Algebra 2 B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following:

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

Benchmark
focal point for instruction within lesson or task

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment
in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment
across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

Strategies to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Standards for Mathematics.

Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Mathematical Thinking and Reasoning Standards
MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following:

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:
Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR. 3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students’ ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. • Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* • Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. • Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance.	• Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. • Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. • Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. • Teacher provides appropriate time for student processing, productive struggle and reflection. • Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. • Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Students represent problems concretely using objects, models and manipulatives. • Students represent problems pictorially using drawings, models, tables and graphs. • Students represent problems abstractly using numerical or algebraic expressions and equations. • Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)*	• Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* • Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. • Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* • Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* • Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology.
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Students complete tasks with flexibility, efficiency and accuracy. • Students use feedback from peers and teachers to reflect on and revise methods used. • Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)*	• Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* • Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. • Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. • Teacher offers multiple opportunities to practice generalizable methods.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Students use content specific language to communicate and justify mathematical ideas and chosen methods. • Students use discussions and reflections to recognize errors and revise their thinking. • Students use discussions to analyze the mathematical thinking of others. • Students identify errors within their own work and then determine possible reasons and potential corrections. • When working in small groups, students recognize errors of their peers and offers suggestions.	• Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* • Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. • Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. • Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Students identify relevant details in a problem in order to create plans and decompose problems into manageable parts. • Students find similarities and common structures, or patterns, between problems in order to solve related and more complex problems using prior knowledge.	• Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. (MTR.1.1) • Teacher provides students opportunities to connect prior and current understanding to new concepts. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* • Teacher allows students to develop an appropriate sequence of steps in solving problems. • Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process.
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. • Students monitor calculations, procedures and intermediate results during the process of solving problems. • Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* • Students reflect on the accuracy of their estimations and their solutions.	• Teacher provides opportunities for students to estimate or predict solutions prior to solving. • Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* • Teacher prompts students to self-monitor by continually asking, "Does this solution or intermediate result make sense? How do you know?" • Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Students connect mathematical concepts to everyday experiences. • Students use mathematical models and methods to understand, represent and solve real-world problems. • Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Students re-design models and methods to improve accuracy or efficiency.	• Teacher provides real-world context to help students build understanding of abstract mathematical ideas. • Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. • Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Teacher provides opportunities for students to apply concepts to other content areas.

Algebra 2 Areas of Emphasis

In Algebra 2, instructional time will emphasize six areas:

(1) developing understanding of the complex number system, including complex numbers as roots of polynomial equations;
(2) extending arithmetic operations with algebraic expressions to include polynomial division, radical and rational expressions;
(3) graphing and analyzing functions including polynomials, absolute value, radical, rational, exponential and logarithmic;
(4) extending systems of equations and inequalities to include non-linear expressions;
(5) building functions using compositions, inverses and transformations; and
(6) developing understanding of probability concepts.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following:

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of the major mathematical topics to all stakeholders.

Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

Number Sense and Operations	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions Inverses and Transformations	Developing Under-standing of Probability Concepts
MA.912.NSO.1.3		x	x			
MA.912.NSO.1.5		x	x			
MA.912.NSO.1.6			x		x	x
MA.912.NSO.1.7			x		x	x

Number Sense and Operations	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions Inverses and Transformations	Developing Under-standing of Probability Concepts
MA.912.NSO.2.1	x	x	x			

Algebraic Reasoning	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions, Inverses and Transformations	Developing Under-standing of Probability Concepts
MA.912.AR.1.1			x		x	
MA.912.AR.1.3		x	x		x	
MA.912.AR.1.5		x	x		x	
MA.912.AR.1.6		x	x		x	
MA.912.AR.1.8	x	x	x	x		
MA.912.AR.1.9		x	x		x	
MA.912.AR.3.2	x	x	x	x	x	
MA.912.AR.3.3		x	x	x	x	
MA.912.AR.3.4		x	x	x	x	
MA.912.AR.3.8		x	x	x	x	
MA.912.AR.3.9		x	x	x	x	
MA.912.AR.3.10		x	x	x	x	
MA.912.AR.4.2			x		x	
MA.912.AR.4.4			x		x	
MA.912.AR.5.2			x		x	
MA.912.AR.5.4			x		x	
MA.912.AR.5.5			x		x	
MA.912.AR.5.7			x		x	
MA.912.AR.5.8			x		x	
MA.912.AR.5.9			x		x	
MA.912.AR.6.1	x					
MA.912.AR.6.5	x	x	x			
MA.912.AR.7.1		x				

MA.912.AR.7.2		x	x		x	
MA.912.AR.7.3		x	x		x	
MA.912.AR.8.1		x	x		x	
MA.912.AR.8.2		x	x		x	
MA.912.AR.8.3		x	x		x	
MA.912.AR.9.2			x	x	x	
MA.912.AR.9.3			x	x	x	
MA.912.AR.9.5			x	x	x	
MA.912.AR.9.7			x	x	x	

Functions	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions, Inverses and Transformations	Developing Under-standing of Probability Concepts
MA.912.F.1.1			x		x	
MA.912.F.1.7			x		x	
MA.912.F.1.9			x			
MA.912.F.2.2			x		x	
MA.912.F.2.3			x		x	
MA.912.F.2.5			x		x	
MA.912.F.3.2			x		x	
MA.912.F.3.4			x		x	
MA.912.F.3.6			x		x	
MA.912.F.3.7			x		x	

Financial Literacy	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions, Inverses and Transformations	Developing Understanding of Probability Concepts

MA.912.FL.3.1			x			
MA.912.FL.3.2			x			
MA.912.FL.3.4			x			

Data Analysis & Probability	Complex Number System	Polynomial Division, Radical and Rational Expressions	Graphing and Analyzing Functions	Linear and Non-Linear Systems of Equations and Inequalities	Building Functions Using Compositions, Inverses and Transformations	Developing Understanding of Probability Concepts
MA.912.DP.2.8						x
MA.912.DP.2.9						x

Number Sense and Operations

MA.912.NSO.1 *Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.*

MA.912.NSO.1.3

Benchmark

MA.912.NSO.1.3 Generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents.

Benchmark Clarifications:
Clarification 1: Within the Algebra 2 course, radicands are limited to monomial algebraic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.7.1, MA.912.AR.7.3

Terms from the K-12 Glossary

- Exponent (Exponential Form)
- Principal Square Roots

Vertical Alignment

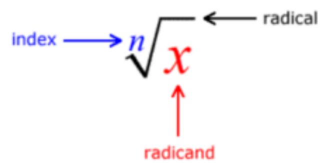
Previous Benchmarks
- MA.912.NSO.1.1, MA.912.NSO.1.2 (Algebra 1, Algebra 1 Honors)
- Depending on a student's mathematics pathways, they may have other previous benchmarks that could apply (see Instructional Strategies).

Next Benchmarks
- MA.912.AR.7.4 (Precalculus Honors)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students extended their understanding of the properties of exponents to include rational exponents. They evaluated numerical expressions and generated equivalent numerical expressions involving rational exponents. In Algebra 1, students also generated equivalent algebraic expressions using the properties of exponents. In Algebra 2, students generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents. Additionally, students solve one-variable radical equations and graph square and cube root functions. In a later course, students will solve and graph mathematical and real-world problems that are modeled with radical functions.

- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Other vocabulary terms may include square root, cube root, nth root, index, radicand, radical form and rational exponent. For a square root, the index is 2 but is not explicitly written.



- Instruction includes naming roots beyond cube root as fourth root, fifth root, etc.
- Explain to students that squaring a positive number and finding the positive square root of a positive number are inverse operations. To extend this thinking to higher powers, there are two cases. The inverse of raising a number to an odd power, n, is finding the nth root. For even powers, we restrict our attention to positive numbers. The inverse of raising a positive number to an even power, n, is finding the positive nth root.
- Instruction includes student discovery of connections between properties of mathematical operations and the inverse relationship between powers and radicals. *(MTR.5.1)*
- Instruction includes creating models of several radicals showing the relationship between the radical expression and the power expression. Because $a^{\frac{1}{2}}$ is a number whose square is a, $\sqrt{a} = a^{\frac{1}{2}}$. Similarly, $\sqrt[3]{a} = a^{\frac{1}{3}}$. In general, $\sqrt[n]{a} = a^{\frac{1}{n}}$ for any integer n greater than 1. *(MTR.2.1)*
    - Instruction includes the algebraic proof behind this pattern. Have students explore this concept further using the cube root and the nth root.

$$\sqrt{a} = a^x$$
$$\left(\sqrt{a}\right)^2 = (a^x)^2$$
$$a^1 = a^{2x}$$
$$1 = 2x$$
$$\frac{1}{2} = x$$

- Because a variable can be positive, negative or zero, sometimes absolute value is needed when simplifying roots with variables. When taking an even root of an even power and the result is an odd power, the absolute value of the result must be used to ensure that the answer is nonnegative.
    - For example, for an odd index: $\sqrt[n]{x^n} = x$ or $\sqrt[3]{5^3} = 5$ and $\sqrt[3]{(-5)^3} = -5$; for an even index: $\sqrt[n]{x^n} = |x|$ or $\sqrt[4]{5^4} = 5$ and $\sqrt[4]{(-5)^4} = 5$.
- Instruction includes decimals as exponents.
    - For example, $x^{3.2} = x^{3\frac{2}{10}} = x^{3\frac{1}{5}} = x^{\frac{16}{5}}$.
- There is often more than one approach to simplifying radical expressions. Spend time exploring the process with students and solicit their solutions and not just their answers. *(MTR.4.1)*

Common Misconceptions or Errors

- For variable expressions with exponents inside radicals with index n, students may try to take the nth root of the exponent instead of dividing it by n.
    - For example, the numerical expression $\sqrt[3]{3^5}$ can be correctly rewritten as $3^{\frac{5}{3}}$ but not as $3^{\sqrt[3]{5}}$.
- Students may incorrectly simplify radicals with a negative radicand depending on whether the index is even or odd.
- Students may assume x represents a positive number and $-x$ must represent a negative number. To address this misconception, read $-x$ as "the opposite of x" to help students understand that if $-x$ is 8 then $x = -8$.
- Students may neglect to insert the absolute value when taking the even root of an even power and the result is an odd power.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.5.1)

Find another equivalent expression for $\sqrt{48x^5y^9}$.

Part A. Compare your expression to that of a neighbor.
Part B. Which expression do you prefer? Explain.
Part C. Convert the original expression to exponential form. Use the laws of exponents to convert this to your preferred alternate expression in exponential form.

Instructional Items

Instructional Item 1

Write the expression $(7x)^{\frac{4}{3}}$ in radical form.

Instructional Item 2

Write the expression $(n^3)^{\frac{3}{2}}$ in radical form.

Instructional Item 3

Write the expression $\sqrt{13ab^5c^4}$ in exponential form.

Instructional Item 4

Write the expression $\sqrt[5]{90xy^2}$ in exponential form.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.5

Benchmark

MA.912.NSO.1.5 Add, subtract, multiply and divide algebraic expressions involving radicals.

Benchmark Clarifications:
Clarification 1: Within the Algebra 2 course, radicands are limited to monomial algebraic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.2.1
- MA.912.AR.7.1, MA.912.AR.7.3

Terms from the K-12 Glossary

- Monomial

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.4 (Algebra 1)

Next Benchmarks

- MA.912.AR.7.4 (Precalculus Honors)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students applied their understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals. In Algebra 2, students perform arithmetic operations with algebraic expressions involving radicals.

- Other vocabulary terms may include like radical expressions and irrational conjugates.
  - Like radical expressions have the same index and same radicand.
  - The term "irrational conjugates" refers to a pair of expressions in the form of $a + b\sqrt{c}$ and $a - b\sqrt{c}$, where a, b and c are rational and $\sqrt{c}$ is irrational.
- Instruction includes the connection between like radicals and like terms.
- Students should understand that sums and differences of like radicals may be combined using the distributive law.
  - For example, $6\sqrt[3]{10} - 17\sqrt[3]{10}$ may be rewritten as $-11\sqrt[3]{10}$.
- Products and quotients of radicals with the same index, may be condensed into one radical.
  - For example, $\frac{\sqrt[3]{10}}{\sqrt[3]{11}}$ may be rewritten as $\sqrt[3]{\frac{10}{11}}$.
  - Another example would be $\sqrt[4]{2} \cdot \sqrt[2]{2} = \sqrt[4]{2} \cdot \sqrt[4]{4} = \sqrt[4]{8}$.
- When the numerator is a monomial with a radical, students can remove the radical from the denominator by multiplying both the numerator and the denominator by another monomial that contains an appropriate radical.

  - For example, if the denominator of an expression is $\sqrt[3]{xy^2}$, the student could multiply both the numerator and the denominator by $\sqrt[3]{x^2y}$ to produce the expression xy in the denominator.
- When dividing by a binomial of the form $a + b\sqrt{c}$, where a, b and c are rational and $\sqrt{c}$ is irrational, students can "rationalize the denominator" by multiplying both the numerator and the denominator by the conjugate $a - b\sqrt{c}$. Explain that the reason they may multiply both the numerator and the denominator by the conjugate is because that is equivalent to multiplying by 1 and will therefore not change the value of the original expression.
  - For example, $3 + 4\sqrt{2}$ and $3 - 4\sqrt{2}$ are irrational conjugates.
  - Instruction includes the idea that the "difference of two squares" is part of this strategy of eliminating the radical in the denominator.
  - Students should understand that the purpose behind rationalizing the denominator is for ease in estimating the quotient when the divisor is a rational number.

Common Misconceptions or Errors

- When students are asked to multiply powers with the same base, they may try to also multiply the base when adding the powers.
  - For example, $3^2 \cdot 3^3 = 3^5$, not 9^5. This may be illustrated by replacing the 3 with x, resulting in $x^2 \cdot x^3 = x^5$, not $(x^2)^5$.
- When rationalizing a denominator that is of the form $a + b\sqrt{c}$, some students may assume that they should multiply the numerator and the denominator by the denominator. To help address this misconception, remind students to multiply the numerator and denominator by the conjugate of the denominator.
- When rationalizing an irrational monomial denominator, students may not multiply by a radical that will produce a perfect root.
- Students may add and subtract radicals that are unlike.
- Students may not realize they need to simplify radical expressions to produce like radical expressions.
- When writing equivalent expressions containing variables, students may ignore the possibility that one of the two expressions may contain a zero in the denominator and the other does not, or one of the two expressions may contain a negative number inside a radical with an even index and the other does not.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Part A. Determine which of the following expressions are like radicals.

$\left(\sqrt[4]{m}\right)^3 \qquad \sqrt[3]{m^4} \qquad 3\sqrt[4]{m} \qquad 7\left(\sqrt[8]{m}\right)^6 \qquad -\frac{(\sqrt{m})^5}{\sqrt[4]{m^7}} \qquad \frac{1}{\sqrt{m}}$

Part B. Add the like radical expressions from Part A.
Part C. Subtract any two like radical expressions from Part A.
Part D. Multiply the like radical expressions from Part A.
Part E. Divide any two like radical expressions from Part A.

Instructional Task 2 (MTR.1.1, MTR.3.1)
Rewrite the following expression in a way that you feel is as simple as possible. Then, compare your expression with a partner's expression.
$\sqrt[6]{x^7} \div \sqrt[6]{x^8y^{18}}$

Instructional Task 3 (MTR.1.1, MTR.3.1)

Part A. Rewrite the following expression in a way that you feel is as simple as possible. Then, compare your expression with a partner's expression. Assume $x \neq 0$ and $y \neq 0$.
$\sqrt[4]{\frac{324x^3y^3}{4x^{-1}y}}$

Part B. Explain why the assumption of $x \neq 0$ and $y \neq 0$ is necessary.
Is there a need to assume that $x > 0$ or $y > 0$? Why or why not?

Instructional Task 4 (MTR.1.1, MTR.3.1)

Part A. Rewrite the following expression in a way that you feel is as simple as possible. Then, compare your expression with a partner's expression. Assume $ab \neq \frac{1}{2}$, $a \geq 0$ and $b \geq 0$.
$\sqrt{a} \div (1 + \sqrt{2ab})$

Part B. Explain why the assumption of $ab \neq \frac{1}{2}$ is necessary.

Instructional Items

Instructional Item 1
Perform the indicated operation. Assume all variables are positive.
$\left(n^{\frac{1}{2}} \cdot n^{\frac{1}{4}}\right) - \sqrt[4]{16n^3}$

Instructional Item 2
Determine the difference of the two expressions. Write your answer as an exact quantity using only a single radical.
$6\sqrt{2z} - 3\sqrt{50z^3}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.6

Benchmark

MA.912.NSO.1.6 Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents.

Benchmark Clarifications:
Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.3.6, MA.912.F.3.7
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Base (of an exponent)
- Exponent (Exponential Form)
- Inverse Functions

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.1 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4, MA.912.C.2.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students began their work with exponential functions. In Algebra 2, students extend their knowledge to the inverse of exponential functions, resulting in logarithms, as well as their understanding of the properties of exponents to include the properties of logarithms as applied to numerical logarithmic expressions.

- Other vocabulary terms may include common logarithm, natural logarithm and change of base formula.
  - Common logarithm is $\log_{10} a$, which is often denoted as $\log a$.
  - Natural logarithm is $\log_e a$, which is often denoted as $\ln a$.
  - Change of base formula is $\log_a b = \frac{\log_c b}{\log_c a}$, where $a > 0$, $b > 0$ and $c > 0$. Special cases include $\log_a b = \frac{\log b}{\log a}$ and $\log_a b = \frac{\ln b}{\ln a}$.
- Instruction includes making the connection between the properties of logarithms and the properties of exponents. Explain that the logarithm base b of y is defined as the exponent x establishing the equivalence of $y = a^x$ given $a > 0$, $y > 0$ and $a \neq 1$. *(MTR.5.1)* Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the “opposite” of the properties of the exponents. This graphic organizer may be used to compare the properties of logarithms to the properties

of exponents.

