarification 2: Quadrilaterals will be filled, outlined or both when identifying.
Clarification 3: Drawing representations must be reasonably accurate.

Connecting Benchmarks/Horizontal Alignment

- MA.3.M.1.1/1.2

Terms from the K-12 Glossary

- Line
- Parallelogram
- Rectangle
- Square

Vertical Alignment

Previous Benchmarks

- MA.2.G.1.1

Next Benchmarks

- MA.4.G.1.1

Purpose and Instructional Strategies

The purpose of this benchmark is to provide opportunities for students to apply their formalized definitions of geometric attributes when identifying and drawing quadrilaterals *(MTR.5.1)*. With the support of vocabulary developed about geometric attributes in benchmark MA.3.GR.1.1, the goal of this benchmark is for students to identify and draw quadrilaterals based on them. In Grade 2, students started to explore and draw quadrilaterals in less formal ways.

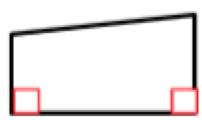
- This benchmark gives students opportunities to build vocabulary around examples of quadrilaterals (e.g., parallelograms, rhombi, rectangles, squares, and trapezoids) based on the attributes that define them. Understanding quadrilaterals will help them make comparisons to non-quadrilaterals *(MTR.4.1)*.
- In Grade 4, students will classify types of angles and identify them in two-dimensional figures. In Grade 5, prior learning about quadrilaterals and triangles is synthesized for students to classify these figures based on their attributes.
- Instruction should include highlighting measurement as an attribute to help categorize quadrilaterals.

Common Misconceptions or Errors

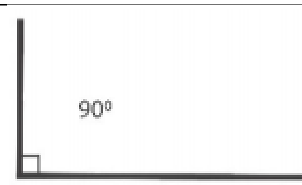
- Students can confuse some pairs of intersecting lines as perpendicular. Encourage students to justify their thinking whenever they reason about geometric concepts. For example, students can use the corners of a standard sheet of paper as a comparison to determine whether a pair of intersecting lines is perpendicular.
- Some students may assume all quadrilaterals must have attributes of squares, rhombi, rectangles, squares, and trapezoids. During instruction, it is important for students to determine that a figure lacking further defining attributes (such as a kite) can still be a quadrilateral.

Strategies to Support Tiered Instruction

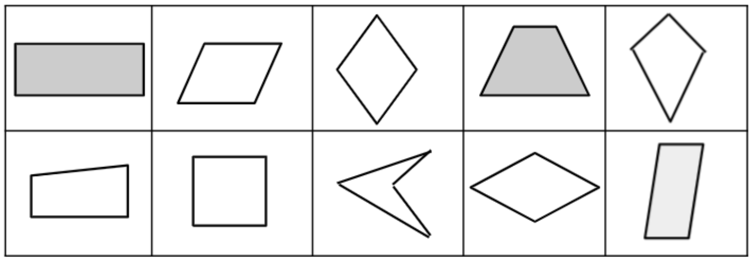
- Instruction includes real-world examples of points, lines, line segments, rays, intersecting lines, perpendicular lines, and parallel lines. The teacher provides images of real-world examples that include geometric figures. Students identify the geometric figure in the example.
  - For example, the teacher provides an image of railroad tracks to represent parallel lines, a speed sign to represent perpendicular lines, a balance beam to represent a line segment, and other common images.
- Instruction includes real-world examples of points, lines, line segments, rays, intersecting lines, perpendicular lines, and parallel lines. The teacher points out items in the classroom that are examples of the geometric terms listed above and has students identify which term it is an example of.
  - For example, if the teacher points out a poster with the number one or the letter l on it, students will say it represents a line segment. If the teacher points out the window, students will say the top and bottom of the window shows parallel lines, while the corners of the window show perpendicular lines.
  - For example, students to find their own examples within in the classroom and explain which geometric term they notice in the figure.



- Teacher provides students with key vocabulary from the glossary to identify right angles to help them identify perpendicular sides in shapes. The teacher also provides a tool such as a square tile or the corner of a standard sheet of paper to help students find right angles. Students then matches quadrilaterals that contain this attribute.
  - For example, the teacher provides a vocabulary card or vocabulary information from the glossary for a right angle, similar to the example shown below. Students then uses the tool provided to locate right angles and identifies which quadrilaterals contain that attribute when provided images of parallelograms, rhombi, rectangles, squares, and trapezoids.

Right Angle	An angle measuring exactly 90°. An angle that forms a square corner.	

- Teacher provides a graphic organizer to help students identify given attributes in figures. Students place the figures under the correct columns and identify quadrilaterals that do not contain any of the attributes stated.
    - For example, the teacher provides sample figures and students draw them in or place the shape cards in the correct columns of the graphic organizer (some figures will fit in more than one column).



4 equal sides	perpendicular sides/right angles	parallel sides	quadrilaterals that do not contain any of the given attributes

- Teacher provides figures that can be classified as quadrilaterals and those that are not (shapes may include: triangles, squares, pentagons, hexagons, square, rectangles, parallelograms, trapezoids, and other quadrilaterals such as a kite). Students sort the figures into two groups, quadrilaterals and non-quadrilaterals and justify their reasoning by explaining how they used the number of sides each figure has to determine their placement.
    - For example, students will add figures to the chart shown below and explain why the figure belongs in that category.

Quadrilaterals	**Non-quadrilaterals**

Instructional Tasks

Instructional Task 1

Draw an example of a quadrilateral that does not have any defining attributes of a square (expect that it has 4 straight sides and 4 vertices). Then explain how you know.

Instructional Items

Instructional Item 1

Which of the following quadrilaterals always have 2 sets of parallel sides? Select all that apply.

a. Square
b. Rectangle
c. Rhombus
d. Parallelogram
e. Trapezoid

Instructional Item 1

Which of the following quadrilaterals always have perpendicular sides? Select all that apply.

a. Square
b. Rectangle
c. Rhombus
d. Parallelogram
e. Trapezoid

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.GR.1.3

Benchmark

MA.3.GR.1.3	Draw line(s) of symmetry in a two-dimensional figure and identify line-symmetric two-dimensional figures.