Properties of Logarithms		Properties of Exponents	
$\log_a b + \log_a c = \log_a bc$	→	$a^b \cdot a^c = a^{b+c}$	Product Property
$\log_a b - \log_a c = \log_a \frac{b}{c}$	→	$\frac{a^b}{a^c} = a^{b-c}$	Quotient Property
$\log_a b^c = c \log_a b$	→	$(a^b)^c = a^{bc}$	Power Property

- Students should understand how to use their one-to-one and inverse properties to evaluate expressions by making decisions on which base is best to use for solving.
  - One-to-One Properties

$a^x = a^y$	$x = y$
$log_a x = log_a y$	$x = y$

  - Inverse Properties

Logarithmic Form	Exponential Form
$log_a 1 = 0$	$a^0 = 1$
$log_a a = 1$	$a^1 = a$
$log_a a^x = x$	$a^{log_a x} = x$

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the Power Property to rewrite the equation as $(\log_b x)(\log_a b) = \log_a x$. Now, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$, and therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties. *(MTR.4.1)*
  - For example, present students with $\log_{1.03} 1.092727$ and ask them for its meaning, "What exponent should be applied to the base 1.03 to achieve the result of 1.092727?"
- Many examples in this course will require students to utilize the change of base formula to evaluate logarithms. Consider the example above.

$$\log_{1.03} 1.092727 = \frac{\log 1.092727}{\log 1.03} \text{ or } \frac{\ln 1.092727}{\ln 1.03}$$

  - Evaluating this expression with a calculator requires the use of common logs or natural logs. Guide students to understand that common logs have a base of 10 while natural logs have a base of e and are easy to enter into a calculator. Have students use a calculator to evaluate the expression to find it is approximately equivalent to 3. Have students confirm this by checking the exponential equivalent expression $1.03^3 = 1.092727$.
  - While students will see more complicated logarithms in this course, it is

appropriate to simplify the logarithms they work with initially in this benchmark until they reach an understanding of the properties of logarithms. Depending on the courses students have taken prior to Algebra 2, this may be their first introduction to logarithms.

- Instruction includes connections to solving one-variable equations involving logarithms or exponential expressions, as well as solving and graphing mathematical and real-world problems that are modeled with exponential and logarithmic functions.
- Students should have practice combining the properties of logarithms to generate equivalent numerical expressions by either simplifying (condensing) or expanding the expression.
- Problem types include logarithms with different bases, including common logarithms and natural logarithms.
- Instruction includes logarithms involving radicals such as $\log\sqrt{2}$.
- Instruction includes that just like with roots, we can simplify or expand logarithms if the argument is factored.

Common Misconceptions or Errors

- Students may see "log" as a variable rather than as an operation.
  - For example, they may see "log" as a common factor in the expression $\log 12 + \log 6$ and mistakenly write $\log(12 + 6)$.
  - For example, they may rewrite the expression $\log(12 \cdot 6)$ and as $\log 12 \cdot \log 6$ instead of $\log 12 + \log 6$. Similarly, they may incorrectly rewrite $\log\left(\frac{12}{6}\right)$ as $\frac{\log 12}{\log 6}$ instead of $\log 12 - \log 6$.
  - For example, they may cancel the "log" from the numerator and the denominator in an expression.
- Students may incorrectly use the change of base formula.
- Students may mistakenly evaluate $\log_a 1$ as 1 instead of 0.
- When expanding a logarithmic expression, students may forget that the power on the argument of the logarithm becomes the coefficient.
  - For example, $\log_5 16^2 = 2\log_5 16$ or $\log_2 16^3 = 3\log_2 16 = 3 \cdot 4 = 12$.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.5.1)

Rewrite $\log 16$ using the following properties.

Part A. Using the Product Property of Logarithms

Part B. Using the Power Property of Logarithms

Instructional Task 2 (MTR.3.1)
Recall that $\log_b(x)$ is, by definition, the exponent by which b must be raised to in order to yield x .
Part A:

- Use this definition to compute $\log_2(2^5)$.
- Use this definition to compute $\log_{10}(0.001)$.
- Use this definition to compute $\ln(e^3)$.
- Explain why $\log_b(b^y) = y$ where $b > 0$.

Part B:

- Evaluate $10^{\log_{10}(100)}$.
- Evaluate $2^{\log_2(\sqrt{2})}$.
- Evaluate $e^{\ln(89)}$.
- Explain why $b^{\log_b x} = x$ where $b > 0$.

Instructional Items

Instructional Item 1
Evaluate the logarithmic expressions using the properties of logarithms.

a. $\log_4 64^{-5}$
b. $\log_3 81^2$
c. $\ln e^3 + \ln e^4$
d. $3\ln e^7 - \ln e^2$
e. $\log_5 150 - \log_5 6$

Instructional Item 2
Match the expression with the logarithm that has the same value.

1. $\log_4 6 - \log_4 2$	A. $\log_4 64$
2. $2\log_4 6$	B. $\log_4 3$
3. $6\log_4 2$	C. $\log_4 12$
4. $\log_4 6 + \log_4 2$	D. $\log_4 36$

Instructional Item 3
Which of the following expressions are equivalent to $\log_4 16$? Select all that apply.

☐ $\frac{\log 4}{\log 16}$
☐ $\frac{\log 16}{\ln 4}$
☐ $\frac{\ln 4}{\ln 16}$
☐ $\frac{\ln 16}{\ln 4}$
☐ $\frac{\ln 4}{\log 16}$
☐ $\frac{\log 16}{\log 4}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.7

Benchmark

MA.912.NSO.1.7 Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.

Benchmark Clarifications:
Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, 5.7, 5.8, 5.9
- MA.912.F.3.6, 3.7
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Base
- Exponent (Exponential Form)
- Expression
- Inverse Functions

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.2 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4, 2.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students began their work with exponential functions. In Algebra 2, students extend their knowledge to the inverse of exponential functions, resulting in logarithms, as well as their understanding of the properties of exponents to include the properties of logarithms as applied to algebraic logarithmic expressions.

- Other vocabulary terms may include common logarithm, natural logarithm and change of base formula.
    - Common logarithm is $\log_{10} a$, which is often denoted as $\log a$.
    - Natural logarithm is $\log_e a$, which is often denoted as $\ln a$.
    - Change of base formula is $\log_a b = \frac{\log_c b}{\log_c a}$, where $a > 0$, $b > 0$ and $c > 0$. Special cases include $\log_a b = \frac{\log b}{\log a}$ and $\log_a b = \frac{\ln b}{\ln a}$.
- Instruction includes making the connection between the properties of logarithms and the properties of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of $y = a^x$ and, $x = log_a y$ given $a > 0$ and $a \neq 1$. *(MTR.5.1)* Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the "opposite" of the properties of the exponents. This

graphic organizer may be used to compare the properties of logarithms to the properties of exponents.

Properties of Logarithms		Properties of Exponents	
$\log_a b + \log_a c = \log_a bc$	→	$a^b \cdot a^c = a^{b+c}$	Product Property
$\log_a b - \log_a c = \log_a \frac{b}{c}$	→	$\frac{a^b}{a^c} = a^{b-c}$	Quotient Property
$\log_a b^c = c \log_a b$	→	$(a^b)^c = a^{bc}$	Power Property

- Students should understand how to use their one-to-one and inverse properties to evaluate expressions by making decisions on which base is best to use for solving.
  - One-to-One Properties

$a^x = a^y$	$x = y$
$log_a x = log_a y$	$x = y$

  - Inverse Properties

Logarithmic Form	Exponential Form
$log_a 1 = 0$	$a^0 = 1$
$log_a a = 1$	$a^1 = a$
$log_a a^x = x$	$a^{log_a x} = x$

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the Power Property to rewrite the equation as $(\log_b x)(\log_a b) = \log_a x$. Students should notice that there the arguments for two of the logs are b and x with each the same base of a. So, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$. Therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.
- Many examples in this course will require students to utilize the change of base formula to evaluate logarithms. Consider the example shown.

$$\log_5 2x = \frac{\log 2x}{\log 5} \text{ or } \frac{\ln 2x}{\ln 5}$$

- Instruction includes connections to solving one-variable equations involving logarithms or exponential expressions, as well as solving and graphing mathematical and real-world problems that are modeled with exponential and logarithmic functions.
- Students should have practice combining the properties of logarithms to generate equivalent algebraic expressions by either simplifying (condensing) or expanding the expression.
- Problem types include logarithms with different bases, including common logarithms and natural logarithms.

Common Misconceptions or Errors

- Students tend to treat "log" as a variable rather than as an operation:
    - For example, they may see "log" as a common factor in the expression $\log x + \log y$ and mistakenly write $\log(x + y)$.
    - For example, they may distribute the "log" in the expression $\log(a \cdot b)$ and rewrite it as $\log a \cdot \log b$ instead of $\log_a m + \log_a n$. Similarly, they may incorrectly rewrite $\log\left(\frac{a}{b}\right)$ as $\frac{\log a}{\log b}$.
    - For example, they may divide both sides of the equation $\log(7x - 12) = 2\log x$ by "log" to mistakenly obtain $7x - 12 = 2x$.
- Students may cancel the "log" from the numerator and the denominator in an expression. Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.
- When expanding a logarithmic expression, students may forget the power on the logarithm becomes the coefficient. They may also forget $\log_a a = 1$.
    - For example, $\log 10\, x^4 = \log_{10} 10\, x^4 = \log_{10} 10 + \log_{10} x^4 = 1 + 4\log x$.
- Students may struggle to see the general structure and relationship between dividing/subtracting and multiplying/adding.
    - For example, when expanding the logarithm of a quotient, students may write $\ln \frac{3x^4}{y} = \frac{\ln 3x^4}{\ln y}$. To address this misconception, remind students: $\ln \frac{a}{b} = \ln a - \ln b$.
    - For example, when expanding the logarithm of a product, remind students: $\log_3 2x = \log_3 2 + \log_3 x$, not $\log_3 2 \cdot \log_3 x$.
- Students may incorrectly use the Change of Base Formula.
- Students may mistakenly evaluate $\log_a 1$ as 1 instead of 0.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Part A. Write three expressions equivalent to $\log_4 \frac{x^3}{5y}$. Compare your list with a partner.

Part B. Work with your partner to find an equivalent expression that is not on either list.

Instructional Task 2 (MTR.4.1, MTR.5.1)

Use the functions below to answer the following questions.

$$f(x) = 10^{0.2x}$$
$$h(x) = 5(\log x)$$

Part A. Use technology to graph the functions f and h on the same coordinate plane. What do you notice?

Part B. Determine $f(2)$ and $h(f(2))$. What do you notice?

Part C. Determine $h(2)$ and $f(h(2))$. What do you notice?

Part D. Discuss with a partner why logarithmic functions are said to be inverses of exponential functions.

Instructional Items

Instructional Item 1

Expand the following logarithmic expressions, using the properties of logarithms.

a. $\log_4 5x$
b. $\ln 6x^3$
c. $\log_3 6\sqrt{x}$

Instructional Item 2

Condense the following logarithmic expressions as a single logarithmic quantity, using the properties of logarithms.

a. $5\ln x + 8\ln y$
b. $\log_5 3 + \frac{1}{3}\log_5 x$
c. $\log_5 3 + \frac{1}{3}\log_5 x$
d. $3\ln x + 4\ln y - 5\ln z$
e. $\frac{3}{2}\log_2 x^6 - \frac{3}{4}\log_2 x^8$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.2 *Represent and perform operations with expressions within the complex number system.*

MA.912.NSO.2.1

Benchmark

MA.912.NSO.2.1	Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.5
- MA.912.AR.1.8
- MA.912.AR.3.2
- MA.912.AR.6.1

Terms from the K-12 Glossary

- Real Numbers

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.3 (Algebra 1)

Next Benchmarks

- MA.912.NSO.2.2, 2.3, 2.4, 2.5, 2.6 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students perform operations on polynomials whose variables are real numbers. In Algebra 2, students extend this work to polynomials whose variables are complex numbers. In later courses, students will extend their work with complex numbers by representing them as points in the complex plane.

- The Complex Number System includes real numbers and imaginary numbers. Real numbers and imaginary numbers are the basic building blocks of the set of complex numbers.
- A complex number is any number that can be written in the standard form $a + bi$, where a and b are real numbers. If $b = 0$, the complex number $a + bi$ is a real number. If $a = 0$, the complex number $a + bi$ is an imaginary number. Imaginary and real numbers are both subsets of the set of complex numbers.
- The imaginary unit i is defined as $i = \sqrt{-1}$.
- For the complex number $a + bi$, a is called the real part and b is called the imaginary part.
- Instruction includes exploring and discussing the powers of i and its cyclical nature. *(MTR.5.1)*.
  - $i^2 = -1$
  - $i^3 = i^2 \cdot i = -1 \cdot i = -i$
  - $i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$
  - $i^5 = i^4 \cdot i = 1 \cdot i = i$
- Other vocabulary terms may include complex conjugates.
  - Complex conjugates are two complex numbers where the real parts are the same and the imaginary parts are opposites.
- When adding and subtracting complex numbers, students should understand that the real parts will be added/subtracted, and the imaginary parts will be added/subtracted. A connection can be made to like terms when adding and subtracting complex numbers.
- The division of two complex numbers can be accomplished by multiplying the numerator and denominator by the complex conjugate of the denominator.
  - For example, $\frac{2+3i}{1-i} = \frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(2+3i)(1+i)}{(1-i)(1+i)} = \frac{-1+5i}{2} = -\frac{1}{2} + \frac{5}{2}i$.
- Instruction includes the similarity between this division procedure and the procedure of rationalizing the denominator when dealing with radicals.
- When dividing by a complex number with an imaginary term, teach students to rationalize the denominator by multiplying by the complex conjugate. Explain that the reason they must multiply both the numerator and the denominator by the complex conjugate is because that expression is equivalent to 1 and will therefore not change the value of the original expression.
  - $3 - 2i$ and $3 + 2i$ are complex conjugates.

  - Instruction includes the connection between using the complex conjugate here to "eliminate" the imaginary term in the denominator and using the irrational conjugate in MA.912.NSO.1.5 to eliminate the radical term in the denominator.
- Students should understand that one purpose behind using the complex conjugate in complex division is to show that operations with complex numbers are closed, so the quotient of two complex numbers can also be written as a complex number in the form $a + bi$.
- Instruction prepares students for rewriting polynomial expressions as the product of polynomials over the real or complex number system. Students will also solve quadratic equations with complex solutions.
- Instruction makes the connection to relevant real-world applications. *(MTR.7.1)*

Common Misconceptions or Errors

- When asked to multiply two radicals with negative radicands, students may think they can first multiply the two negative radicands in order to obtain a positive radicand. This rule only applies to nonnegative numbers, so they must simplify each expression before multiplying.
  - For example, $\sqrt{-6} \cdot \sqrt{-24} = i\sqrt{6} \cdot 2i\sqrt{6} = 2i^2\sqrt{36} = 2(-1)(6) = -12$
- Students may neglect to simplify i^2 to -1.
- Students may neglect to subtract both the real and imaginary parts when subtracting two complex numbers.
- Students may not distribute terms correctly when multiplying complex numbers.
- Students may find the additive inverse and not the complex conjugate when dividing complex numbers.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

Complex numbers are closed under addition, subtraction, multiplication and division. Closure means that performing any of these operations on complex numbers will result in a complex number.

Part A. Given the complex numbers $2 + 3i$ and $1 - 4i$, perform all four arithmetic operations and show that each operation results in a complex number by putting each result into the form $a + bi$.

Part B. Given the complex numbers $a + bi$ and $c + di$, show that complex numbers are closed under all four operations.

Instructional Items

Instructional Item 1

Write $\sqrt{-125}$ in standard form, $a + bi$.

Instructional Item 2

Write i^{35} in standard form, $a + bi$.

Instructional Item 3

Write $(14 + 6i) - (22 - 12i)$ in standard form, $a + bi$.

Instructional Item 4
Write $(2+5i)(7-4i)$ in standard form, $a+bi$.

Instructional Item 5
Write the following in standard form, $a+bi$.

Part A. $\frac{8}{5-2i}$

Part B. $\frac{8-3i}{6i}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Algebraic Reasoning

MA.912.AR.1 *Interpret and rewrite algebraic expressions and equations in equivalent forms.*

MA.912.AR.1.1

Benchmark

MA.912.AR.1.1 Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

Algebra I Example: Derrick is using the formula $P = 1000(1 + .1)^t$ to make a prediction about the camel population in Australia. He identifies the growth factor as (1 + .1), or 1.1, and states that the camel population will grow at an annual rate of 10% per year.

Example The expression 1.15^t can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t}$ which is approximately equivalent to 1.012^{12t}. This latter expression reveals the approximate equivalent monthly interest rate of 1.2% if the annual rate is 15%.

Benchmark Clarifications:
Clarification 1: Parts of an expression include factors, terms, constants, coefficients, and variables.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.5.5, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.8.2, MA.912.AR.8.3
- MA.912.FL.3.2

Terms from the K-12 Glossary

- Coefficient
- Equation
- Expression
- Factors

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.2 (Algebra 1)
- MA.912.AR.2.2, MA.912.AR.2.5 MA.912.AR.2.6 (Algebra 1)
- MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.7
- (Algebra 1)
- MA.912.AR.4.1 (Algebra 1)
- MA.912.AR.5.3, MA.912.AR.5.6 (Algebra 1)

Next Benchmarks

- MA.912.NSO.2.4 (Precalculus)
- MA.912.AR.6.6 (Precalculus)
- MA.912.AR.7.4 (Precalculus)
- MA.912.AR.10.5 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students generated and interpreted equivalent linear, absolute value, quadratic and exponential expressions, and equations. In Algebra 2, students extend this work to include radical, rational, and logarithmic equations or expressions.