Benchmark Clarifications:
Clarification 1: Instruction develops the understanding that there could be no line of symmetry, exactly one line of symmetry or more than one line of symmetry.
Clarification 2: Instruction includes folding paper along a line of symmetry so that both halves match exactly to confirm line-symmetric figures.

Connecting Benchmarks/Horizontal Alignment

- MA.3.FR.1.1
- MA.3.AR.3.1

Terms from the K-12 Glossary

- Line of Symmetry

Vertical Alignment

Previous Benchmarks

- MA.2.GR.1.3

Next Benchmarks

- MA.6.GR.1.1
- MA.8.GR.2.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to draw lines of symmetry and identify line-symmetric figures. In Grade 2, students identify lines of symmetry in a two-dimensional figure by partitioning (e.g., folding) it and matching its halves. In addition, students in Grade 3 also developed the understanding that a figure can have no lines of symmetry, exactly 1 line of symmetry, or more than 1 line of symmetry.

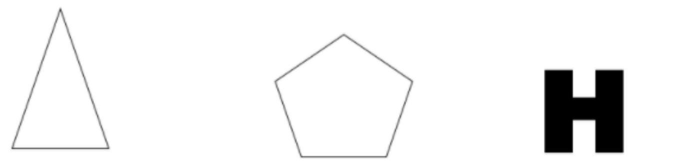
- During instruction, teachers should continue encouraging students to partition figures and match their halves to identify line(s) of symmetry *(MTR.2.1)*.
- Instruction can also ask students to build generalizations about which two-dimensional figures are line symmetric and why. For example, students could argue that all squares share similar defining attributes and only differ in size, therefore all squares will be line-symmetric *(MTR.2.1, MTR.4.1)*.
- Instruction builds a foundation for exploring reflections in middle school.

Common Misconceptions or Errors

- Students can miss identifying all lines of symmetry in line-symmetric figures. Encourage classroom discussions and have students justify their arguments about lines of symmetry using their partitioned representations.

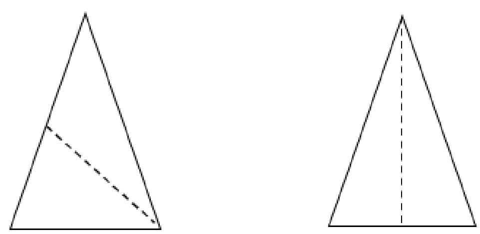
Strategies to Support Tiered Instruction

- Teacher provides figures that have at least one line of symmetry and tells how many lines of symmetry the figure has. Students draw lines to show where the lines of symmetry would be.
    - For example, the teacher provides students with images similar to those shown below and has students draw the number of lines of symmetry given and explain how they know the lines they draw are lines of symmetry.



1 line of symmetry | 5 lines of symmetry | 2 lines of symmetry

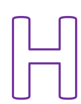
- Teacher provides a figure partitioned in different ways with dotted lines. Students fold the image along the dotted line and determine if it is a line of symmetry (do the two sides match).
    - For example, the teacher gives a triangle like the one shown below. Students fold along the dotted lines and determine if it shows a line of symmetry or not.



Instructional Tasks

Instructional Task 1

Mika says that the uppercase letter H below has 1 line of symmetry. Errol says that the uppercase letter H has 2 lines of symmetry. Who is correct? Show your thinking.



Instructional Items

Instructional Item 1

Select all the figures that have at least one line of symmetry.

a. A

b. G

c. H

d. R

e. Q

Instructional Item 2

How many lines of symmetry does the following figure have?

A

Instructional Item 3

A figure is shown.

X

How many lines of symmetry does the figure have?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.GR.2 *Solve problems involving the perimeter and area of rectangles.*

MA.3.GR.2.1

Benchmark

MA.3.GR.2.1	Explore area as an attribute of a two-dimensional figure by covering the figure with unit squares without gaps or overlaps. Find areas of rectangles by counting unit squares.

Benchmark Clarifications:
Clarification 1: Instruction emphasizes the conceptual understanding that area is an attribute that can be measured for a two-dimensional figure. The measurement unit for area is the area of a unit square, which is a square with side length of 1 unit.
Clarification 2: Two-dimensional figures cannot exceed 12 units by 12 units and responses include the appropriate units in word form (e.g., square centimeter or sq.cm.).

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2

Terms from the K-12 Glossary

- Rectangular Array

Vertical Alignment

Previous Benchmarks
- MA.2.AR.3.2
- MA.2.GR.2.1

Next Benchmarks
- MA.4.GR.2.1/2.2

Purpose and Instructional Strategies

The purpose of this benchmark is to provide the foundation for students to understand area measurement. In Grades 1 and 2, students learned about linear measurement using number lines, rulers, and calculating perimeter. In Grade 3, students build on their knowledge of measurement and multiplicative reasoning to explore and understand area measurement. Instruction emphasizes that area is a two-dimensional measurement, therefore it is measured in units that are also two-dimensional – unit squares with side lengths that measure one unit. Area is calculated using unit squares that cover a shape without gaps or overlap *(MTR.5.1)*.

- The expectation of this benchmark is for students to calculate area of rectangles by **counting** unit squares *(MTR.2.1)*.
- Instruction allows for students to draw conclusions about connections to arrays and to determine more efficient counting strategies for calculation, leading to the use of a multiplication formula in 3.GR.2.2 *(MTR.4.1, MTR.5.1)*.

Common Misconceptions or Errors

- Students may miscount unit squares when they are laid out in a figure. Encourage students to mark unit squares as they are counted.
- Students can confuse why area is measured in “square units.” Use this exploratory benchmark for students to relate area measurement to the counting of squares. This benchmark provides the opportunity for students to build vocabulary necessary for area measurement.