- Instruction includes making the connection to linear, absolute value, exponential, rational, radical, and logarithmic expressions or equations.
  - Students should be able to identify factors, terms, constants, coefficients, and variables in expressions and equations.
    - Go beyond these popular parts of an expression and equation: the growth/decay factor in exponential functions, rate of change in linear functions, interest, etc.
  - Look for opportunities to interpret these components in context and make these discussions part of daily instruction. This benchmark is not intended to be taught in isolation, rather it is intended to create a consistent conversation point across the instruction of other benchmarks.
  - When solving contextual problems (which comprise a large portion of this course), ask students to interpret the parts in different steps of the solution process.
    - For example, if Jamie is calculating the future worth of an investment that compounds annually. She can use the compound interest formula below.

$$A = 1500(1 + 0.029)^{12}$$

$A = 1500(1 + 0.029)^{12}$	Principal, Initial amount in the growth factor, Amount of growth per time period, Years of growth
$A = 1500(1.029)^{12}$	Principal, Growth Factor (102.9% of principal each year), Years of growth
$A = 1500(1.40923849245)$	Principal, Total growth Factor (140.9% of initial principal)
$A = 2113.86$	Value of investment after 12 years

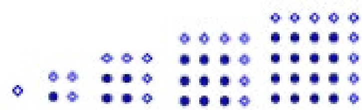
Common Misconceptions or Errors

- Students may not be able to identify parts of an expression and equation or interpret those parts within context. To address this, ensure these are embedded throughout instruction and discussions.
    - For example, building in questions to identify these parts and discussing their connection to the context they represent in a routine way will help students to make these connections.
- Students may not see or consider equivalent expressions that could be created through the properties of exponents or logarithms. Prompt students to think in this direction when appropriate.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

The algebraic expression $(n-1)^2 + (2n-1)$ can be used to calculate the number of symbols in each diagram below. Explain what n likely represents. Check your explanation by calculating the value of the expression for different values of n.



Part A. Which part of any one diagram is represented by $(n-1)^2$?
Part B. Which part of that same diagram is represented by $(2n-1)$?
Part C. How does this relate to the fact that the sum of the first n odd numbers is n^2?

Instructional Task 2 (MTR.3.1, MTR.7.1)

The volume of a prism is $V = BH$.

Part A. For a triangular prism, the formula is $V = \frac{1}{2}bhH$. Which part represents B in the formula for a prism? What do b and h represent?

Part B. For a hexagonal prism, the formula is $V = \frac{1}{2}6saH$. Which part represents B in the formula for a prism? What do a and $6s$ represent?

Part C. For an equilateral triangular prism, the formula is $V = \frac{\sqrt{3}}{4}s^2H$. Which part represents B in the formula for a prism? What does s represent? How does this formula relate to the formula in part A?

Instructional Items

Instructional Item 1

Identify the growth factor in the expression $2(1.75)^x$.

Instructional Item 2

The function $h(t) = -5t^2 + 10t + 7.5$, models the height of a diver above the water (in meters), t seconds after the diver leaves the board.

Part A. Determine the height of the diving board.
Part B. When will the diver enter the water?
Part C. What is the approximate initial velocity when the diver leaves the diving board?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.3

Benchmark

MA.912.AR.1.3 Add, subtract, and multiply polynomial expressions with rational number coefficients.

Benchmark Clarifications:
Clarification 1: Instruction includes an understanding that when any of these operations are performed with polynomials the result is also a polynomial.
Clarification 2: Within the Algebra 1 course, polynomial expressions are limited to 3 or fewer terms.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.3.2

Terms from the K-12 Glossary

- Area Model
- Coefficient
- Distributive Property
- Exponent
- Polynomials

Vertical Alignment

Previous Benchmarks
- MA.912.NSO.1.1 (Algebra 1)
- MA.912.AR.1.7 (Algebra 1)

Next Benchmarks
- MA.912.AR.6.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students were introduced to polynomial expressions, including classifying by degree and number of terms, and performing some operations, for up to three terms. In Algebra 2, students extend their knowledge of polynomial expressions. In this course, the number of terms in a polynomial may increase as students prepare to divide using long division and synthetic division and solve polynomials with higher degrees.
- Instruction includes making the connection to dividing polynomials and the understanding that division does not have closure.
- Reinforce like terms during instruction (using different colors can be a strategy to help identify them as unique from one another).
- Instruction includes the use of manipulatives, such as algebra tiles, and various strategies, including the area model, properties of exponents and the distributive property.*(MTR.2.1)*

- Area Model
  The expression $(2x^2 + 1.5x + 6)(3x + 4.2)$ is equivalent to $6x^3 + 12.9x^2 + 24.3x + 25.2$ and can be modeled below:

	$2x^2$	$1.5x$	6
$3x$	$6x^3$	$4.5x^2$	$18x$
4.2	$8.4x^2$	$6.3x$	25.2

- Multiplying Vertically

$$\begin{array}{r} 2x^2 + 1.5x + 6 \\ \times \quad 3x + 4.2 \\ \hline 8.4x^2 + 6.3x + 25.2 \\ 6x^3 + 4.5x^2 + 18x \qquad\qquad \\ \hline 6x^3 + 12.9x^2 + 24.3x + 25.2 \end{array}$$

- Using the Distributive Property to multiply horizontally
  Instruction should not rely upon the use of tricks or acronyms, like FOIL (as FOIL only relates to two binomials).

Common Misconceptions or Errors

- Students may not understand the meaning of closure or the operations it applies to with polynomials.
- Students may not understand like terms or the properties of exponents.
- Students might think that polynomials are limited to monomials, binomials and trinomials with just one variable. To address this misconception, provide practice with polynomials having multiple variables and more than three terms.
- Students may not correctly apply the distributive property when multiplying polynomials, leaving out one or more terms. To address this misconception, encourage the use of the area model so that it is easier for students to account for all terms.
- Students may incorrectly distribute the exponent when expanding a binomial raised to a power. To address this misconception, remind students that the exponent indicates how many times the base is to be multiplied. Encourage students to rewrite the problem showing the expansion.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Part A. Determine the sum of $-8x^3 + 3x^2 - 2x + 5$ and $\frac{1}{6}x^2 + 7x^3 + \frac{8}{7}$. Explain the method used in determining the sum.
Part B. Discuss whether the addition of polynomials will always result in another polynomial. Why or why not?
Part C. Determine the difference of $3x^2 + 5$ and $x^2 - 0.25x + 1.24$. Explain the method used in determining the difference.
Part D. Discuss whether the subtraction of polynomials will always result in another polynomial. Why or why not?
Part E. Determine the product of $2x^2 + 5x$ and $\frac{2}{9}x^2 - \frac{11}{2}x + 1$. Explain the method used in determining the product.
Part F. Discuss whether the multiplication of polynomials will always result in another polynomial. Why or why not?
Part G. Determine the quotient of $9x^2 - 3x + 12$ and $3x$. Explain the method used in determining the quotient.
Part H. Discuss whether the division of polynomials will always result in another polynomial. Why or why not?

Instructional Items

Instructional Item 1
Simplify $(3x^3 + 6x - 9) - (x^2 - 5x + 7)$.

Instructional Item 2
Find the product: $(m - 7)(2m + 8)(-4m - 1)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.5

Benchmark

MA.912.AR.1.5	Divide polynomial expressions using long division, synthetic division, or algebraic manipulation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.6.1, MA.912.AR.6.2

Terms from the K-12 Glossary

- Algorithm
- Area Model
- Polynomial

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.3, MA.912.AR.1.4 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students learned to divide a polynomial by a monomial. In Algebra 2, students divide polynomial expressions using long division, synthetic division or algebraic manipulation.

- Instruction includes the connection to addition, subtraction and multiplication of polynomials to develop the understanding of closure, and the connection to properties of exponents.
- Instruction includes proper vocabulary and terminology such as dividend, divisor and quotient.

$$\begin{array}{ccccc} \text{dividend} & & \text{divisor} & & \text{quotient} \\ \downarrow & & \downarrow & & \downarrow \\ 6 & \div & 2 & = & 3 \end{array} \quad \text{or} \quad \begin{array}{l} \text{dividend} \rightarrow \\ \\ \text{divisor} \rightarrow \end{array} \frac{6}{2} = 3 \leftarrow \text{quotient}$$

- Instruction includes the connection to long division of whole numbers to develop the understanding that polynomial division follows a very similar procedure, and that when the remainder is zero, the divisor and quotient are factors of the dividend. *(MTR.5.1)*
- Synthetic division can only be used to divide a polynomial by a binomial of degree 1, such as $x - k$. The terms in both the divisor and the dividend must be in descending order.
  - For example, divide $-x^3 + 5x^2 + 6$ by $x - 3$. In this example, the value of k is 3.
    - Write the coefficients of the dividend in order of descending exponents, including a 0 for any missing terms. Write the k-value to the left of the vertical bar.

$$\begin{array}{r|rrrr} 3 & -1 & 5 & 0 & 6 \\ & & & & \\ \hline \end{array}$$

    - Bring down the leading coefficient. Multiply the leading coefficient by the value of k and write the product below the next leading coefficient and add.

$$\begin{array}{r|rrrr} 3 & -1 & 5 & 0 & 6 \\ & & -3 & & \\ \hline & -1 & 2 & & \end{array}$$

    - Multiply the sum (2 in this example) by the value of k and write the product under the next leading coefficient. Repeat this process for the remaining coefficients.

$$\begin{array}{r|rrrr} 3 & -1 & 5 & 0 & 6 \\ & & -3 & 6 & 18 \\ \hline & -1 & 2 & 6 & 24 \end{array}$$

- The last number in the bottom row is the remainder and the others are the coefficients of the quotient. In this example, the quotient is $-1x^2 + 2x + 6 + \frac{24}{x-3}$

- Synthetic division can also be used when dividing a polynomial by a binomial in the form $ax - b$. In this case, first factor out the constant a from the divisor to obtain $a(x - \frac{b}{a})$. Then use synthetic division with $\frac{b}{a}$ as the k value to divide the dividend by $x - \frac{b}{a}$. Then divide the quotient obtained from this synthetic division by the constant a to obtain the quotient of the original problem.

- It may be useful for students to use grid paper to help them organize their work when they divide polynomials. The vertical and horizontal alignment may help reduce the risk of error.
- Instruction includes connections to the Remainder Theorem. *(MA.912.AR.6.2)*
- It is not the expectation of this benchmark that students perform division using a specific method or strategy. Students should be able to compare various methods and use the one that is most efficient for them, depending on the task.
- Instruction includes the use of manipulatives, such as algebra tiles, and various strategies, including the area model and properties of exponents.
- Area Model
  - For example, when dividing $2x^2 + 7x + 5$ by $x + 2$, students can create a table with the same number of rows as the number of terms in the divisor (as shown below).

x		
$+2$		

  - Then, students can write the first term of the dividend in the upper left rectangle. Have students determine what the length must be if the area of that rectangle (shaded yellow) is $2x^2$.

x	$2x^2$	
$+2$		

  - Once students determine that its length is $2x^2 \div x$ which is $2x$, they can write the length of the rectangle.

	$2x$	
x	$2x^2$	
$+2$		

  - Next, students can use this information to determine the area of the upper right rectangle (shaded green). Students should notice that the second term of the dividend is $7x$, which means that when you add the like terms diagonally you need to get $7x$. The area of this rectangle is $7x - 4x = 3x$.

	$2x$	3
x	$2x^2$	$3x$
$+2$	$4x$	

- Lastly, students can calculate the length of the column, and then the area of the bottom right rectangle to determine if there is a remainder. Notice the bottom right corner is not 5, but 6. Therefore, there is a remainder of -1. Can you see why this is -1 instead of $+1$?

	$2x$	3
x	$2x^2$	$3x$
$+2$	$4x$	6

Common Misconceptions or Errors

- Students may forget to insert a placeholder for polynomial divisors or dividends with missing terms, meaning those terms have a coefficient of zero.
- When using long division, students sometimes forget to multiply each monomial in the quotient by every term in the divisor. To address this misconception, remind students to use the distributive property to multiply by each term.
- Students may forget to write the polynomials in standard form.
- Students may forget that synthetic division can only be used when you are dividing by a linear factor.
- When doing long division, students may add instead of subtracting the terms.
- When doing synthetic division, students may forget that the leading coefficient of the divisor should be 1. If the leading coefficient is not 1, the student may not know how to adjust the problem by factoring out the leading coefficient so that synthetic division can still be used.
- When doing synthetic division, students may forget to reverse the sign of the constant in the divisor.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Part A. For the following expressions, determine if synthetic division can be used. Explain your reasoning. Then divide each expression using long division or synthetic division.

A. $(x^3 + 1) \div (x + 1)$
B. $(x^3 - 2x^2 - 9x + 18) \div (x^2 - 6)$
C. $(x^3 - 1) \div (2x - 1)$

Part B: Determine if the divisor is a factor of the dividend for A through C.

Instructional Items

Instructional Item 1

Divide using polynomial division.

$$(x^3 - 2x^2 - 9x + 18) \div (x^2 - 5x + 6)$$

Instructional Item 2
Divide using polynomial division.

$$(2x^4 + 3x^3 + 5x - 1) \div (x^2 - 2x + 2)$$

Instructional Item 3
Divide using polynomial division.

$$(x^3 + 4x^2 + 6x + 4) \div (x + 2)$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.6

Benchmark

MA.912.AR.1.6 Solve mathematical and real-world problems involving addition, subtraction, multiplication, or division of polynomials.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.6.2

Terms from the K-12 Glossary

- Area Model
- Polynomial

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.3, MA.912.AR.1.4 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students performed operations on polynomials limited to 3 or fewer terms. In Algebra 2, students solve mathematical and real-world problems involving operations with polynomials. In later courses, students will explain and apply theorems for polynomials to solve mathematical and real-world problems.

- Instruction includes working with real-world contexts. *(MTR.7.1)*

Common Misconceptions or Errors

- Students may only subtract the first term when subtracting one polynomial from another.
- Students may not correctly apply the distributive property when multiplying polynomials, leaving out one or more terms. To address this misconception, encourage the use of the area model so it is easier for students to account for all terms.

- Students may incorrectly distribute the exponent when expanding a binomial raised to a power. To address this misconception, remind students that the exponent indicates how many times the base is to be multiplied. Encourage students to rewrite the problem showing the expansion.

Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.4.1)

A polygon has an area represented by $A = 4x^2 + 12x + 9$.

Part A. If the polygon is a rectangle and the base is $2x + 3$, what would the height be?
Part B. Determine an expression for the perimeter of the polygon in part A.
Part C. If the polygon is a triangle with base $2x + 3$, what would the height be?
Part D. If the triangle is a right triangle, determine an expression for the perimeter of the polygon in part C.

Instructional Items

Instructional Item 1

The length of a rectangle is $(3x^2 - 5x + 2)$ units, and the width is $(4x - 6)$ units. Find the perimeter and the area of the rectangle.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.8

Benchmark

MA.912.AR.1.8 Rewrite a polynomial expression as a product of polynomials over the real or complex number system.

Benchmark Clarifications:
Clarification 1: Instruction includes factoring a sum or difference of squares and a sum or difference of cubes.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.2
- MA.912.AR.6.1, MA.912.AR.6.5

Terms from the K-12 Glossary

- Polynomial

Vertical Alignment

Previous Benchmarks

- MA.8. NSO.1.7
- MA.8.AR.1.1, MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.7 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students rewrote polynomials, up to 4 terms, as a product of polynomials over the real number system. In Algebra 2, students will rewrite polynomials as a product of polynomials over the complex number system.

- Instruction includes student understanding that factoring of polynomials relates to the multiplication of polynomials in the same way that factoring of whole numbers relates to the multiplication of whole numbers.
- Instruction includes special cases such as difference of squares, sum of squares, sum and difference of cubes, and perfect square trinomials.
- Instruction builds upon student prior knowledge of factors including the special cases.
    - Students will likely be exploring the sum of squares for the first time in this course. The connection should be made to the difference of two squares.
    - For example, when factoring $x^2 + 4$:
      $$x^2 + 4 = x^2 - (-4)$$
      $$= x^2 - \sqrt{-4}^2$$
      $$= x^2 - (2i)^2.$$
      This is in the form $a^2 - b^2$, which factors into $(a - b)(a + b)$.
      So, $x^2 + 4 = (x - 2i)(x + 2i)$.
    - The sum of two squares: $a^2 + b^2 = (a + bi)(a - bi)$.
- Instruction includes the use of models, manipulatives and recognizing patterns when factoring. Students should be fluent in perfect squares up to 225 and perfect cubes up to 125, from MA.8.NSO.1.7 in grade 8, to recognize these special patterns.

Common Misconceptions or Errors

- Students may have trouble choosing which factoring technique to use. To address this, consider having students create a flow chart/decision tree when determining how to factor the polynomial they are working with.
- Students may believe they have factored completely, rather than factoring over the complex number system.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.5.1)

Part A. Given the polynomial $x^4 + 16y^4z^8$, rewrite it as a product of polynomials.
Part B. Discuss with your partner the strategy used. How do your polynomial factors compare to your partner's factors.

Instructional Task 2 (MTR.3.1, MTR.5.1)

Part A. What are the factors of the cubic function $f(x) = 8x^3 + 1$?
Part B. Determine the real and complex roots of the function in Part A.

Instructional Items

Instructional Item 1

Given the polynomial $x^8 + 4y^4$, rewrite it as a product of polynomials over the complex number system.

Instructional Item 2
Given the polynomial $x^2-10x+24$, rewrite it as a product of polynomials.

Instructional Item 3
Given the polynomial $x^3-3x^2-9x+27$, rewrite it as a product of polynomials.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.9

Benchmark

MA.912.AR.1.9 Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection to fractions and common denominators.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.8.1, MA.912.AR.8.2, MA.912.AR.8.3

Terms from the K-12 Glossary

- Least Common Multiple
- Rational Expression

Vertical Alignment

Previous Benchmarks
- MA.7.NSO.2
- MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.4 (Algebra 1)

Next Benchmarks
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In middle grades students perform operations with rational numbers, including fractions. In Algebra 1, students rewrote polynomial expressions in different forms by factoring, distributing, and/or combining like terms. In Algebra 2, they extend the work they did with operations with rational numbers and rewriting polynomial expressions to operations with rational algebraic expressions.

- Instruction includes the connection to fractions and common denominators. A rational expression, same as a fraction, can be written as a ratio. Instead of a ratio of two numbers, rational expressions are a ratio of two polynomial expressions.
- The properties of fractions can be applied to rational expressions.
  - Simplifying rational expressions
    Factor the numerator and denominator and cancel common factors.
  - Multiply rational expressions
    It may be helpful to first factor the numerator and the denominator, then multiply the numerators and multiply the denominators. Cancel any common factors.

  - Divide rational expressions
    Students can multiply the first expression by the reciprocal of the second expression and follow the multiplication steps.
  - Adding and subtracting rational expressions
    Students can find a common denominator and then add or subtract the numerators.
- The properties of exponents can be applied to rational expressions, especially the quotient property.
  - Students need to make the connection between $\frac{x^5}{x^2}$ and $\frac{(x+3)^5}{(x+3)^2}$.
- If polynomial expressions are not written in factored form, using the equivalent expression in factored form is helpful in determining the least common denominator.
- Instruction includes writing the restrictions or excluded values. It is important to remind students that we list restrictions based on the original expression not the simplified expression. Students are looking for what makes the denominator equal to zero as that value or those values must be excluded. This is important for students and will help with understanding extraneous solutions when solving rational expressions later.

Common Misconceptions or Errors

- When simplifying rational expressions, students cancel out terms instead of factors. Remind students to always factor before canceling.
- When canceling out the entire numerator, students forget to write 1 as the numerator.
- When multiplying rational expressions, students cross multiply instead of multiplying straight across.
- When adding and subtracting rational expressions, students tend to forget to find a common denominator.
- When adding and subtracting rational expressions, students mistakenly follow the steps of multiplication (i.e., add across the denominators and/or add across the numerators).
- Students may incorrectly divide out variable terms that are not factors.
  - For example, $\frac{x-2}{x-4} \neq \frac{-2}{-4}$.

Instructional Tasks

Instructional Task 1(MTR.7.1)
The U.S. Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficiency for city driving and one for highway driving.

  - For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high combined fuel economy. This number is not the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for x mpg in the city and y mpg on the highway, is computed as $\frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)}$.

Part A. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.

Part B. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function $\frac{a(x)}{b(x)}$.

Part C. Rewrite your answer from Part B in the form of $q(x) + \frac{r(x)}{b(x)}$ where and $q(x)$, (x) and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.

Part D. Use your answer in Part C to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately x+5.

Instructional Task 2 (MTR.5.1, MTR.7.1)

Lauren walked 4 miles to the store to buy a new bike at an average speed of x miles per hours. She returned home riding her bike and her average speed was 2 miles per hour faster than walking.

Part A. Write an expression for the time Lauren takes to get to the store.

Part B. Write an expression for the time Lauren takes to get home from the store.

Part C. Write an expression for the total time she spent on both trips.

Part D. If Lauren spent 3 hours on both trips, what is Lauren's average walking speed?

Instructional Items

Instructional Item 1

Perform the operation(s) on the rational expression. Write your answer as a single rational expression.

a. $\frac{x^2-5x-24}{6x+2x^2} \cdot \frac{5x^2}{8-x}$

b. $\frac{9-x^2}{x^2+5x+6} \div \frac{2x-6}{5x+10}$

c. $\frac{5}{3x} + \frac{2}{7x} - \frac{1}{2x}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3 *Write, solve and graph quadratic equations, functions and inequalities in one and two variables.*

MA.912.AR.3.2

Benchmark

MA.912.AR.3.2	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real and complex number systems.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is to solve by factoring techniques, taking square roots, the quadratic formula and completing the square.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.2.1
- MA.912.AR.1.3, MA.912.AR.1.8

Terms from the K-12 Glossary

- Domain
- Range
- Quadratic Function

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.7 (Algebra 1)
- MA.912.AR.3.1, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7 (Algebra 1)

Next Benchmarks

- MA.912.T.1.3 (Precalculus)
- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.C.3.3, MA.912.C.3.4, MA.912.C.3.5 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

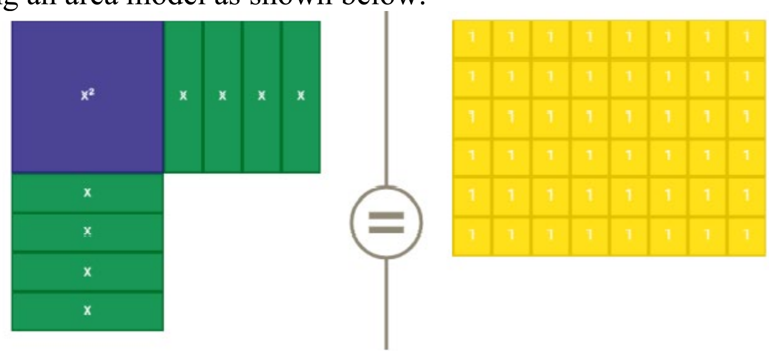
Purpose and Instructional Strategies

In Algebra 1, students solved quadratic equations in one variable over the real number system through various methods, including factoring techniques, taking the square root, using the quadratic formula and completing the square. In Algebra 2, students extend this learning to write and solve one-variable quadratic equations over the complex number system. In Precalculus Honors and Geometry Honors, solving quadratics is useful in connection with the Law of Cosines. In Calculus, solving quadratics is useful in finding intervals of increase or decrease and intervals of concavity for polynomials of degree 3 and 4.

- Instruction emphasizes the understanding that solving a quadratic equation in one variable is the same as the process of determining x-intercepts, roots, or zeros of the graph of a quadratic function.
- While the derivation of the quadratic formula is not an expectation of this benchmark, students can develop the quadratic formula by using completing the square to isolate x in

the equation $ax^2 + bx + c = 0$, making the connection to solving literal equations.

- Instruction includes evaluating the discriminant ($b^2 - 4ac$) to determine whether there is one real solution (equals zero), two real solutions (equals a positive number) or two non-real solutions (equals a negative number).
- Discuss the connection between the number of solutions of a quadratic equation and the graph of a quadratic function. Guide students to see that real solutions result in roots or x-intercepts on the graph of the quadratic function, and that quadratic functions that produce non-real roots never touch the x-axis, since the x-axis is a real number line.
- Instruction on completing the square includes the use of algebra tiles or area model drawings. Students should understand the visual nature of completing the square and connect it to their prior work with area models. *(MTR.2.1)*
  - For example, consider $x^2 + 8x = 48$ and solve by factoring to show that the two solutions are $x = -12$ and $x = 4$. Then ask the question, "If we look at the binomial $x^2 + 8x$ on the left, what would we have to add to it to make a perfect square trinomial?" Write the equation $x^2 + 8x + \_\_ = 48 + \_\_$ on the board to represent the question. Now, represent the equation using algebra tiles or by drawing an area model as shown below.



  - Lead student discussion about the quantity of 1 unit by 1 unit tiles needed to "complete the square" on the left-hand side. Once they state the need for 16 tiles, point out that 16 tiles must also be added to the right to maintain equivalence, then write the number 16 into both blank boxes. Students can then factor the perfect square trinomial to arrive at $(x + 4)^2 = 64$. Have student solve using square roots to find the same solutions of $x = -12$ and $x = 4$.
- Instruction includes the Complex Conjugate Theorem, where students understand that if $a + bi$ is a solution, then $a - bi$ is also a solution.
- In many contexts, students may generate solutions that may not make sense when placed in context. Be sure students assess the reasonableness of their solutions in terms of context to check for this. *(MTR.6.1)*
  - For example, the time it takes for a ball to drop from a height of 28 feet can be modeled by $0 = -16t^2 + 28$. Students solve this equation to find that $t \approx \pm 1.32$ seconds. Through discussion, students should see that -1.32 seconds does not make sense in context and therefore should be omitted. *(MTR.4.1)*
- Enrichment of this benchmark includes determining if a quadratic is a perfect square trinomial.
  - For example, given the equation $0 = 4x^2 - 12x + 9$, students can identify a, b and c as 4, −12 and 9, respectively. Students should recognize that a and c are perfect

squares, where a is $2^2 = 4$ and c is $(-3)^2 = 9$. Since the coefficient of b is twice the product of the square roots ($-12 = 2(2)(-3)$), it can be determined that the given equation is a perfect square trinomial.

- Instruction builds upon students' prior knowledge of factors including the special cases.
- Students will likely be exploring the sum of squares for the first time in this course. The connection should be made to the difference of two squares.
  - For example, when factoring $x^2 + 4$:
$$x^2 + 4 = x^2 - (-4)$$
$$= x^2 - \sqrt{-4}^2$$
$$= x^2 - (2i)^2.$$
This is in the form $a^2 - b^2$, which factors into $(a - b)(a + b)$.
So, $= (x - 2i)(x + 2i)$.
  - The sum of two squares: $a^2 + b^2 = (a + bi)(a - bi)$.

Common Misconceptions or Errors

- When completing the square, students may forget to use the addition or subtraction property of equality to add or subtract values from both sides of the equation. To address this misconception, remind these students that additions or subtractions from one side of an equation must be replicated on the other to maintain equivalency.
- When completing the square and removing a common factor from the x^2 and x term, students may forget to consider that factor when adding/subtracting from the other side of the equation.
  - For example, when solving $10 = 12t^2 + 48t + 55$, students may ultimately add 4 rather than 48 to the left side. Help students to see that there are 12 sets of 4 ultimately being added to the right and therefore, there must be twelve sets added to the left as well.
- When factoring or using the quadratic formula, students may forget to set one side of the equation equal to zero.
  - For example, students may see equations such as $x^2 - 4x + 6 = 27$ and use the number 2, −4 and 6 as a, b and c, respectively, in the quadratic formula, arriving at incorrect solutions. To address this misconception, graph the related function and ask students if the solutions they calculated correspond to the roots of the parabola. Once they see they do not, have students set the equation equal to zero and recalculate.
- When students use the quadratic formula and get a negative number under the radical, they may think they computed incorrectly. To address this misconception, ensure students know this means the solutions are non-real (or imaginary) and the parabola will not touch the x-axis.
- When the radicand is negative, students may incorrectly simplify.
  - For example, students may simplify $\sqrt{-16}$ as -4, rather than $4i$.
- When using the square root or completing the square methods for solving a quadratic equation, students may forget to include the positive and negative roots.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.6.1, MTR.7.1)
A rectangular park has a width of 18 feet and a length of 30 feet. The park is expanded so that the length and width both increase by x feet and the area of the park doubles.

Part A. Write an equation to model the area of the expanded park.
Part B. By how much do the length and width of the park increase?

Instructional Items

Instructional Item 1
Solve the quadratic equation $x^2 - 13x - 50 = 0$ over the complex number system.

Instructional Item 2
Solve $x^2 - 18x + 5 = 0$ over the complex number system.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.3

Benchmark

MA.912.AR.3.3	Given a mathematical or real-world context, write and solve one-variable quadratic inequalities over the real number system. Represent solutions algebraically or graphically.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.2
- MA.912.AR.6.5
- MA.912.AR.9.5, MA.912.AR.9.7

Terms from the K-12 Glossary

- Domain
- Range
- Quadratic Function

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.6 (Algebra 1)
- MA.912.AR.3.1 (Algebra 1)

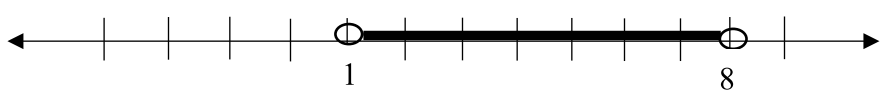
Next Benchmarks

- MA.912.T.1.3 (Precalculus)
- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.C.3.3, MA.912.C.3.4, MA.912.C.3.5 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students wrote and solved one-variable linear inequalities. In Algebra 2, students write and solve one-variable quadratic inequalities. In later courses, students will use some of these techniques to determine critical values and test intervals for positive or negative values to identify the solution set(s).

- Within a real-world context, it is important to consider the reasonableness of solutions.
  - For example, if the x-value represents diameter, x cannot be a negative value because negative diameters do not exist.
- When solving quadratic inequalities algebraically, replace the inequality symbol with an equal sign. The two values obtained from solving the quadratic will be the critical values of the inequality.
  - For example, if the original inequality is $x^2 - 9x + 8 < 0$, rewrite it as $x^2 - 9x + 8 = 0$. If you solve by factoring, you will get $(x - 1)(x - 8) = 0$ and the solutions would be 1 and 8. Graph the solutions on a number line with an open circle at 1 and 8, which would be the critical values. These dots will separate the number line into three sections. Test an x-value in each section to determine whether that value satisfies the inequality. If it does, shade that area and if it does not, do not shade that area. In this example, the only values that would satisfy the inequality would be between (but not including) 1 and 8, therefore the answer would be written algebraically as $1 < x < 8$.



Test a point in this region, 0 for example: $0^2 - 9(0) + 8 \text{ is not less than } 0$

Test a point in this region, 3 for example: $3^2 - 9(3) + 8 \text{ is less than } 0$

Test a point in this region, 9 for example: $9^2 - 9(9) + 8 \text{ is not less than } 0$

Common Misconceptions or Errors

- Students may have trouble remembering which inequality symbols represent open circles and which represent closed circles.
- Students may plug in the test x-value incorrectly and therefore shade in the wrong intervals.
- Students may not realize that the open circle represents points that are not included in the solution.
- Students may not know what to do if there are no roots or only one root. Help students see that if there are no roots, the entire number line is the test interval and if there is only one root, there will be two test intervals.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

A city league soccer team decides to fence the soccer field where they play each weekend. They have 3,000 feet of fencing to use to enclose the rectangular field. The area of the field is at least 81,000 square feet.

Part A. Write an inequality that represents the situation.
Part B. Solve the inequality.
Part C. Graph the solution set.
Part D. Determine whether all of the possible dimensions in the solution set allow enough room for a standard soccer field.

Instructional Items

Instructional Item 1

Solve the inequality and graph the solution on a number line: $x^2 + 30 > 13x$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.4

Benchmark

MA.912.AR.3.4 Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Algebra 1 Example: Given the table of values below from a quadratic function, write an equation of that function.

x	-2	-1	0	1	2
$f(x)$	2	-1	-2	-1	2

<u>Benchmark Clarifications:</u>

Clarification 1: Within the Algebra 1 course, a graph, written description or table of values must include the vertex and two points that are equidistant from the vertex.

Clarification 2: Instruction includes the use of standard form, factored form and vertex form.

Clarification 3: Within the Algebra 2 course, one of the given points must be the vertex or an x-intercept.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.7
- MA.912.F.1.1, MA.912.F.1.7

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function

- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks
- MA.912.F.1.8 (Algebra 1)

Next Benchmarks
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students wrote quadratic functions from a graph, written description, or table using the vertex and two points that are equidistant from the vertex. In Algebra 2, students will continue this learning, however one of the given points must be the vertex or an x-intercept.

- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand when one form might be more useful than another depending on the context.
- Standard Form
  Can be described by the equation $y = ax^2 + bx + c$, where a, b and c are any real number. This form can be useful when identifying the y-intercept.
- Factored Form
  Can be described by the equation $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are real numbers and the roots, or x-intercepts. This form can be useful when identifying the x-intercepts, roots or zeros.
- Vertex Form
  Can be described by the equation $y = a(x - h)^2 + k$, where the point (h, k) is the vertex. This form can be useful when identifying the vertex.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes the connection to completing the square and literal equations to rewrite an equation from standard or factored form to vertex form.
- Students should also explore the pattern of perfect squares, or their multiples, from the vertex when exploring points on the graph.
  - For example, for a horizontal distance of one unit from the vertex, a vertical change of 2 units would indicate an a value of 2. This may be beneficial when determining the value of a in a quadratic function, given a graph.
- Instruction includes the use of graphing software or technology.
  - For example, when determining the value of a, consider using graphing software to allow students to use sliders to quickly observe this. This provides an opportunity for students to notice patterns regarding the value of a and the concavity and stretch of the parabola.
- Instruction includes writing a quadratic equation given the vertex or an x-intercept.
  - For example, if the vertex given is (0, 4) and the parabola passes through (4,2), use $y = a(x - h)^2 + k$ and plug in the vertex $(h = 0, k = 4)$:
    $$y = a(x - 0)^2 + 4$$
    $$y = ax^2 + 4.$$
    Use the other point to solve for a: $2 = a(4)^2 + 4$

$$2 = 16a + 4$$
$$-2 = 16a$$
$$a = -\frac{1}{8}.$$

- The equation of the parabola in vertex form is $y = -\frac{1}{8}x^2 + 4$.

Common Misconceptions or Errors

- When writing functions in vertex form, students may confuse the sign of h.
  - For example, students may see a vertex of $(-1, -2)$ and an a value of 3 and write the function as $y = 3(x - 1)^2 - 2$ instead of $y = 3(x + 1)^2 - 2$. To address this, help students recognize that because h is subtracted from x in vertex form, it will change the sign of that coordinate. Show students a graph of both functions to confirm and make the connection to transformation of functions.
- When writing a quadratic function given an x-intercept and another point, students may use the x-intercept as the vertex.
- Students may be unsure which form of a quadratic function to use with the information given.

Instructional Tasks

Instructional Task 1(MTR.3.1, MTR.7.1)

Savannah and Lauren are playing volleyball.

Part A. Savannah hits the ball in the air from a height of 40 inches. The ball reaches a maximum height of 9 feet after 2 seconds. Write a function that represents the height of the ball over time.
Part B. Which units of measure were chosen to represent the height of the volleyball in Part A?
Part C. Lauren returns the ball by hitting it into the air from a height of 38 inches. Savannah and her teammates miss the ball, and it hits the ground after 4.3 seconds. Write a function that represents the height of the ball over time.
Part D. Which units of measure were chosen to represent the height of the volleyball in Part A?
Part E. Whose ball reached the greatest height?