Strategies to Support Tiered Instruction

- Instruction includes modeling how to number the unit square tiles, so students do not miscount when finding area.
    - For example, the teacher provides students with figures created with squares and has them number each square as they count.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

- Instruction includes creating figures with no gaps or overlaps that have a given area. Students mark each unit square with a number as they count to check that the area of the figure they create has the correct area.
    - For example, the teacher provides students with grid paper and ask them to create a figure with an area of 24 square units. Student count and label 24 connected squares on the grid paper and then shade in the entire figure (see example below).

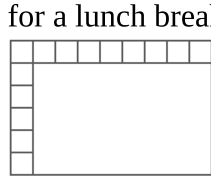
	1	2	3	4	5	6	7	8	
	9	10	11	12	13	14	15	16	
	17	18	19	20	21	22	23	24	

- Instruction includes measuring the area of given figures by covering them with 1-inch square tiles, leaving no gaps or overlaps. Students count the total number of squares it takes to completely cover the figure and explain how that number represents the area in square units of the figure.
    - For example, the teacher provides a sheet with figures that can be covered perfectly using the square tiles. Students tile the figure and count the square tiles to identify the area.
- Instruction includes students creating their own figures by connecting square tiles with no gaps or overlaps and counting the tiles.
    - For example, the teacher provides a set of 1-inch tiles and asks students to build a figure with an area of 18 square inches. After students have created the figure, they will count and number each tile to ensure they have an area of 18 square inches.

Instructional Tasks

Instructional Task 1

Kendra used unit squares with 1-centimeter side lengths to find the area of the rectangle below. She started, but then stopped for a lunch break.

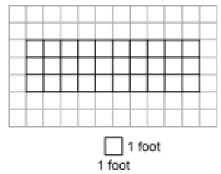


a. What is the area of Kendra's figure?
b. Explain how you counted.

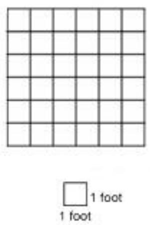
Instructional Items

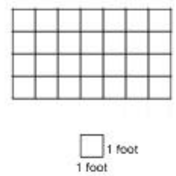
Instructional Item 1

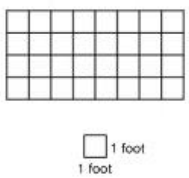
Alex put the tiles shown on his floor.

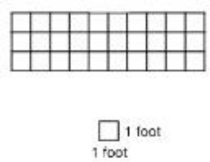


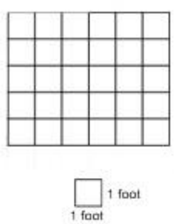
Part A. What is the area in square feet of the portion that Alex has covered?
Part B. What is the area in square feet of the entire floor?
Part C. The area of Alex’s floor is 30 square feet. Select all the floors that could be Alex’s.

A. 

B. 

C. 

D. 

E. 

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.GR.2.2

Benchmark

MA.3.GR.2.2	Find the area of a rectangle with whole-number side lengths using a visual model and a multiplication formula.

Benchmark Clarifications:
Clarification 1: Instruction includes covering the figure with unit squares, a rectangular array or applying a formula.
Clarification 2: Two-dimensional figures cannot exceed 12 units by 12 units and responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.3/2.4

Terms from the K-12 Glossary

- Rectangular Array

Vertical Alignment

Previous Benchmarks

- MA.2.AR.3.2
- MA.2.GR.2.2

Next Benchmarks

- MA.4.GR.2.1/2.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to calculate the area of rectangles presented visually as arrays or by using a multiplication formula *(MTR.5.1)*.

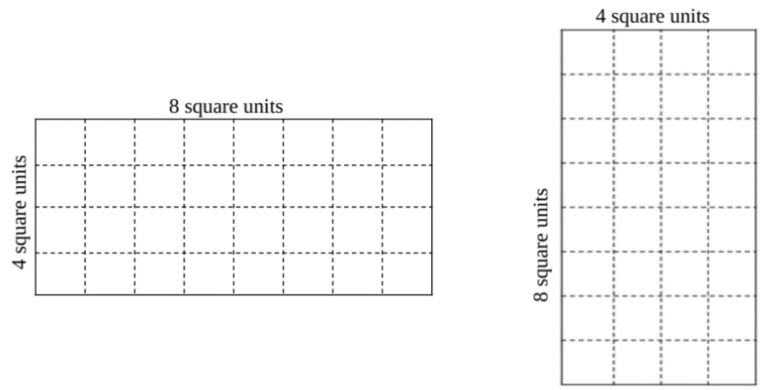
- The benchmark MA.3.GR.2.1 expects students to calculate the area of rectangles by counting unit squares that covered them with no gaps or overlaps. As students count, they will likely connect their calculations to rectangular arrays and connect understanding that multiplication is a more efficient strategy for calculating than counting or adding unit squares.
- Instruction should encourage students to discover a multiplication formula based on patterns they have observed through practice and classroom discussions. This will make a multiplication formula more meaningful for students conceptually *(MTR.5.1)*. Teachers can help students formalize the formula into an equation, like $A = l \times w$. In this benchmark, memorization of a multiplication formula is the goal *(MTR.3.1)*.

Common Misconceptions or Errors

- When using a formula, students may be confused about which dimension to label the length and width in a rectangle. During instruction, teachers should make connections to the commutative property of multiplication to emphasize that the order in which dimensions are multiplied will not change the rectangle's area, and therefore the length and width can be labeled flexibly.

Strategies to Support Tiered Instruction

- Instruction includes the teacher modeling how to draw in rows and columns to cover a figure based on the side lengths given. Students then count the total number of square units that make up the figure and write a multiplication equation to represent it. Teachers help students make the connection to the Commutative Property of Multiplication by having them create and compare figures with the same factors for their rows and columns, just switched. Emphasize that the order in which dimensions are multiplied will not change the rectangle's area, and therefore the length and width can be labeled flexibly.
    - For example, when provided with a figure with the dimensions of 4×8, students draw in the rows and columns as shown by the dotted lines. The teacher then asks students to do the same for an image with the dimensions 8×4 and has them compare the area of the two figures.