Instructional Items

Instructional Item 1

Write a quadratic function in standard form whose graph has vertex (3, 5) and passes through the point (0, -2).

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.8

Benchmark

MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

Algebra 1 Example: The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$, where t is the number of years since 1972. In what year does the car's value start to increase?

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.
Clarification 2: Instruction includes the use of standard form, factored form and vertex form.
Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 4: Within the Algebra 1 course, notations for domain, range and constraints are limited to inequality and set-builder.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.2.1
- MA.912.F.1.7
- MA.912.F.2.2, MA.912.F.2.3

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function
- Function Notation
- Intercept
- Quadratic Function
- Range
- Set-Builder Notation
- *x*-intercept
- *y*-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.3.1, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved problems that were modeled by quadratic functions, and graphed and interpreted their key features. In Algebra 2, students will extend their knowledge of these key features to include the use of interval notation.

- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the equation $y = ax^2 + bx + c$, where a, b and c are any real number. This form can be useful when identifying the y-intercept.
  - Factored Form
    Can be described by the equation $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are real numbers and the roots, or x-intercepts. This form can be useful when identifying the x-intercepts, or roots.
  - Vertex Form
    Can be described by the equation $y = a(x - h)^2 + k$, where the point (h, k) is the vertex. This form can be useful when identifying the vertex.
- Instruction includes the use of graphing technology.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality Notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-Builder Notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
  - Interval Notation
    Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.
    - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$. If the domain is all values such that the absolute value is greater than 2, then it can be represented as $(-\infty, -2) \cup (2, \infty)$.
- Depending on a student's pathway, they may not have worked with interval notation (as

it was not an expectation in Algebra 1) before this course. Instruction includes making connections between inequality notation and interval notation.

- For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
- For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
- For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a graphic organizer where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} 1∈ M 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

- Instruction provides opportunities to make connections between the domain and range and other key features.
  - For example, a coffee shop uses the function, $P(x) = -80x^2 + 480x - 540$ to model the profit they can earn in thousands of dollars in terms of the price per cup of coffee, in dollars. In determining the domain and range that includes prices that yield a positive profit, one would also have to identify the vertex (or maximum) and the roots of the function. Students should realize that they can do this by transforming the given expression into vertex form.

Common Misconceptions or Errors

- Students may find themselves stuck initially, unsure of where to start. In conversations with these students, prompt them to reflect on what they know about the context and how they can use that information to determine the requested information (MTR.1.1).
  - For example, students may have an equation in standard form and need to interpret the vertex in context. Prompting students to consider how they've calculated vertices in the past should lead them to choose to either convert the

equation into vertex form or use the line of symmetry to help determine the vertex.

- Students may not recognize that vertex form is useful for determining maximums or minimums and this may hinder some students from appropriately connecting concepts in problems.
  - For example, students may not understand the meaning of the vertex and will solve the original quadratic function by setting it equal to 0.
- Students may interpret $f(x) = 0$ as equivalent to $f(0)$.
- Students may struggle interpreting key features and graphing from various forms of quadratic functions.
  - For example, students may be asked to find the vertex and may believe that completing the square is the only method to use. Prompting students to consider different forms will help with understanding the meaning of key features.
- Students may not include the value zero as a zero part of the domain or range if one of the factors is x.
- Students may not give solutions that are feasible in a real-world context.
- Students may forget to include the negative sign when calculating the x-coordinate of the vertex or axis of symmetry when b is negative. Remind students to include the negative sign in the calculation when b is negative.

Instructional Tasks

Instructional Task 1 (MTR.6.1, MTR.7.1)

The height (y) of a soccer ball x seconds after it is kicked can be modeled by the function $y = -16x^2 + 32x$.

Part A. Graph the function.
Part B. Find and interpret the domain and range. Discuss constraints based on the context of the problem.
Part C. Find and interpret the vertex based on the context of the problem.
Part D. When is the function increasing or decreasing, and what do those intervals mean in the context of the problem?
Part E. Is this function ever negative? Explain.
Part F. When does the soccer ball hit the ground?

Instructional Task 2 (MTR.7.1)

The function, h, defined by $h(t) = -5t^2 + 10t + 7.5$, models the height of a diver above the water (in meters), seconds after the diver leaves the board.

Part A. How high above the water is the diving board?
Part B. When does the diver hit the water?
Part C. What is the maximum height the diver reaches during the dive?
Part D. After how long will the diver reach the same height as when he started?

Instructional Items

Instructional Item 1

Graph the function $f(x) = 3x^2 + 8x - 5$.

Instructional Item 2
Identify the x-intercepts of the function $y = -3(x+2)(x-5)$.

Instructional Item 3
A ball is thrown into the air from the top of a hill. The height, h , of the ball in feet, at x seconds is modeled by the function $h(x) = -8x^2 + 4x + 20$. What is the maximum height, in feet, reached by the ball?

Instructional Item 4
The profit of cookies sold at Luca's Bakery is represented by the function $f(x) = -500(x - 0.55)^2 + 400$, where x is the price in dollars of each cookie.

Part A. What is the domain of the function within the context of the problem?
Part B. What values of x in the domain lead to a positive profit?

Instructional Item 5
The Chrysler Building, which opened in 1930, is one of New York's oldest buildings with a roof height of 925 feet (282 meters). Suppose you could conduct an experiment by dropping a small object from the roof of the Chrysler Building. Using the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 in feet, how long would it take for the object to reach the ground, assuming there is no air resistance?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.9

Benchmark

MA.912.AR.3.9	Given a mathematical or real-world context, write two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description.

Benchmark Clarifications:
Clarification 1: Instruction includes the use of standard form, factored form and vertex form where any inequality symbol can be represented.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.2, MA.912.AR.9.3, MA.912.AR.9.5, MA.912.AR.9.7

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function
- Function Notation
- Quadratic Function
- Range
- x-intercept

- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.7, MA.912.AR.2.8 (Algebra 1)
- MA.912.AR.3.1 (Algebra 1)

Next Benchmarks

- MA.912.T.1.3 (Precalculus)
- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.C.3.3, MA.912.C.3.4, MA.912.C.3.5 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students wrote two-variable linear inequalities to represent relationships between quantities from a graph or written description within a mathematical or real-world context. In Algebra 2, students will write two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description given a mathematical or real-world context. In later courses, students will use some of these techniques to determine critical values and test intervals for positive or negative values to identify the solution set(s).

- Instruction includes making connections to various forms of quadratic inequalities to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the inequality $y > ax^2 + bx + c$, where *a, b* and *c* are any real number, and any inequality symbol can be used. This form can be useful when identifying the y-intercepts and if the graph opens concave up or concave down.
  - Factored Form
    Can be described by the inequality $y > (x - r_1)(x - r_2)$, where r_1 and r_2 are real numbers and the roots, or *x*-intercepts. This form can be useful when identifying the roots, or *x*-intercepts.
  - Vertex Form
    Can be described by the inequality $y > a(x - h)^2 + k$, where the point (h, k) is the vertex. This form can be useful when identifying the vertex.
- Instruction includes the connection to one-variable quadratic equations.

Common Misconceptions or Errors

- Students may write quadratic equations rather than inequalities.
- Students may have trouble writing inequalities in various forms from graphs of quadratics.
- Students often choose the incorrect inequality symbol when interpreting graphs or contexts. Help these students develop the habit of using a test point to check their work. Any point can be used, but many students find it easiest to use the origin (0, 0) as it often makes mental calculation much quicker.

Instructional Tasks

Instructional Task 1(MTR.6.1, MTR.7.1)

Residents that live near a park want to put fencing around a rectangular area to create a dog park. The dog park must have a perimeter of 500 feet and an area of at least 6,000 feet.

Part A. Use perimeter and area formulas to write a quadratic inequality describing the possible lengths of the dog park.

Part B. Solve the quadratic inequality.

Part C. Choose a length in the solution region and find the width. Then check that the dimensions satisfy the original area inequality.

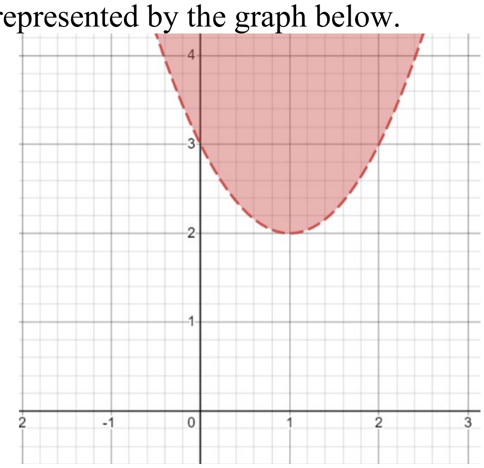
Instructional Items

Instructional Item 1

A rectangular flower bed has a perimeter of 60 feet and must have an area of at least S square feet. Write a quadratic inequality that represents the situation.

Instructional Item 2

Write a quadratic inequality represented by the graph below.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.10

Benchmark

MA.912.AR.3.10 Given a mathematical or real-world context, graph the solution set to a two-variable quadratic inequality.

Benchmark Clarifications:

Clarification 1: Instruction includes the use of standard form, factored form and vertex form where any inequality symbol can be represented.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.2, MA.912.AR.9.3, MA.912.AR.9.5, MA.912.AR.9.7

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function
- Quadratic Function
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.7, MA.912.AR.2.8 (Algebra 1)
- MA.912.AR.3.1 (Algebra 1)

Next Benchmarks

- MA.912.T.1.3 (Precalculus)
- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.C.3.3, MA.912.C.3.4, MA.912.C.3.5 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students have graphed the solutions of one-variable absolute value inequalities and two-variable linear inequalities. In Algebra 2, students graph the solution set of a two-variable quadratic inequality given a mathematical or real-world context. In later courses, students will use some of these techniques to determine critical values and test intervals for positive or negative values to identify the solution set(s).

- Instruction includes the connection to linear inequalities in two variables and their solutions on a graph. This connection includes instruction about when to use a dashed line versus a solid line based on the inequality given.
- Instruction includes the use of a test point to determine which region to shade on the graph.
- Instruction includes the connection to graphing quadratic equations by first setting the quadratic equal to zero, then solving in the same method as you would if it was an equation.
- Instruction should include the use of graphing technology.
- Instruction includes graphing the two-variable quadratic inequalities listed in MA.912.AR.3.9.

Common Misconceptions or Errors

- Students may get confused about whether to use a dashed or solid curve when graphing.
- If test points are used incorrectly, that could result in an error in shading.
- Students may get confused about a point being part of the solution if the curve is solid, but not part of the solution if the curve is dashed.
- Students may have a hard time interpreting appropriate solutions in a real-world situation.

Instructional Tasks

Instructional Task 1(MTR.3.1, MTR.7.1)

Residents that live near a park want to put fencing around a rectangular area to create a dog park. The dog park must have a perimeter of 500 feet and an area of at least 6,000 feet.

Part A. Use perimeter and area formulas to write a quadratic inequality describing the possible lengths of the dog park.

Part B. Solve the quadratic inequality.

Part C. Graph the quadratic inequality.

Instructional Task 2 (MTR.3.1, MTR.7.1)

Profit on an item is sales minus costs. Sales and costs are based on the price set, P. Your company wants to manufacture a new toy. From research, you expect that per 100 units, you will make:

Sales $= (400 - 1.2P)P$ and have

Costs $= 550 + 160(400 - 1.2P)$.

Part A. Write the equation for the expected profit.

Part B. Graph the solution that indicates that the expected profit is positive.

Instructional Items

Instructional Item 1

A rope used at a construction site can safely support a weight W in pounds if $W \leq 6500d^2$, where d is the diameter of the rope in inches.

Part A. Graph the quadratic inequality.

Part B: What does the shaded area represent?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4 *Write, solve and graph absolute value equations, functions and inequalities in one and two variables.*

MA.912.AR.4.2

Benchmark

MA.912.AR.4.2	Given a mathematical or real-world context, write and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.10

Terms from the K-12 Glossary

- Absolute Value
- Compound Inequality

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.6, MA.912.AR.2.7 MA.912.AR.2.8 (Algebra 1)
- MA.912.AR.4.1 (Algebra 1)

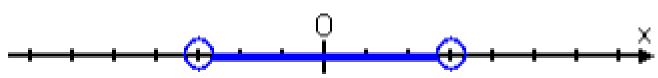
Next Benchmarks

- MA.912.C.1.1, MA.912.C.1.9 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved one-variable absolute value equations and in Algebra 1 Honors, students solved absolute value inequalities in one variable. In Algebra 2, students write and solve one-variable absolute value inequalities and represent solutions both algebraically and graphically. In future courses, including Precalculus Honors and Calculus Honors, students will focus on the concept of two points being very close to each other. This concept is often expressed in terms of absolute value inequalities.

- Instruction includes making the connection between solving absolute value equations to solving absolute value inequalities. The solution to an absolute value equation is typically one or two points, but the solution to an absolute value inequality is typically going to be intervals.
- Instruction includes making the connection to the graphical representation of solutions and using words to help determine solutions.
  - For example, the equation $|x| < 3$ can be read as "Which values of x are less than 3 units away from zero?" This can be represented algebraically as $-3 < x < 3$ and graphically, the solution is shown on the number line below.



- Instruction includes making the connection to compound inequalities.
  - When solving the inequality $|x - 1| \leq 3$, students may rewrite the absolute value inequality by separating the inequality into two inequalities to remove the absolute value bars:
$$x - 1 \leq 3 \quad \text{and} \quad -(x - 1) \leq 3.$$
  - Students may also rewrite the absolute value inequality by writing as a single compound inequality:
$$-3 \leq x - 1 \leq 3.$$
- Instruction encourages students to discuss their thinking with their peers (MTR.4.1). Instructional focus should include conversations about the solution set algebraically and graphically. Often students will be able to reason out their thinking but will struggle with representing their thinking mathematically. Encourage this process and ask questions that will help with solving the task. (MTR.1.1) Are there multiple ways for students to represent this problem mathematically? (MTR.2.1)
- Instruction includes student understanding that compound inequalities with "or" create a combining (or a union) of the solutions of the individual inequalities while compound

inequalities with “and” create an overlap (or an intersection) of the solutions of the individual inequalities.

Common Misconceptions or Errors

- Students may confuse the inequality symbols.
- Students may forget that multiplication or division by a negative number will change the direction of the inequality symbol.
- Students may misrepresent the inequality symbols on a number line and/or incorrectly identify the solutions graphically.
- Students may not create two inequalities from an absolute value inequality.
- Students may incorrectly identify the solutions graphically.
- Students may represent the absolute value inequality using the incorrect compound inequality. To address this misconception, have students develop a graphic organizer to organize steps.
- In interval notation, students may misinterpret the symbol for the union or intersection of two sets and indicate values incorrectly as being included in the solution set.

Instructional Tasks

Instructional Task 1 (MTR.7.1)
Great Gardens Landscaping has been hired to build gardens around a new resort. They designed plans for two different rectangular gardens. In the first plan the length is twice that of the width and in the second plan the length is 2 feet longer than the width. The perimeters of the gardens cannot differ by more than 3 feet.

Part A. Write and solve an inequality to determine the possible widths of the garden. Check that your solutions are feasible within the context of the problem.
Part B. What are the dimensions of the two gardens using the smallest allowable width?
Garden 1: Garden 2:
Part C. Using the largest allowable width, what is the area of the garden with the larger area?

Instructional Task 2 (MTR.7.1)
Ms. Pruitt’s candy company sells boxes of their famous chocolate-covered peanut butter bites. The candy is sold in boxes that must be within 95 grams of weighing 1,465 grams. Each piece weighs 42.5 grams and the box weighs 116 grams. Calculate and graph the possible quantities of chocolate-covered peanut butter bites that could be in a single box.

Instructional Task 3 (MTR.6.1, MTR.7.1)
Tom is a machinist at Ancient City Choppers. To ensure the highest reliability of the motorcycle engines produced at the shop, Tom machines each cylinder to a 4.01’’ diameter with a tolerance of 0.002”.

Part A. Write an absolute value inequality that could be used to determine the acceptable range of each cylinder’s diameter.
Part B. Express that range graphically.

Instructional Items

Instructional Item 1

There is a cloud layer centered at 4 miles above the sea that is 1,000 feet thick. At what elevations, x, given in miles, will a plane in that area be out of the clouds?

Instructional Item 2

Determine the solutions to the absolute value inequality and represent the solutions on a number line.

$$|2x + 3| \leq 6$$

Instructional Item 3

The manufacturing plant for a popular sugar brand fills each of its bags with 16 ounces of sugar to be sent to local grocery stores. After the bags are filled, another machine weighs them to ensure accuracy. If the bag weighs 0.7 ounces more or less than the desired weight, the bag is rejected. Write an absolute value inequality to find the heaviest and lightest bag the machine will approve. Include a verbal and graphical explanation of possible values that are acceptable.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4.4

Benchmark

MA.912.AR.4.4	Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior and symmetry.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.10
- MA.912.F.1.7

Terms from the K-12 Glossary

- Absolute Value
- Coordinate Plane
- Domain
- Function
- Intercept
- Piecewise Function
- Range

- Set-Builder Notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks
- MA.912.AR.4.3 (Algebra 1)

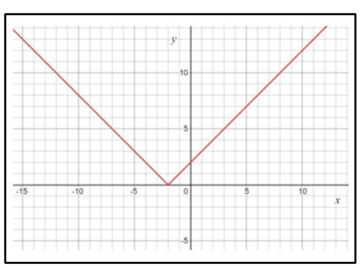
Next Benchmarks
- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.7.4 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students graphed absolute value functions when given a table, equation or written description, and determined its key features. In Algebra 2, students solve and graph problems that are modeled with absolute value functions, interpret key features, and determine constraints depending on the context.