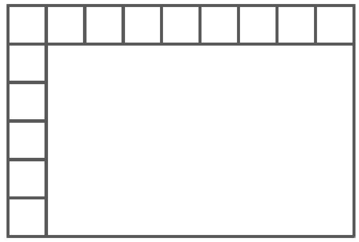


- Teacher provides dimensions for a given rectangle and students use square tiles to build the figure in two ways. Students then count the number of tiles in each row and in each column and creates a multiplication expression. Next, the students count the total number of tiles used to make the figure and recognize that as the area of the figure.
    - For example, the teacher asks students to create a rectangle with a length of 5 and a width of 7. Students use the square tiles to create two rectangles applying the Commutative Property of Multiplication and writing multiplication equations to match. Then, students count the total number of tiles to check that the area they found for their equation is correct.

Instructional Tasks

Instructional Task 1

Kendra used unit squares with 1-centimeter side lengths to find the area of the rectangle below. She started, but then stopped for a lunch break.

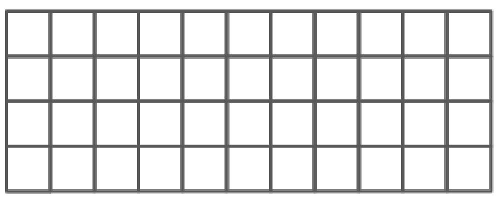


Part A. Write two equations that can be used to find the area of Kendra's rectangle.
Part B. What is the area of Kendra's rectangle?
Part C. Which has greater area, the rectangle above or a square with side lengths of 8 centimeters? Explain.

Instructional Items

Instructional Item 1

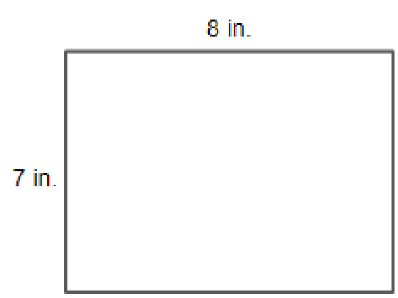
The rectangle below is composed of unit squares. Which equations can be used to find the area of the rectangle?



a. $A = 4 \times 10$
b. $A = 10 \times 4$
c. $A = 4 + 11$
d. $A = 4 \times 11$
e. $A = 11 + 11 + 11 + 11$
f. $A = 11 \times 4$

Instructional Item 2

What is the area of the rectangle below?



*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.

MA.3.GR.2.3

Benchmark

MA.3.GR.2.3	Solve mathematical and real-world problems involving the perimeter and area of rectangles with whole-number side lengths using a visual model and a formula.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is not to find unknown side lengths.
Clarification 2: Two-dimensional figures cannot exceed 12 units by 12 units and responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4
- MA.3.AR.1.2
- MA.3.M.1.1/1.2

Terms from the K-12 Glossary

- Perimeter
- Rectangle

Vertical Alignment

Previous Benchmarks
- MA.2.GR.1.1/1.2

Next Benchmarks
- MA.4.GR.2.1/2.2

Purpose and Instructional Strategies

The purpose of this benchmark is for students to solve mathematical and real-world problems using the perimeter and area of rectangles using a visual model and/or a formula for each.

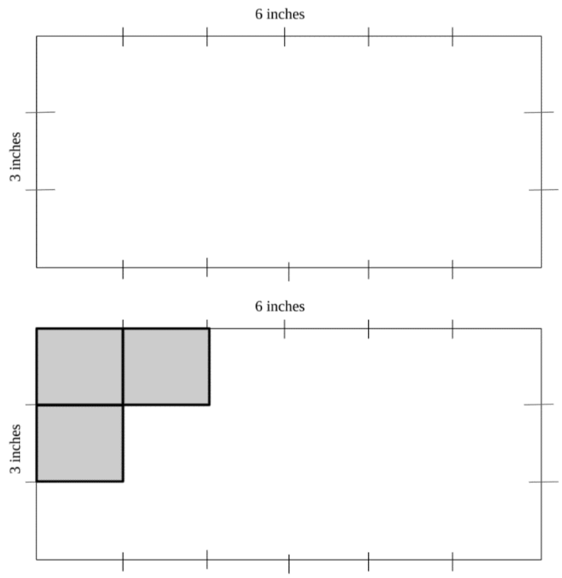
- In the provided mathematical and real-world problems, instruction should include cases where students use a ruler to measure lengths before determining its perimeter and/or area *(MTR.3.1)*.
- Mathematical problems include visual models of rectangles, while examples of real-world problems could include photos or classroom objects (e.g., measuring the area of one face on a tissue box). Students will not be expected to find unknown side lengths until Grade 4 *(MTR.7.1)*.
- This benchmark gives students the chance to measure perimeter and area together and understand their differences – perimeter as a one-dimensional length measurement and area as a two-dimensional measurement. (Note: Though students explored and measured perimeter in Grade 2, they were not expected to determine a formula.) *(MTR.5.1)*
- As recommended for MA.3.GR.2.2 for a multiplication formula for area, classroom instruction should include activities that allow students to build formulas for perimeter based on patterns they observe (e.g., $P = l + l + w + w, P = 2l + 2w$) before expecting them to memorize. Student-created formulas will build conceptual understanding around a formula before memorizing it *(MTR.4.1)*.

Common Misconceptions or Errors

- Students may confuse area and perimeter and use incorrect formulas to find measurements. During instruction, the teacher should continue to emphasize the difference between perimeter as a one-dimensional measurement of length and area as a two-dimensional measurement that covers a shape with unit squares. The teacher can use visuals to show the perimeter (e.g., yarn, string stretched around the rectangle) and area (e.g., square counters, square-shaped sticky notes, square-shaped crackers covering it) to help students differentiate between the measurements.

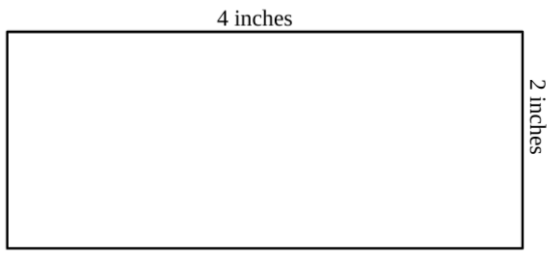
Strategies to Support Tiered Instruction

- Instruction includes opportunities to explore both area and perimeter of given figures and make connections to the formulas to find each. The teacher provides students with dimensions for a figure that has whole number side lengths that can be measured using inches or centimeters. Students use a ruler to measure the side lengths and place tick marks for each whole number unit. Students then label each side length and use the formula for perimeter to calculate. Next, students use the formula for area to find the area and then use the tick marks made when measuring to draw in the rows and columns to check their work.
    - For example, the teacher asks students to draw a figure with a length of 3 inches and a width of 6 inches. Students use a ruler to draw the figure and place tick marks along each side for each inch. Students then use the formula to find the perimeter (P = $3 + 3 + 6 + 6$). Next, students use the tick marks made when measuring to draw in the rows and columns to cover the figure with square inches and then use the formula to find area (A = 3×6).



- Teacher provides a figure that has whole number side lengths that can be measured using inches. Students use a visual representation such as string or yarn to measure the distance around the figure and then measure the length of the string to make the connection to perimeter being a one-dimensional measurement. Students then use a different visual representation such as 1-inch tiles or square sticky notes to cover the figure to find the area and make the connection to area being a two-dimensional measurement.

- For example, the teacher provides an image like the example below. Students use a piece of string to measure the distance around the figure and then use a tape measure to measure the length of the string. Or students can use the string to measure the 2 sides, then add the 2 lengths and multiply by 2 to determine the perimeter. Students will then use square tiles to cover the image to determine the area.



Instructional Tasks

Instructional Task 1

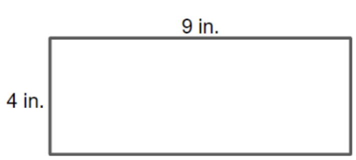
Find the whole number length and whole number width of every rectangle with an area of 18 square feet. Record the length, width and perimeter of each rectangle in the table.

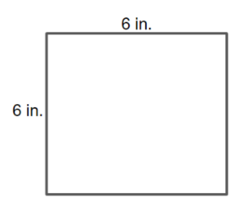
Area	Length	Width	Perimeter
18 sq. ft.			
18 sq. ft.			
18 sq. ft.			
18 sq. ft.			
18 sq. ft.			
18 sq. ft.			

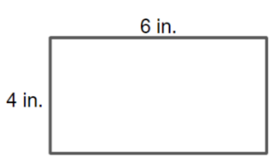
Instructional Items

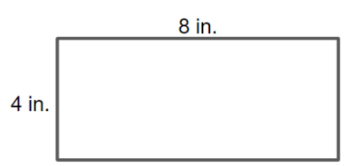
Instructional Item 1

Which of the following rectangles has a perimeter of 24 inches and an area of 36 square inches?

a. 

b. 

c. 

d. 

Instructional Item 2

A rectangle is 12 centimeters long and 9 centimeters wide. What is the area of the rectangle?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Benchmark

MA.3.GR.2.4 Solve mathematical and real-world problems involving the perimeter and area of composite figures composed of non-overlapping rectangles with whole-number side lengths.

Example: A pool is comprised of two non-overlapping rectangles in the shape of an "L". The area for a cover of the pool can be found by adding the areas of the two non-overlapping rectangles.

Benchmark Clarifications:
Clarification 1: Composite figures must be composed of non-overlapping rectangles.
Clarification 2: Each rectangle within the composite figure cannot exceed 12 units by 12 units and responses include the appropriate units in word form.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4
- MA.3.AR.1.2
- MA.3.M.1.1/1.2

Terms from the K-12 Glossary

- Composite Figure
- Perimeter
- Rectangle

Vertical Alignment

Previous Benchmarks
- MA.2.G.2.2

Next Benchmarks
- MA.4.GR.2.1

Purpose and Instructional Strategies

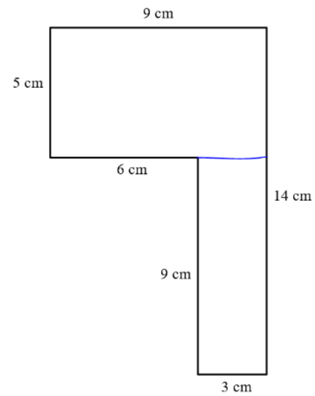
The purpose of this benchmark is for students to solve mathematical and real-world problems involving the perimeter and area of composite figures with whole-number side lengths. This benchmark builds on the work with perimeter done in Grade 2. The area of each rectangle in a composite figure is expected to be within the appropriate multiplication limits for Grade 3 – up to 12 units by 12 units. All side lengths of composite figures should be given, though cases can be provided where students are expected to use a ruler to measure before finding a composite figure's perimeter and/or area.

- Students have previous experience with decomposing larger rectangles into smaller rectangles to find individual areas. When students utilized the distributive property in area models to multiply 2-digit factors by a 1-digit factors, students decomposed (broke apart) the 2-digit number as the sum of its tens and ones. Students learned that the product was the sum of smaller rectangles' areas. Students likely used area models to build fluency within 12×12 as well. During instruction of this benchmark, teachers should have students make connections to their previous learning as they begin decomposing the composite figures *(MTR.2.1, MTR.5.1)*.
- Instruction of measuring area of composite figures should include opportunities for students to justify how they decompose their composite figures into 2 or more rectangles before calculating. As students share the different ways they decompose their figures, they identify that any correct decomposition will yield the correct calculation *(MTR.2.1, MTR.2.1, MTR.3.1)*.

Common Misconceptions or Errors

- Students can confuse the side lengths when determining area calculations once a composite figure has been decomposed. For example, a potential way to decompose the figure from the task below is seen on the right. A student may not yet understand that once decomposed in this way, the length of $14\ cm$ on the right side of the figure is now the sum of $5\ cm + 9\ cm$. The student may continue to multiply by $14\ cm$ to find the areas of each rectangle instead. Likewise, they may multiply $5\ cm$ by $6\ cm$ (instead of $9\ cm$) to find the area of the upper rectangle. During instruction, encourage students to label how side lengths change once a composite figure has been decomposed.