- When making connections to transformations of functions, use graphing software to explore absolute value functions stemming from the parent equation to see the effects on the graph. Allow students to make predictions. *(MTR.4.1)*
- Students will be working with functions in the form, $f(x) = a|x - b| + c$, where a, b and c are real numbers. Instruction includes helping students understand how a is related to the slopes of the two pieces of an absolute value function and that (b, c) is the vertex.
- Instruction includes making the connection between the algebraic and graphical representations. In doing so, students can tie together key feature concepts of absolute value functions and make connections to linear functions and their key features.
- Instruction reinforces the definition of absolute value as a number's distance from zero on a number line. For absolute value functions, noticing where the argument of the absolute value bars will be zero can be helpful in ensuring that students graph correctly.
    - For example, if students are asked to graph $y = |x + 2|$, students should recognize that the absolute value will be a V-shaped graph and the vertex will occur at the value of x that makes the math statement of the function, $|x + 2|$, equal to zero. Notice in the table below the value that makes the function zero is -2 and to graph using a table, students can choose x values to the left and right of that critical value, -2.

x	$\lvert x+2\rvert$
-4	2
-3	1
-2	0
-1	1
0	2



- Instruction allows for the flexibility to graph absolute value functions using a variety of ways.

- For example, the function $y = |x + 2|$ can be graphed using transformations, recognizing that this graph shifts two units to the left and will open upward because there's a positive a value.
- For example, the function $y = |x + 2|$ can be graphed by plotting points from the vertex, using the "slope." In this case, once the vertex is plotted, use the slope to graph the part that is on the right side of the vertex and use symmetry to graph the part that is on the left side of the vertex.
- For example, for the function $y = |x + 2|$, students can create a T-chart and solve for the values that satisfy the function. It is important that students include negative inputs in their T-chart, so they are not misled as to what the graph looks like. Students can use their understanding of the parent function of $y = |x|$, whose V-shaped graph has a vertex at the origin. In creating and plotting points, students will use the vertex to find the center of the graph and then choose additional x-values to use in the table to the left and right of the vertex.

- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality Notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-Builder Notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
  - Interval Notation
    Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.
    - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Depending on a student's pathway, they may not have worked with interval notation (as it was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in

interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
    - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- In Algebra 2, students are introduced to new mathematical symbols. Consider having a graphic organizer where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in$ M 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

- Instruction includes the use of x-y notation and function notation for absolute value functions.
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that absolute value graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by absolute value functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given. *(MTR.4.1)*
- Depending on a student's pathway in the future, it will be important to understand that absolute value functions are piecewise linear functions.
- When addressing real-world contexts, the absolute value is used to define the difference or change from one point to another. Connect the graph of the function to the real-world context so the graph can serve as a model to represent the solution. *(MTR.6.1, MTR.7.1)*
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Problem types include applying distance to real-world problems, showing how much a value deviates from a standard.
- Instruction includes opportunities for students to compare functions and show similarities between key features. It is also important that students see that the whole graph is symmetric across the vertex and if one point is found on one half of the graph, we can use its reflection on the other half.

Common Misconceptions or Errors

- Students may think that the distributive property should be used if a constant is in front of the absolute value.
  - For example, for $f(x) = 4|x - 2|$, students will incorrectly distribute the 4 to the absolute value function when evaluating.
- Students may not fully understand the connection to all the key features (emphasize the use of technology to help with student discovery) and how to represent them using the proper notation.
- Students may confuse the transformations when graphing.
  - For example, for $f(x) = |x - 2|$, students may incorrectly graph the vertex two units to the left rather than two units to the right.

Instructional Tasks

Instructional Task 1 (MTR.5.1, MTR.7.1)
A car starts traveling on the interstate at 9 a.m. at a speed of 70 miles per hour. The car's distance from an interesting landmark in miles, y, is represented by the function $y = |100 - 70x|$, where x is the number of hours since 9 a.m. The car will stop at 2 p.m.

Part A. Find and interpret the y-intercept of the graph of the function.
Part B. Find and interpret the vertex of the graph of the function.
Part C. Find the intervals where the function is increasing and decreasing and give an interpretation of the intervals.
Part D. In context of the problem, what is the domain and range?

Instructional Task 2 (MTR.3.1, MTR.5.1)
Part A. Graph $g(x) = -|x - 4| + 2$.
Part B. Compare the graph from Part A to its parent function $f(x) = |x|$.
Part C. Use the information from Part A and B to determine the domain and range of the function $g(x)$.

Instructional Items

Instructional Item 1
Graph $f(x) = 2|x - 1| + 3$ and determine when the function is positive, negative, increasing or decreasing.

Instructional Item 2
Part A. Graph $g(x) = -|x - 4| + 2$.
Part B. Compare the graph from Part A to its parent function $f(x) = |x|$.
Part C. Determine the domain and range of the function $g(x)$.

Instructional Item 3
Electrical parts, such as resistors and capacitors, come with specified values of their operating parameters: resistance, capacitance, etc. However, due to imprecision in manufacturing, the actual values of these parameters vary somewhat from piece to piece, even when they are supposed to be the same. The best that manufacturers can do is to try to guarantee that the variations will stay within a specified range, often $\pm1\%$, $\pm5\%$ or $\pm10\%$. Suppose we have a

resistor rated at 680 ohms, $\pm 5\%$. Write an absolute value function to represent the range of possible values of the actual resistance.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5 *Write, solve and graph exponential and logarithmic equations and functions in one and two variables.*

MA.912.AR.5.2

Benchmark

MA.912.AR.5.2	Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.7.1
- MA.912.AR.8.1
- MA.912.F.1.1
- MA.912.F.2.2, MA.912.F.2.3
- MA.912.F.3.2
- MA.912.FL.3.1, MA.912.FL.3.2, MA.912.FL.3.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Base of an Exponent
- Exponent (Exponential Form)
- Exponential Function

Vertical Alignment

Previous Benchmarks

- MA.912.F.1.8 (Algebra 1)
- MA.912.FL.3.4 (Algebra 1)
- MA.912.NSO.1.1 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students worked with the properties of exponents and solved one-variable linear, quadratic, and exponential equations. In Algebra 2, students will learn that there are similar rules for logarithms and some exponential equations can be solved by using logarithms. In later courses, students will use derivatives to solve different types of function problems.

- Instruction includes the use of various methods to solve one-variable exponential equations.

    - For example, students can use the properties of exponents or logarithms to determine the solution for the equation $2^x = 32$.
        - Students can rewrite the equation as $2^x = 2^5$, noticing that since the base of both sides of equations are the same, then the exponents must be equivalent to one another. Therefore, $x = 5$.
- Instruction includes solving logarithmic equations by changing from a logarithmic expression to an exponential expression.
    - For example, when solving $\log_3(4x - 7) = 2$, we can obtain an exact solution by changing the logarithm to exponential form creating the equivalent equation $4x - 7 = 3^2$. Students should recognize that this is a one-variable linear equation $4x - 7 = 9$ and, therefore, $x = 4$.
    - Instruction includes checking the viability of solutions and identifying extraneous solutions. *(MTR.6.1)*

Common Misconceptions or Errors

- Students may not understand the relationship between logarithm and exponential functions and may treat them as separate concepts.
- Students may forget to check for extraneous solutions.
    - For example, when asked to solve $\log_x 36 = 2$, students may rewrite this as an exponential equation $36 = x^2$ and get $x = \pm 6$. The base of the logarithm is always positive, so we discard the -6.
- Students might struggle to recognize the inverse relationship between exponentiation and logarithms, leading to difficulties in transitioning between the two.
- Students may incorrectly apply the change of base formula when solving logarithmic equations, resulting in errors in determining the value of x.
- Students might overlook the importance of checking the viability of solutions, leading to potential acceptance of extraneous solutions. This is particularly critical in logarithmic equations, where the argument of the logarithm must be positive.
- Students might mistakenly interpret logarithmic equations as linear equations, leading to errors in solving and identifying the correct values for x.
    - Example: In the equation $\log_4(2x + 2) = 2$, a student mistakenly treats it as a linear equation and solves for x incorrectly as $x = 3$ instead of $x = 7$.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Between 7:00 a.m. and 9:00 a.m., cars arrive at Starbucks drive-thru at the rate of 24 cars per hour (0.40 car per minute). The following function can be used to determine the probability that a car will arrive within t minutes of 7:00 a.m.

$$C(t) = 1 - e^{-0.4t}$$

Part A. Determine the probability that a car will arrive within 5 minutes of 7 a.m. (that is, before 7:05 a.m.).

Part B. Determine the probability that a car will arrive within 30 minutes of 7 a.m.

Part C. Within how many minutes of 7:00 a.m. would the probability a car will arrive be 78%?
Part D. What does the value $C(t)$ approach as t becomes unbounded in the positive direction?
Part E. Graph $C(t)$ using graphing technology.

Instructional Items

Instructional Item 1
Determine the value of x that satisfies the equation $3^{4x-1} = \frac{1}{27}$.

Instructional Item 2
Solve for x in the equation $2^x = 11$.

Instructional Item 3
Solve for x in the equation $\log_6(2x-3) = \log_6(x+2)$.

Instructional Item 4
Determine the value of x that satisfies the equation $2\log_{10} 6 - \frac{1}{3}\log_{10} 27 = \log_{10} x$.

Instructional Item 5
Solve for x in the equation $\log_2 x = 3$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.4

Benchmark

MA.912.AR.5.4 Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, exponential functions are limited to the forms $f(x) = ab^x$, where b is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.
Clarification 2: Within the Algebra 1 course, tables are limited to having successive nonnegative integer inputs so that the function may be determined by finding ratios between successive outputs.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.7.1
- MA.912.AR.8.1
- MA.912.F.1.1, MA.912.F.2.2, MA.912.F.2.3
- MA.912.F.3.2
- MA.912.FL.3.1, MA.912.FL.3.2, MA.912.FL.3.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Exponential Function
- Exponents
- Function
- Percent of Change

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.1 (Algebra 1)
- MA.912.AR.5.3 (Algebra 1)
- MA.912.F.1.8 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students wrote exponential functions that modeled relationships characterized by having a constant percent of change per unit interval. In Algebra 2, students continue this work, not limited to the forms $f(x) = a(b)^x$. Additionally, students further develop their understanding of this feature of exponential functions from Algebra 1 by emphasizing real-world context and applications.

- Instruction includes exponential functions that are not necessarily in the form $f(x) = a(b)^x$.

- Examples include:
  - $f(x) = -6(2.1)^{2-x}$
  - $f(x) = (1.16)^{4x}$
- Instruction includes guiding students to use laws of exponents to convert examples like these to the form $f(x) = a(b)^x$.
  - $f(x) = -6(2.1)^{2-x} = -6(2.1^2)(2.1)^{-x} = -26.26(\frac{1}{2.1})^x$
  - $f(x) = (1.16)^{4x} = (1.16^4)^x = (1.81063936)^x$

- Instruction includes guidance on how to determine the initial value or the percent rate of change of an exponential function when it is not provided.
- Instruction includes interpreting percentages of growth and decay from exponential functions.
  - For example, the function $f(x) = 500(1.16)^x$ represents 16% growth over one unit of time.
    - Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.
  - Additionally, the function $f(x) = 500(0.72)^x$ represents 28% decay of an initial value.
    - Guide students to discuss the meaning of the number 0.72 as a percent. They should understand it represents 72%. Taking 72% of an initial value decreases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential decay.
- Use prior knowledge of transformations on functions to explore different forms of exponential functions.
  - For example, the function $f(x) = 16\left(\frac{1}{2}\right)^x$ can be written as $f(x) = 4\left(\frac{1}{2}\right)^{x-2}$ from the given table of values, showing the horizontal shift.

x	0	1	2	3
$f(x)$	16	8	4	2

- Instruction includes students using Euler's number, symbolized by the letter e, as the base in many applied exponential functions. Euler's number, e, is irrational and referred to as the natural base. It can be approximated to 2.72 and is used when the relationship between two quantities is growing exponentially at a continuous rate.
- Instruction incudes the use of technology.

Common Misconceptions or Errors

- Students may not recognize that if b is greater than 1 that it represents growth and if b is less than one that it represents decay.
- Students may struggle differentiating between a constant rate of change and common ratio to determine which type of function they are writing.
- Students may confuse written form of decimals and percentages, e.g., 5% growth (1 +

.05) is equal to 105 % not equal to 150%.

- Remind students to use multiple points when given a table to help find the exponential function.
- Students may say the decay rate for $f(x) = 500(0.72)^x$ is 72%
- Students may incorrectly assume that the value of b itself represents percentage change rather than the growth or decay factor.
    - For example, if $f(x) = 200(1.05)^x$, students may mistakenly think that the percentage growth is 1.05%, instead of recognizing that it represents a 5% increase.
- Students may struggle to distinguish between the initial value (a) and the growth/decay factor (b).
    - For example, in the function $g(x) = 3(2)^x$, students might incorrectly identify 3 as the growth factor, not realizing that it is the initial value.
- Students may misinterpret the role of the exponent x in exponential functions, leading to incorrect understanding of the relationship between the input and output.
    - For example, in $h(x) = 10(0.8)^x$, students might mistakenly believe that increasing x results in an increase in the output, rather than a decay.
- Students may struggle to convert between decimal and percentage forms when interpreting growth or decay rates.
    - For example, given $p(x) = 300(1 - 0.15)^x$, students might incorrectly interpret the decay rate as 0.15%, instead of recognizing it as a 15% decrease.
- Students may overlook the implications of having zero or negative exponents in the context of exponential functions.
    - For example, in $k(x) = 5(2)^{-x}$, students might mistakenly believe that the function represents exponential growth, not realizing that it represents exponential decay due to the negative exponent.
- Students may struggle to understand the significance of Euler's number (e) as the base in certain exponential functions.

Instructional Tasks

Instructional Task 1(MTR.7.1, MTR.2.1)

Raylene and Caleb were working on separate biology experiments. Each student documented their cell counts over time in the chart below.

Raylene's Experiment	
# of minutes	*# of cells*
3	2400
6	1200
9	600
12	300
15	150

Caleb's Experiment	
# of minutes	*# of cells*
0	160
1	240
2	360
3	540
4	910

Part A. Do the number of cells in Caleb's experiment increase at a percentage rate of change? If so, what is the percentage rate? If not, describe what is happening to the number of cells. Does this change represent exponential growth or decay? Justify your answer.

Part B. Write exponential functions to represent the relationship between the quantities for each student's experiment. In which experiment is the number of cells changing more rapidly? Justify your answer.

Part C. Graph these functions and determine their key features.

Instructional Items

Instructional Item 1
In 2020, the world population was approximately 7.98 billion, and in 2016, the world population was approximately 7.46 billion. Assuming the percent rate of change per year of the world's population is constant from 2016 until 2025, find the world's population in 2025.

Instructional Item 2
The function $\mathrm{g(t)} = 300(0.85)^{\mathrm{t}}$ models the decay of a substance over time. Identify the initial quantity and the percentage decay.

Instructional Item 3
Consider the function $\mathrm{f(x)} = 500(2)^{-\mathrm{x}}$. Rewrite in the form $f(x) = a(b)^x$ and determine the percent change and initial value.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.5

Benchmark

MA.912.AR.5.5	Given an expression or equation representing an exponential function, reveal the constant percent rate of change per unit interval using the properties of exponents. Interpret the constant percent rate of change in terms of a real-world context.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.7.1
- MA.912.AR.8.1
- MA.912.F.1.1, MA.912.F.2.2, MA.912.F.2.3
- MA.912.F.3.2
- MA.912.FL.3.1, MA.912.FL.3.2, MA.912.FL.3.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Exponential Function
- Exponents

- Function
- Percent Rate of Change

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.1 (Algebra 1)
- MA.912.AR.5.3 (Algebra 1)
- MA.912.F.1.8 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students generated equivalent algebraic expressions using the properties of exponents and wrote exponential functions that modeled relationships characterized by having a constant percent of change per unit interval. In Algebra 2, students reveal the constant percent rate of change per unit interval and interpret the constant percent rate of change in terms of a real-world context.

- Instruction includes using the properties of exponents.
- Instruction includes determining the percent rate of change for different time periods.
    - For example, the function $f(x) = 200(1.06)^x$ could represent the value of an investment with an annual 6% rate of increase where x is expressed in years. Using the laws of exponents this can be written as $f(x) = 200((1.06)^{\frac{1}{12}})^{12x} = 200(1.0049)^{12x}$. Since x is given in years, *12x* represents the number of months so the percent increase per month is 0.49%.
- Instruction includes connecting prior knowledge of constant rate of change, including linear functions and making comparisons.
- When generating equivalent expressions to reveal the constant percent rate of change per unit interval, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies. *(MTR.2.1)*
- Instruction includes interpreting percentages of growth/decay from exponential functions and observe that b can be used to determine a percentage.
    - For example, the function $f(x) = 500(1.16)^x$ represents 16% growth of an initial value.
- Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.
- Instruction includes interpreting percentages of growth and decay from exponential functions.
    - For example, the function $f(x) = 500(1.16)^x$ represents 16% growth over one unit of time.
        - Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.

    - Additionally, the function $f(x) = 500(0.72)^x$ represents 28% decay of an initial value.
        - Guide students to discuss the meaning of the number 0.72 as a percent. They should understand it represents 72%. Taking 72% of an initial value decreases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential decay.
- Instruction includes real-world problems on compound interest and continuously compounded interest in connection with MA.912.FL.3.4.

Common Misconceptions or Errors

- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Students may struggle with representing the growth factor as a percent, rather than a whole number.
- Consider the expression $3^{2x} \times 3^x$. Some students might incorrectly simplify this as 3^{2x^2} instead of recognizing that $3^{2x} \times 3^x$ should be simplified using the rule $a^m \times a^n = a^{m+n}$. The correct simplification is 3^{3x} .
- Given the exponential function $p(t) = 500(1.08)^t$, students might mistakenly represent the growth factor 1.08 as 8% instead of recognizing it as a factor that results in an 8% increase per time unit. The correct interpretation is that the population grows by 8% each time unit.
- For a decay scenario modeled by $Q(x) = 200(0.95)^x$, students might incorrectly express the decay rate as 0.95. To address this, students can express this function as $200(1 - 0.05)^x$ to see that the decay rate is $r = 0.05$(not 0.95).