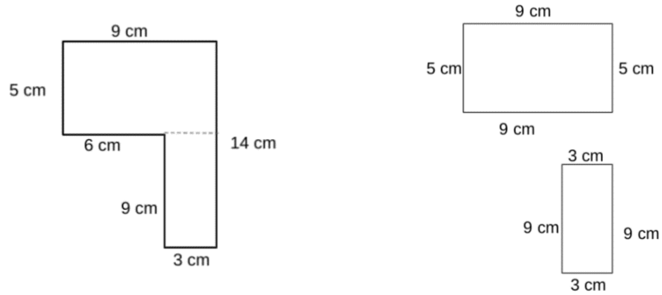
Find the area of the figure. Write the area on the line.



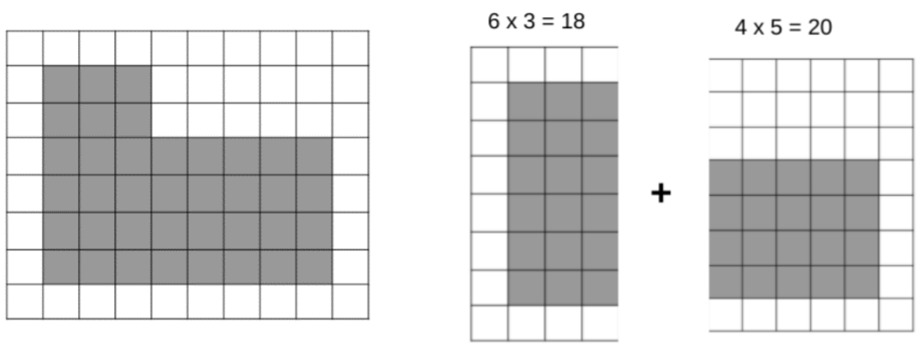
- Students may add all sides of the parts of the figure once it is decomposed to determine the perimeter of the composite figure. For example in the figure shown, the student may find the perimeter to be $(9\ cm + 9\ cm + 5\ cm + 5\ cm) + (9\ cm + 9\ cm + 3\ cm + 3\ cm) = 52\ cm$, when the correct answer is $9\ cm + 14\ cm + 3\ cm + 9\ cm + 6\ cm + 5\ cm = 46\ cm$.

Strategies to Support Tiered Instruction

- Instruction includes decomposing figures in multiple ways, finding the area of each individual rectangle and then finding the sum of the two rectangles.
    - For example, the teacher provides students with a composite figure and asks them to decompose the figure into two rectangles. Students draw and label the two rectangles as separate parts to show their understanding of how the side lengths have changed once the figure was decomposed. Then, students find the area of each individual rectangle. By drawing the two separate rectangles, students identify which measurements to use due to the decomposing of the figures. One example of how the figure can be decomposed is shown. Students may come up with other ways.

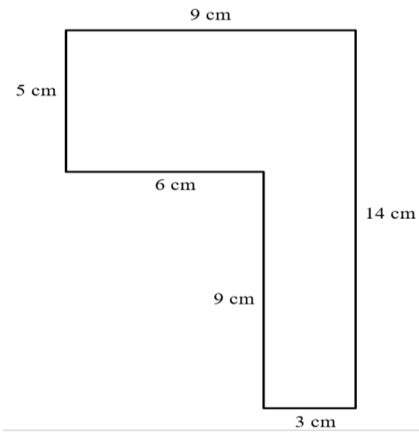


- Teacher provides composite figures created with unit squares. Students cut the figures to decompose them into two separate rectangles and label the dimensions for each figure. Students then find the sum of the area of the two rectangles.
    - For example, teacher provides students with figures similar to the one below. Students determine how they can decompose them into two rectangles (there could be more than one way). Students then cut the figure apart to show the two rectangles and writes multiplication equations to represent the area of each part. Finally, students find the sum of the two areas and determines if the area is the same as the whole figure.
    - Students can find the perimeters of the two separate rectangles and then determine if the sum of the two perimeters is the same as the perimeter of the composite figure.



Instructional Tasks

Instructional Task 1

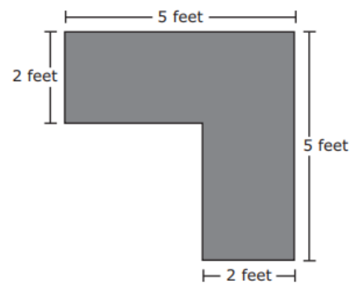


What is the perimeter? ____________
What is the area? ___________

Instructional Items

Instructional Item 1

A drawing of the top of a desk is shown.



What is the area of the top of the desk?

a. 14 square feet
b. 16 square feet
c. 20 square feet
d. 25 square feet

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.DP.1 *Collect, represent and interpret numerical and categorical data.*

MA.3.DP.1.1

Benchmark

MA.3.DP.1.1	Collect and represent numerical and categorical data with whole-number values using tables, scaled pictographs, scaled bar graphs or line plots. Use appropriate titles, labels and units.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is to complete a representation or construct a representation from a data set.
Clarification 2: Instruction includes the connection between multiplication and the number of data points represented by a bar in scaled bar graph or a scaled column in a pictograph.
Clarification 3: Data displays are represented both horizontally and vertically.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4
- MA.3.GR.1.1

Terms from the K-12 Glossary

- Bar Graph
- Categorical Data
- Whole Number

Vertical Alignment

Previous Benchmarks
- MA.2.DP.1.1

Next Benchmarks
- MA.4.DP.1.1

Purpose and Instructional Strategies

The purpose of this benchmark is for students to represent numerical and categorical data using tables, scaled pictographs, scaled bar graphs, or line plots, using appropriate titles, tables, and units. Though there are many skills included in this benchmark, students bring background knowledge from Grades 1 and 2 when they collected, categorized and represented data in tables, pictographs, and bar graphs. In Grade 2, students were expected to represent data with appropriate titles, labels and units.

- Before instruction begins, teachers should provide students with opportunities of reading and solving problems using scaled graphs before being asked to draw one. These skills will assist students with determining what they already know. This will save instructional time that can be focused on the Grade 3 extensions explained in the next paragraph *(MTR.3.1)*.
- Instruction should include opportunities for students to collect and display their own numerical and categorical data *(MTR.7.1)*.
- In Grade 3, two extensions of previous understandings about collecting and representing data occur. First, categorical data represented in pictographs and bar graphs are scaled. Students use their understanding of multiplication to read the data representations appropriately. Second, students represent numerical data in line plots, which shows the frequency of data on a number line *(MTR.2.1)*.
- During instruction, it is important to remind students that scales on graphs should begin with 0.
- Because the expectation is to represent data with whole-number values, number lines do not need to be partitioned into fractional parts. Students will represent fractional values beginning in Grade 4.
- During instruction, it is important that students have the opportunity to display data horizontally and vertically. Their work with GR.1.1 will be beneficial in making graphs that are accurate representations.

Common Misconceptions or Errors

- Students may confuse which types of data (categorical or numerical) can be displayed with a data representation. In Grades 1 and 2, students graphed frequency of categorical data in pictographs and bar graphs. Representing frequency in numerical data graphed via line plots is a new expectation in Grade 3. During instruction, expect students to justify the representations they choose based on the data collected.
- Students tend to count each square as one for intervals on bar graphs that are not single units.

Strategies to Support Tiered Instruction

- Instruction includes how to decide which way to display data (numerical vs. categorical). The teacher provides examples of when to use pictographs and bar graphs, and when to use line plots.
  - For example, students measure the lengths of pencils to the nearest $\frac{1}{2}$ inch. Because the students are finding a numerical measurement, this data would be graphed on a line plot.
- Instruction includes how to decide which way to display their data (numerical vs. categorical). The teacher provides examples of when to use pictographs and bar graphs, and when to use a line plot. Also, the teacher provides instruction regarding how numerical data refers to data that is in the form of numbers and categorical data is a type of data that is divided into groups.
  - For example, categorical data could be favorite colors, types of pets at home, or hair color. Types of numerical data could be ages of students, numbers of siblings at home, or the results of the measurement of objects.
- Instruction includes opportunities to count the correct intervals on a scaled bar graph. The teacher provides instruction for identifying the scale and showing students how to read the bars according to the scale.

Instructional Tasks

Instructional Task 1

The data below shows the ages of students in an art class and their favorite colors.

Name	Age	Favorite Color
Addison	10	Blue
Brett	9	Red
Carson	10	Yellow
Dewayne	9	Blue
Elliott	8	Blue
Frankie	9	Green
Glenn	11	Yellow
Horace	9	Blue
Isaiah	10	Red
Jorge	8	Red

Part A. Represent the ages of the students in the art class using a line plot.

Part B. Represent the favorite colors of the students in an art class using a scaled pictograph.

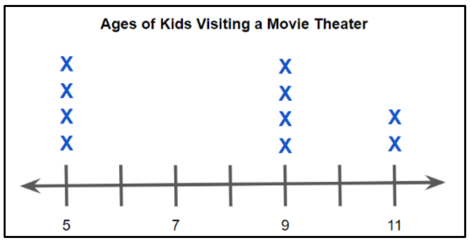
Instructional Items

Instructional Item 1

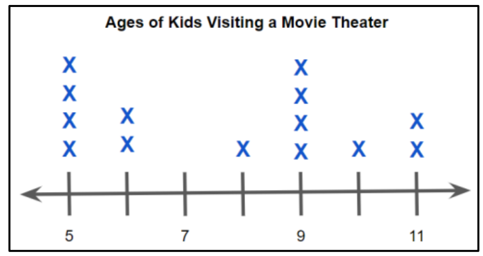
Rebecca surveyed the ages of kids visiting a movie theater and displayed the data using a line plot. The customers' ages are below. Which line plot correctly displays the data that Rebecca collected?

Ages of Kids Visiting a Movie Theater
5, 11, 9, 5, 6, 5, 9, 9, 8, 10, 6, 11, 9, 5

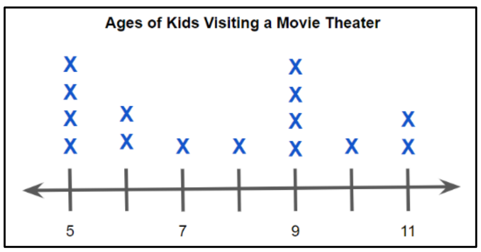
a.



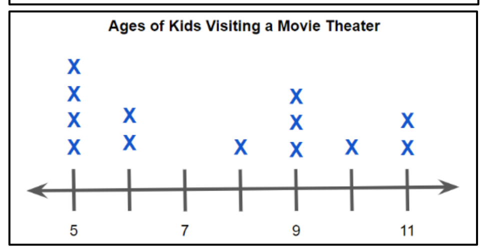
b.



c.



d.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.3.DP.1.2

Benchmark

MA.3.DP.1.2 Interpret data with whole-number values represented with tables, scaled pictographs, circle graphs, scaled bar graphs or line plots by solving one- and two-step problems.

Benchmark Clarifications:

Clarification 1: Problems include the use of data in informal comparisons between two data sets in the same units.

Clarification 2: Data displays can be represented both horizontally and vertically.

Clarification 3: Circle graphs are limited to showing the total values in each category.

Connecting Benchmarks/Horizontal Alignment

- MA.3.NSO.2.2/2.4

Terms from the K-12 Glossary

- Bar Graph
- Categorical Data
- Circle Graph
- Whole Number

Vertical Alignment

Previous Benchmarks

- MA.2.DP.1.2

Next Benchmarks

- MA.4.DP.1.3

Purpose and Instructional Strategies

The purpose of this benchmark is for students to interpret data displayed in scaled pictographs, circle graphs, scaled bar graphs and line plots. Like MA.3.DP.1.1, the purpose of this benchmark builds on data interpretation skills from Grades 1 and 2. In Grade 1, students interpreted data represented with tally marks and pictographs, and in Grade 2, students also interpreted data represented in pictographs and bar graphs. Additionally, students solved addition and subtraction problems using the data representations.