Instructional Tasks

Instructional Task 1 (MTR.4.1,MTR.7.1)

Four physicists describe the amount of a radioactive substance in grams, Q, left after t years below.

Function 1: $Q = 300e^{-0.0577t}$

Function 2: $Q = 300\left(\frac{1}{2}\right)^{\frac{t}{12}}$

Function 3: $Q = 300(0.9439)^t$

Function 4: $Q = 252.290(0.9439)^{t-3}$

Part A. Compare the four different functions describing the radioactive substance.
Part B. Determine whether the described functions are approximately equivalent.
Part C. What is the constant percent rate of change annually of the radioactive substance?
Part D. Why do you think each of the four physicists described the amount of radioactive substance differently? What information does each tell you?

Instructional Task 2 (MTR.7.1)
A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x) = 5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

Part A. Exactly how many fish did the fisherman release into the lake?
Part B. Find b if you know the lake contains 33 fish after eight weeks.
Part C. Instead, now suppose that $P(x) = 5b^x$ and $b = 2$. What is the weekly percent growth rate in this case? What does this mean in everyday language?
Part D. How does the weekly percent growth rate compare in Part B and Part C?

Instructional Items

Instructional Item 1
The equation $y = 42{,}500(0.9198)^x$ represents the value of a car x years after its initial purchase. The average rate of depreciation for vehicles is often measured in 5-year intervals. Write an equivalent expression to show the constant percent rate of change over 5 years.

Instructional Item 2
The function $V(t) = 250{,}000(1.038)^t$ represents the value (V) of a home t years after its initial purchase. Determine the annual percent rate of change.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive*

MA.912.AR.5.7

Benchmark

MA.912.AR.5.7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.

Example: The graph of the function $f(t) = e^{5t+2}$ can be transformed into the straight line $y = 5t + 2$ by taking the natural logarithm of the function's outputs.

<u>Benchmark Clarifications:</u>
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 3: Instruction includes understanding that when the logarithm of the dependent variable is taken and graphed, the exponential function will be transformed into a linear function.
Clarification 4: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.7.1
- MA.912.AR.8.1
- MA.912.F.1.1, MA.912.F.2.2, MA.912.F.2.3
- MA.912.F.3.2
- MA.912.FL.3.1, MA.912.FL.3.2, MA.912.FL.3.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Domain
- Exponent
- Exponential Function
- Function
- Function Notation
- Range
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.1 (Algebra 1)
- MA.912.AR.5.3 (Algebra 1)
- MA.912.F.1.8 (Algebra 1)

Next Benchmarks

- MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students worked with exponential functions in limited forms. In Algebra 2, students extend this work to solve and graph problems modeled with exponential functions, determining key features and constraints in terms of the context.

- Problem types include creating a function as a tool to determine requested information or providing the graph or function that models the context.
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically.
- Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between two variables.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
  - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a

value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.
- If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

  - Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that exponential graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by exponential functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.

Common Misconceptions or Errors

- Students may confuse finding the common logarithm and natural logarithm.
- Students may not fully understand how to interpret proper function notation when determining the key features of an exponential function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

The population of a city is $K = 178{,}563e^{0.011t}$ where $t = 0$ represents time in years, relative to the year 2012.

Part A. Find the population of the city in the years 2022 and 2030.

Part B. Discuss whether it makes sense within the context of the problem for t to be a negative value.

Part C. Discuss what might be a reasonable domain for this function within the context of the problem.

Instructional Items

Instructional Item 1

Consider the function $y = 2e^{0.3x} - 4$

Determine any asymptotes for the graph of the function.

Instructional Item 2

An exponential function is given.

$$y = (17)^{2m}$$

Part A. Does this function represent exponential growth or decay?

Part B. Select four values for m and create a graph.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.8

Benchmark

MA.912.AR.5.8 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features.

<u>Benchmark Clarifications:</u>

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.

Clarification 2: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.7
- MA.912.AR.4.4
- MA.912.F.1.1
- MA.912.F.2.2, MA.912.F.2.3, MA.912.F.2.5
- MA.912.F.3.6, MA.912.F.3.7

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Exponential Function
- Function
- Function Notation
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.5 (Algebra 1)
- MA.912.AR.3.7 (Algebra 1)
- MA.912.AR.9.6 (Algebra 1)
- MA.912.F.1.6 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.6 (Precalculus)
- MA.912.C.2.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students graphed exponential functions and determined their key features, including asymptotes and end behavior. In Algebra 2, students develop an understanding of logarithmic functions, the inverse function of exponential functions. Further study of logarithms will take place in Precalculus and Calculus.

- Instruction includes making the connection between exponents and logarithms as inverse functions and operations. *(MTR.5.1)*
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
  - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the

symbol, ∪, for union.

- If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Instruction includes functions with various bases, including base 10 and base e.

Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithmic function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases,

ask reflective questions to help students examine the meaning of the domain and range in the problem.

- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Given $f(x) = \log_3(x)$ and $g(x) = 3^{-x}$. Complete Parts A–F.

Part A. Graph each function.
Part B. State the domain of each function.
Part C. State the range of each function.
Part D. Determine the asymptote(s).
Part E. Compare the functions. Be sure to include mathematical terms such as intercepts, increasing, decreasing, positive, negative and/or end behavior.
Part F. Discuss your comparison with a partner. How were your comparisons the same? How were they different?

Instructional Items

Instructional Item 1

Given $f(x) = \log_7(x - 3) + 1$

Part A. Graph the function.
Part B. State the domain of the function.
Part C. State the range of the function.
Part D. Determine the asymptote(s).

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.9

Benchmark

MA.912.AR.5.9 Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.3.8
- MA.912.AR.9.10

- MA.912.F.1.1
- MA.912.F.2.2, MA.912.F.2.3, MA.912.F.2.5
- MA.912.F.3.6, MA.912.F.3.7

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Exponential Function
- Function
- Function Notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.1, MA.912.NSO.1.2 (Algebra 1)
- MA.912.AR.2.5 (Algebra 1)
- MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.6 (Precalculus)
- MA.912.AR.7.4 (Precalculus)
- MA.912.C.2.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved problems modeled with linear, exponential and quadratic functions. In Algebra 2, students solve problems modeled with logarithmic functions. Further study of logarithms will take place in Precalculus and Calculus.

- Instruction features a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or should provide the graph or function that models the context.
- When making connections to transformations of functions, use graphing software to explore logarithmic functions applying transformations to see the effects on the graph. Encourage students to make predictions. *(MTR.4.1)*
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
    - Interval notation should be written with the lower bound first, then the upper

bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.

- If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} 1∈ M 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Instruction includes determining when to use base 10 or natural logs based on the context given.

Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithm function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.7.1)

A video uploaded to social media initially had 75 views one minute after it was posted. The total number of views to date has been increasing exponentially and can be modeled by the function $y = 75e^{0.2t}$, where t represents time measured in days since the video was posted.

Part A. Convert the exponential to logarithmic form.
Part B. How many days will it take until 5000 people have viewed this video?
Part C. State a realistic domain and interpret in the context of the problem.
Part D. Sketch a graph that represents days 5–10.

Instructional Items

Instructional Item 1

Wanda invests $10,000 in a company with earnings of 4% per year.

Part A. Write an equation in logarithmic form that describes the situation.
Part B. How long will it take for the investment to earn an additional $20,000?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.6 *Solve and graph polynomial equations and functions in one and two variables.*

MA.912.AR.6.1

Benchmark

MA.912.AR.6.1	Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.2

Terms from the K-12 Glossary

- Function
- Function Notation
- Polynomials

Vertical Alignment

Previous Benchmarks
- MA.912.AR.3.1 (Algebra 1)

Next Benchmarks
- MA.912.AR.6.3, MA.912.AR.6.4 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved quadratic equations by factoring. In Algebra 2, students solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems given a mathematical or real-world context. Polynomials play an important role in future mathematics courses.

- Instruction includes student understanding that factoring can be thought of as undoing the
- multiplication of polynomial expressions.
- Students have had prior work factoring and solving quadratic expression using various factoring techniques. Instruction should make connections back to that work.
- Instruction includes special cases such as difference of squares, sum of squares, sum and difference of cubes, and perfect square trinomials.
- Instruction builds upon student prior knowledge of factors including the special cases.
  - Students will likely be exploring the sum of squares for the first time in this course. The connection should be made to the difference of two squares.
  - For example, when factoring $x^2 + 4$:

$$x^2 + 4 = x^2 - (-4)$$
$$= x^2 - \sqrt{-4}^2$$

$= x^2 - (2i)^2.$
This is in the form $a^2 - b^2$ which factors into $(a - b)(a + b)$.
So, $x^2 + 4 = (x - 2i)(x + 2i)$.

   - The sum of two squares: $a^2 + b^2 = (a + bi)(a - bi)$.
- Instruction includes the use of models, manipulatives and recognizing patterns when factoring.
   - Students should be fluent in perfect squares up to 225 and perfect cubes up to 125, from MA.8.NSO.1.7 in grade 8, to recognize these special patterns.

Common Misconceptions or Errors

- Students may factor as if they were factoring quadratics without taking exponents into consideration.
- Students may forget to set each factor equal to zero to find the solutions.
- Students may forget to rearrange the polynomial equation so that one side is equal to zero before they begin to solve.
- Students may have trouble recognizing which factoring technique is appropriate to the expression. To address this misconception, consider having students create a decision tree.

Instructional Tasks

Instructional Task 1 (MTR.7.1)
A wooden block is shaped like a rectangular prism with a volume of 320 cubic centimeters. The block is $2x$ centimeters tall, $(x - 3)$ centimeters long and $(x - 4)$ centimeters wide.

Part A. Write a polynomial equation in *x* whose solution can be used to determine the dimensions of the block.
Part B. Find the dimensions of the block using technology if necessary to solve this equation for *x*.
Part C. Is there more than one solution for *x* over the real number system?
Part D. Can you replace the volume of 320 with a different volume so that the resulting mathematical equation would result in more than one real solution?
Part E. Are all the solutions in part D reasonable within the context of the problem?

Instructional Items

Instructional Item 1
Solve the polynomial equation for *x* over the complex number system.
$3x^3 + 12x^2 + 12x = 0$

Instructional Item 2
Solve the polynomial equation for *x* over the complex number system.
$x^4 - 5x^2 = 36$

Instructional Item 3
Solve the polynomial equation for *x*. Include all real and non-real solutions.
$x^4 - 10x^3 + 25x^2$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.6.5

Benchmark

MA.912.AR.6.5	Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.1
- MA.912.F.2.2, MA.912.F.2.3

Terms from the K-12 Glossary

- End Behavior
- Function
- Function Notation
- Intercept
- Polynomials
- x-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.F.1.1, MA.912.F.1.6(Algebra 1)
- MA.912.AR.3.7, MA.912.AR.3.8 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.C.3.3 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

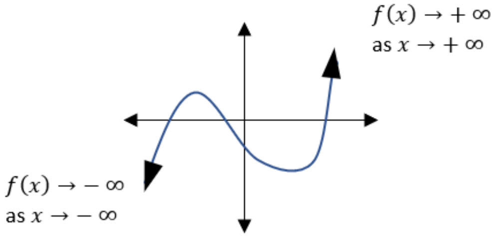
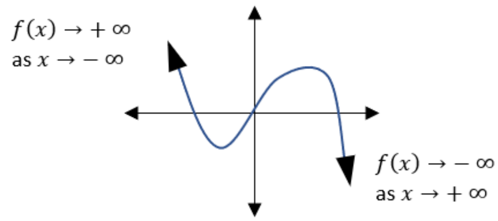
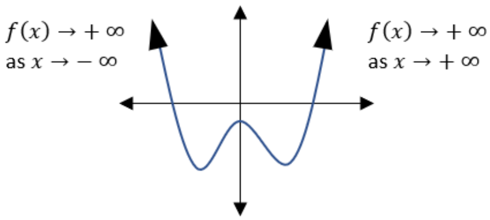
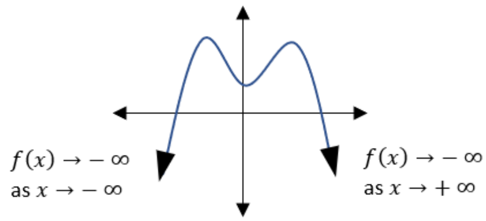
Purpose and Instructional Strategies

In Algebra 1, students interpret end behavior as a key feature of quadratic functions. In Algebra 2, students will sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity, and knowledge of end behavior. In future courses, students will graph polynomial functions of degree 3 or higher given a table, equation, or written description, and interpret key features of those functions.

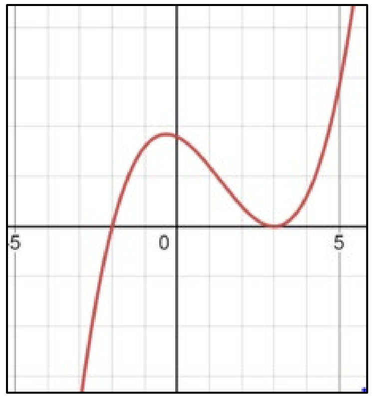
- Instruction includes teaching students how to properly read the notations for end behavior:

Math Symbol	(Name) Meaning	Example
$\rightarrow$	Approaches or goes to	As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$ As x approaches negative infinity, the function approaches negative infinity

- Instruction includes knowledge of end behavior using graphs and words.

Degree: odd **Leading coefficient:** positive  	**Degree:** odd **Leading coefficient:** negative 
Degree: even **Leading coefficient:** positive  	**Degree:** even **Leading coefficient:** negative 

- Instruction includes use of the notation with x first and with $f(x)$ first.
  - For example, end behavior could be written the way it is in the table above, or could be written in the following way: As $x \to -\infty, f(x) \to -\infty$.
- Instruction includes describing the left side of the graph using the notation $x \to -\infty$ and describing the right side of the graph using the notation $x \to +\infty$.
- Instruction includes describing the end behavior of the function using the notation $f(x) \to -\infty$ (down) and $f(x) \to +\infty$ (up). Make the connection between the direction in which the arrows point on the ends of the function.
- Instruction includes the connection between the zeros/*x*-intercepts being the points at which the function touches the x-axis.
- Students may be given the polynomial in factored form or standard form.
- Instruction includes connections to MA.912.AR.6.2 and may require students to factor to find the zeros.
- Consider having students draw the arrows at the end of the function in the correct direction before graphing the other part of the function.
- Instruction includes ensuring the polynomial function is written in standard form before determining the degree and leading coefficient.
- Instruction includes examining multiplicity in terms of the behavior at the zero. Instruction should make connections that the degree of the factor represents the type of function behavior that will be observed at that zero.
  - For example, the function $f(x) = (x-3)^2(x+2)$ has a zero at $x = 3$ with a multiplicity of 2 since the degree of that factor is 2, and a zero at $x = -2$ with a multiplicity of 1 since the degree of that factor is 1. This means that at the zero, (3,0), the graph will touch the *x*-axis without passing through, and at the zero, (-2,0), the graph will pass through the *x*-axis, as shown below.



- Consider having students identify the zeros, or critical values, and make an interval table to determine when the function is positive or negative. An example for the polynomial above is shown. These types of tables are used in future courses.

x	$(x-3)$	$(x-3)$	$(x+2)$	$f(x)$
$(-\infty, -2)$	$-$	$-$	$-$	$-$
-2				0
$(-2, 3)$	$-$	$-$	$+$	$+$
3				0
$(3, \infty)$	$+$	$+$	$+$	$+$

Common Misconceptions or Errors

- Students may confuse the notation for end behavior by thinking the $x \rightarrow$ describes the direction of the arrows on the end of the function.
- Students may read the graph of the function left to right, rather than looking at each side of the graph and determine the direction in which the function is going toward each end.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1, MTR.4.1)

Part A. Use zeros, multiplicity and end behavior to sketch the graph of the polynomial function $f(x) = -(x-2)^2(x+1)(x-1)(x-5)$. Justify why you sketched the graph the way you did.

Part B. Compare your graph to your partners. How are they the same? How are they different?

Instructional Items

Instructional Item 1

Sketch the graph of $f(x) = (x+3)^2(x+1)(x-2)$.

Instructional Item 2

Use knowledge of end behavior and the y-intercept to show that the function $f(x) = -3x^4 + 5x^3 - 2x^2 + 8x + 2$ has at least two zeros.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.7 *Solve and graph radical equations and functions in one and two variables.*

MA.912.AR.7.1

Benchmark

MA.912.AR.7.1	Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.3, MA.912.NSO.1.5
- MA.912.AR.3.2
- MA.912.AR.5.2
- MA.912.AR.6.1
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Equation

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.4 (Algebra 1)
- MA.912.AR.2.1 (Algebra 1)
- MA.912.AR.3.1 (Algebra 1)
- MA.912.AR.4.1 (Algebra 1)

Next Benchmarks

- MA.912.AR.7.4 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved one-variable linear, quadratic and absolute value equations. In Algebra 2, students solve one-variable radical equations, identify any extraneous solutions, and interpret the solutions as viable or non-viable in context. In later courses, students will solve and graph mathematical and real-world problems modeled with radical functions.