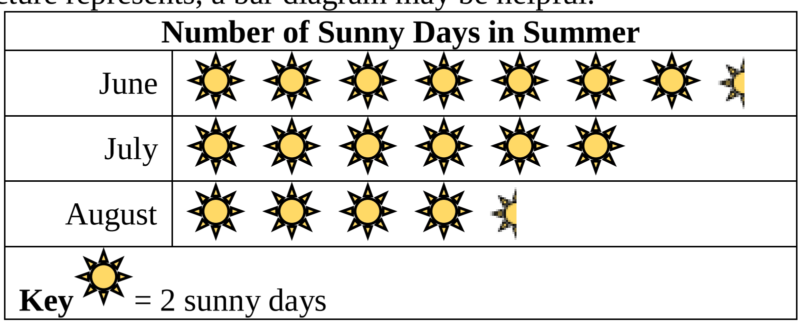
- In Grade 3, students will interpret categorical data represented in scaled pictographs and bar graphs, whole-number numerical data represented in line plots, and whole-number category totals in circle graphs (e.g., instead of percentages). To interpret the represented data, they will solve one- and two-step problems from a given data set or compare two data sets in the same units *(MTR.5.1)*.
- Instruction should include opportunities for students to interpret their own numerical and categorical data *(MTR.7.1)*.
- Students could use addition, subtraction, multiplication or division to solve the problems. This benchmark should be taught with MA.3.DP.1.1 (collecting and representing data) *(MTR.2.1*, *MTR.4.1*, *MTR.5.1)*.

Common Misconceptions or Errors

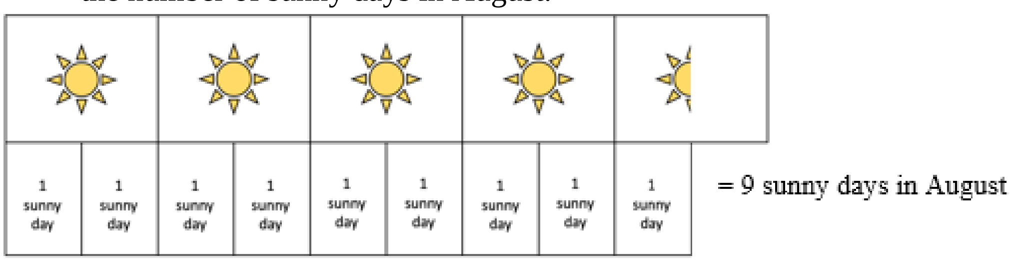
- Students may confuse the values in scaled pictographs and bar graphs. They should always utilize the given key when determining frequency of each category.

Strategies to Support Tiered Instruction

- Instruction includes opportunities to determine the values in a scaled pictograph, pointing out the importance of paying close attention to the key of the pictograph. The key outlines how much each of the pictures on the graph will represent. Students connect multiplication strategies to this concept. Instruction includes opportunities to practice counting by 2s, 5s, and 10s, to be successful with this benchmark. To help students see the connection between the key and what each picture represents, a bar diagram may be helpful.
    - Instruction includes opportunities to determine the values in a scaled pictograph, pointing out the importance of paying close attention to the key of the pictograph. The key outlines how much each of the pictures on the graph will represent. Students connect multiplication strategies to this concept. Instruction includes opportunities to practice counting by 2s, 5s, and 10s, to be successful with this benchmark. To help students see the connection between the key and what each picture represents, a bar diagram may be helpful.



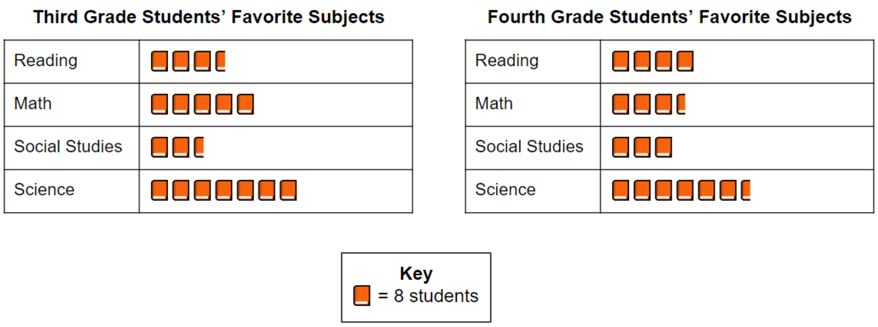
    - For example, students use the key for the pictograph and a bar model to determine the number of sunny days in August.



Instructional Tasks

Instructional Task 1

The pictographs show favorite subjects in third and fourth grades at Palm Elementary School.



Part A. Write an equation that shows how many fourth graders chose reading as their favorite subject.

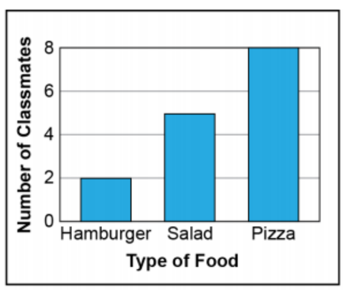
Part B. How many third graders chose social studies as their favorite subject?

Part C. How many more students prefer math in third grade than fourth grade?

Instructional Items

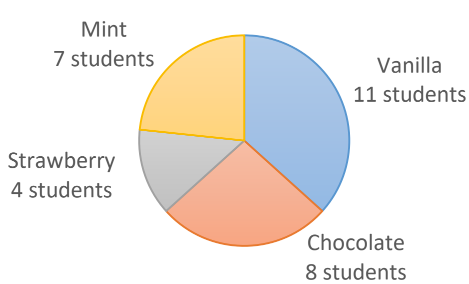
Instructional Item 1

John surveys his classmates about their favorite foods, as shown in the bar graph. How many more classmates chose pizza as their favorite food than classmates who chose salad?



Instructional Item 2

Molly surveys her class about their favorite ice cream flavors, as shown in the circle graph. How many students picked a favorite ice cream flavor other than vanilla?



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*