- A radical equation is an equation having a radical expression where the variable is in the radicand. Radical equations may also be written using variables with rational exponents.
- The focus for Algebra 2 is square root and cube root equations, as noted in MA.912.AR.7.2 and AR.7.3.
- Students have been working with the Laws of Exponents since middle school and will now apply these rules to solving radical equations. Students can experiment with equivalent numerical expressions to verify that raising both sides of an equation to the same power will produce an equivalent equation.
- Students will need to understand the inverse relationship between radicals and exponents as well as how to convert between radical form and rational exponent form. *(MTR.2.1)*
- Students should feel comfortable finding and representing the solutions of a radical equation algebraically, graphically and numerically (with a table). *(MTR.2.1)*

- Instruction includes simple equations as well as those having variables on both sides of the equal sign.
  - For simple radical equations, students will use their strategies for solving equations to isolate the radical with the variable on one side of the equation. Then, students should use their knowledge of the Laws of Exponents to remove this radical, by raising both sides of the equation to the power of the index.
  - When multiple radicals are in an equation, students must isolate one radical before raising both sides of the equation to the power of its index. Then proceed to isolating the next radical and use the same procedure.
- Working with numerical expressions can also be used to address extraneous solutions, which are solutions that arise during the process of solving a radical equation but are not in fact viable solutions to that equation.
  - For example, beginning with a false statement such as $3 = -3$ and then squaring both sides of the equation to produce $9 = 9$, can demonstrate non-viable solutions. *(MTR.6.1)*
- Instruction includes exploring radicals with both even and odd indices to determine when extraneous solutions may arise.

Common Misconceptions or Errors

- Students may incorrectly square or cube only one term when they should be squaring or cubing both sides of an equation.
- Students may struggle to solve quadratic equations that arise within the process of solving a radical equation.
- Students may neglect to check their solutions to determine if they are viable.
- Students may incorrectly assume all radicals are square roots and square both sides of the radical equation rather than using the index as a reference.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.6.1)

Part A. Find the solution(s) of the equation $\sqrt{(4x+1)} + \sqrt{(x-2)} = 9$.

Part B. Does the equation have any extraneous solutions? If so, name those solutions.

Instructional Task 2 (MTR.4.1, MTR.7.1)

The speed (s) of a tsunami can be calculated by taking the square root of the product of the acceleration of gravity (g), in meters per square second (m/s^2) and the depth of the water (d) in meters.

$$s = \sqrt{g \cdot d}$$

Part A. What information is needed to determine the depth of the water where the speed of a tsunami is traveling at 475 miles per hour (mph)? Research additional information if not given.

Part B: Compare your research with a partner. Do you agree? If not, find another partner pair to discuss and come to a consensus.

Part C. Calculate the depth of the water where the speed of a tsunami is travelling at 475 miles per hour (mph).

Instructional Items

Instructional Item 1

Solve the equation $\sqrt[3]{(2m-7)} = 5$ for m.

Instructional Item 2

Solve the equation $\sqrt{(x+5)} = x - 1$ for x.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.7.2

Benchmark

MA.912.AR.7.2 Given a table, equation or written description of a square root or cube root function, graph that function and determine its key features.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and relative maximums and minimums.

Clarification 2: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.5
- MA.912.AR.5.8
- MA.912.AR.6.5
- MA.912.AR.8.2
- MA.912.F.2.2, MA.912.F.2.3, MA.912.F.2.5
- MA.912.F.3.6

Terms from the K-12 Glossary

- Domain
- End Behavior
- Equation
- Function
- Intercept
- Range (of a Relation or Function)
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.4 (Algebra 1)
- MA.912.AR.2.4 (Algebra 1)
- MA.912.AR.3.7 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.4 (Precalculus)
- MA.912.AR.7.4 (Precalculus)
- MA.912.F.3.8, MA.912.F.3.9

- MA.912.AR.4.3 (Algebra 1)
- MA.912.AR.5.6 (Algebra 1)
- MA.912.F.2.1 (Algebra 1)

(Precalculus)

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students graphed linear, quadratic, absolute value and exponential functions, along with determining their key features. In Algebra 2, students will expand on this work with square root and cube root functions. In later courses, students will solve and graph mathematical and real-world problems modeled with radical functions.

- Instruction should include connections to inverse functions, such as square root functions to quadratic functions with a restricted domain and connecting cube root functions with cubic functions. Compare the domains and ranges for students to make connections between these types of parent functions.
- Set the value under the square root greater than or equal to zero in order to determine the domain of a square root function. This will help students determine what values could be used when creating a table.
- Students are encouraged to use a graphing utility when exploring graphs of square root and cube root functions. The graphs of square root functions should assist with students' reasoning about their domain and range.
    - Instruction includes graphing square root and cube root functions with transformations.
- Build on students' knowledge of key features of linear, quadratic, absolute value and exponential functions. Prior to graphing the functions, have students make predictions about where square root and cube root functions are increasing, decreasing, positive or negative, as well as predicting the possible end behavior.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
    - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, $\cup$, for union.
        - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
    - Instruction includes making connections between inequality notation and interval notation.
        - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.

- For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
- For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} 1∈ M 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

Common Misconceptions or Errors

- Students may incorrectly restrict the domain of a cubic function, or not restrict the domain of a quadratic function, to determine the key features of a cube/square root function. To address this misconception, remind students of what determines a relation as a function or not.
- When determining increasing or decreasing intervals, students may accidentally use the y-value, instead of the x-value.
  When determining increasing or decreasing intervals, students may incorrectly include the turning point in their interval.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1)

Part A. Create a table of values for each of the following equations: $y = \sqrt[3]{x}$, $y = \sqrt[3]{(x-4)}$, and $y = \sqrt[3]{(8x)}$.
Part B. Graph each of the functions listed in Part A.
Part C. List the domain and range for each of the functions in Part A.
Part D. Describe any intercepts, relative maximums or relative minimums, and end behavior of the functions in Part A.

Part E. Describe in a simple way how these graphs are related to each other.

Instructional Items

Instructional Item 1

Graph $\sqrt[3]{x-4}$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.7.3

Benchmark

MA.912.AR.7.3 Solve and graph mathematical and real-world problems that are modeled with square root or cube root functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and relative maximums and minimums.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.5
- MA.912.AR.1.1
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.8.3
- MA.912.F.1.1, MA.912.F.1.7
- MA.912.F.3.6

Terms from the K-12 Glossary

- Domain
- End Behavior
- Function
- Intercept
- Range (of a Relation or Function)
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.912.NSO.1.4 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.6 (Precalculus)

- MA.912.AR.1.1 (Algebra 1)
- MA.912.AR.2.5 (Algebra 1)
- MA.912.AR.3.8 (Algebra 1)
- MA.912.AR.4.3 (Algebra 1)
- MA.912.F.1.1 (Algebra 1)
- MA.912.F.1.6 (Algebra 1)
- MA.912.AR.7.4 (Precalculus)
- MA.912.F.3.8, MA.912.F.3.9 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1 students solved and graphed mathematical and real-world problems modeled with linear, quadratic and absolute value functions, including interpreting key features. In Algebra 2, students will solve and graph mathematical and real-world problems modeled with square root or cube root functions, including interpreting their key features. In later courses, students will build on their work with square root and cube root functions to include radical functions where the radical index is not two or three.

- When making connections to transformations of functions, use graphing software to explore cube and square root functions applying transformations to see the effects on the graph. Encourage students to make predictions. *(MTR.4.1)*
- Instruction should focus on a variety of real-world and mathematical contexts. When using real-world problems, students are expected to interpret the key features in context. *(MTR.7.1)*
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
    - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.
        - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
    - Instruction includes making connections between inequality notation and interval notation.
        - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
        - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
        - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one

of the end values.

- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x\mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x\mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3)\cup(4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1\in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x\in\mathbb{R}$ x is an element of the set of all real numbers

Common Misconceptions or Errors

- Students may not understand how to determine if a solution is extraneous.
- Students may not completely understand the relationships between roots and exponents so they may use the wrong root or exponent to isolate the variable when solving the equation.
- Students may not understand how to use a graph or table to justify what they have found algebraically.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.7.1)

When designing a road, the radius of a curve is important to the safety of travelers in cars. The radius of the curve helps determine the maximum velocity that cars may travel while on the curve. The equation $v=\sqrt{2.5r}$ can be used to determine the maximum velocity a car can safely travel on an unbanked curve, where the radius, r, is measured in feet and velocity, v, is measured in miles per hour.

Part A. What is the maximum velocity a car can safely travel if the radius of the unbanked curve is 1440 feet?

Part B. What is the radius of the turn if the maximum velocity the car can travel is 50 mph?

Part C. Identify the key features of this function and interpret them in context.

Instructional Task 2 (MTR.3.1, MTR.4.1, MTR.6.1)

Keyla and Maxwell solved the following radical equation. When Keyla checked her work, was she correct in concluding she had found the solution? When Maxwell checked his work, was he correct in concluding that there was no solution? Why or why not?

Keyla

$\sqrt{5-x} = \sqrt{x+11}$

$\left(\sqrt{5-x}\right)^2 = \left(\sqrt{x+11}\right)^2$

$5-x = x+11$

$-2x = 6$

$x = -3$

Check:

$\sqrt{5-(-3)} = \sqrt{-3+11}$

$\sqrt{8} = \sqrt{8}$

Correct!

Maxwell

$\sqrt{5-x} = \sqrt{x+11}$

$\left(\sqrt{5-x}\right)^2 = \left(\sqrt{x+11}\right)^2$

$5-x = x+11$

$2x = 6$

$x = 3$

Check:

$\sqrt{5-3} = \sqrt{3+11}$

$\sqrt{2} \neq \sqrt{14}$

No solution.

Instructional Items

Instructional Item 1

Solve the equation $\sqrt{x+6} = \sqrt{2} + \sqrt{x}$.

Instructional Item 2

Daniel and Clair are sailing on a sailboat. They find the hull speed to be 10 nautical miles per hour. What is the length of the sailboat's waterline given the formula $h = 1.34\sqrt{l}$, where h is the hull speed (the fastest speed that a sailboat can travel) and l is the waterline length (the length of the line made by the water's edge when the boat is full)?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.8 *Solve and graph rational equations and functions in one and two variables.*

MA.912.AR.8.1

Benchmark

MA.912.AR.8.1 Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

Benchmark Clarifications:
Clarification 1: Within the Algebra 2 course, numerators and denominators are limited to linear and quadratic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.3, MA.912.AR.1.5, MA.912.AR.1.6, MA.912.AR.1.9
- MA.912.AR.3.2
- MA.912.AR.5.2
- MA.912.AR.6.1
- MA.912.AR.7.1

Terms from the K-12 Glossary

- Equation
- Least Common Multiple
- Linear Expression
- Quadratic Expression
- Rational Expression

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.3, MA.912.AR.1.4(Algebra 1)
- MA.912.AR.2.1 (Algebra 1)
- MA.912.AR.3.1 (Algebra 1)
- MA.912.AR.4.1 (Algebra 1)

Next Benchmarks

- MA.912.AR.8.3 (Precalculus)
- MA.912.C.1.3 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved one-variable linear, quadratic and absolute value equations. In Algebra 2, students solve one-variable rational equations, limited to linear and quadratic expressions, identify any extraneous solutions, and interpret the solutions as viable or non-viable in context. In other courses, students will solve one-variable rational equations without limitations on the types of expressions.

- Instruction should include connections to operations with fractions when solving rational equations.
    - For example, when solving rational equations that involve addition or subtraction,

students can find common denominators to assist in solving. Given the equation $\frac{1}{(x-2)}+\frac{1}{(x+3)}=\frac{1}{x}$, students can determine a common denominator of $(x-2)(x+3)$ to rewrite the equation as $\frac{(x+2)}{(x-2)(x+3)}+\frac{(x-2)}{(x-2)(x+3)}=\frac{1}{x}$. Students should recognize that since the fractions on the left side of the equation have the same denominator, the equation can be rewritten as $\frac{2x+1}{(x-2)(x+3)}=\frac{1}{x}$. From here students can either solve by multiplying both sides of the equation by one or both denominators, or can find common denominators between fractions on either side of the equal sign and then set the numerators equivalent to one another.
  - Using the equivalent factored form of a quadratic expression is helpful in determining the least common denominator.
- Using tables to organize information may be beneficial when writing one-variable rational equations.
  - For example, a person has 2 routes to choose from when commuting to work. Route A averages 40 miles per hour (mph) while Route B averages 50 mph. If both routes are the same distance, but Route B saves 6 minutes driving time, how long is this person's commute to work?

Route	Rate (mph)	Time (hours)	Distance (miles)
Route A	40	$\frac{x}{40}$	x
Route B	50	$\frac{x}{50}$	x

- Instruction includes how to determine what the solutions to the equation are and whether they are viable. *(MTR.6.1)* Students should have experience with rational equations that produce different types of solutions: more than one, exactly one and extraneous.
- Instruction provides opportunities for students to discuss why extraneous solutions may arise with rational equations. *(MTR.4.1)*
- Instruction gives students the opportunity to discuss constraints and what effect those constraints have on the solution(s) to the equations. *(MTR.4.1)*

Common Misconceptions or Errors

- Students may struggle to determine if a common denominator is needed to solve a rational equation.
- If students determine a common denominator is needed, they may struggle to decide what factors are needed to be multiplied to the numerator and the denominator while keeping the rational equation equivalent.
- Students may forget to test the solutions in the original equation to determine if it is extraneous or not.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Phu and Sandrine are helping to mow a neighbor's lawn. If Phu worked alone, it would take him 12 hours. If they work together, it would only take 4 hours.

Part A. Write an equation that can be used to determine how long it would take Sandrine to mow the lawn alone. Be sure to define the variable.
Part B. Solve the equation from Part A.
Part C. Compare your equation and answer with a partner. Compare any similarities and differences.
Part D. Work with your partner to find another method for determining how long it would take Sandrine to mow the lawn alone.

Instructional Task 2 (MTR.5.1)
Create a rational equation with the following:

a. One solution
b. Two solutions
c. Infinitely many solutions
d. An extraneous solution

Instructional Items

Instructional Item 1
Lenni bought 11 pounds of deli meat, turkey and ham, to make sandwiches for the beach cleanup volunteers. He spent \$83.94 for the turkey and \$53.95 for the ham, and the cost per pound of turkey is \$3.20 more than that of ham. Write and solve an equation that can be used to determine how many pounds of each deli meat Lenni bought.

Instructional Item 2
Determine the solution(s) of the equation $\frac{4x-16}{x^2+2x-8} = \frac{8}{2x+8}$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.8.2

Benchmark

MA.912.AR.8.2	Given a table, equation or written description of a rational function, graph that function and determine its key features.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
Clarification 2: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
Clarification 3: Within the Algebra 2 course, numerators and denominators are limited to linear and quadratic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.5, MA.912.AR.1.9
- MA.912.AR.5.8

- MA.912.AR.6.5
- MA.912.AR.7.2
- MA.912.F.2.2, MA.912.F.2.3, MA.912.F.2.5

Terms from the K-12 Glossary

- Domain
- End Behavior
- Equation
- Function
- Intercept
- Linear Expression
- Quadratic Expression
- Range (of a Relation or Function)
- Rational Expression
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.4 (Algebra 1)
- MA.912.AR.3.7 (Algebra 1)
- MA.912.AR.4.3 (Algebra 1)
- MA.912.AR.5.6 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.4 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students graphed linear, quadratic, absolute value and exponential functions, along with determining their key features. In Algebra 2, students will expand on this work with rational functions. In future courses, students will graph rational functions and determine key features with no limitations on the expressions in the numerator and denominator.

- Begin with a basic parent function $f(x) = \frac{1}{x}$, including table of values, to start graphing as well as to discuss domain and range. Ensure students identify the pattern of the *y*-values as the *x*-values increase.
- Students will use rules for transformations to graph rational functions.
- Students are encouraged to use a graphing utility when exploring graphs of rational functions and their key features.
- Instruction includes discussion around attempting to calculate the *x*- and *y*-intercepts.
    - Ask students what values of *x* would give the function a value of 0. A rational function will be 0 only when the numerator has a value of 0, which will not occur for the parent function, but will for other rational functions.
- Build on students' understanding of asymptotes from working with exponential functions in Algebra 1. Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.

- Instruction includes distinguishing between vertical, horizontal and oblique (or slant) asymptotes, and how to determine each for a rational function, as well as identifying any holes in the graph of the function.
    - Explore graphically a variety of rational function to discern patterns and reasoning for these occurrences.
        - For example, graph $f(x) = \frac{1}{x-3}$ to show a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$. Next, graph $f(x) = \frac{x}{x-3}$ and $f(x) = \frac{3x}{x-3}$ to show the changes in the horizontal asymptote. From here, students can make conjectures about how to calculate the horizontal asymptote.
    - Graph $f(x) = \frac{1}{x^2-9}$ to show multiple vertical asymptotes at $x = 3$ and $x = -3$, and the horizontal asymptote remains at $y = 0$. Then graph $f(x) = \frac{x-3}{x^2-9}$ to illustrate the removal of one asymptote, but ask students to determine if $x = 3$ is still a viable input for the function. In doing so, students should see when holes arise in the graph of a rational function.
    - Finally, graph $f(x) = \frac{x^2}{x-3}$ to illustrate an oblique asymptote. Incorporate instruction of MA.912.AR.1.5 when appropriate for calculating an oblique asymptote.
    - Additional functions can be explored graphically to further solidify student conjectures about determining horizontal and oblique asymptotes. A graphic organizer or other tool may be useful once the three cases of comparing the degree of the numerator and denominator have been fully explored.
- When graphing by hand, ensure students identify several coordinate points on the graph of the rational function for each segment of the domain separated by vertical asymptotes. This will assist in determining the shape of each piece of the function.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
    - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, $\cup$, for union.
        - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
    - Instruction includes making connections between inequality notation and interval notation.
        - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.

    - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
    - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

Common Misconceptions or Errors

- Students may use graphing technology incorrectly, such as neglecting to use parentheses around expressions for the numerator or denominator.
- Students may struggle with identifying asymptotes, especially when they are not represented by an axis.
- Students may reverse the horizontal and vertical asymptotes.
- Students may confuse the conditions for calculating horizontal and oblique asymptotes.
- Students may not be able to identify the degree of the numerator or denominator of a rational function.
- When simplifying a rational expression, students may divide individual terms rather than the entire factor.
- If students determine a common denominator is needed, they may struggle to decide what factors need to be multiplied to the numerator and the denominator while keeping the rational equation equivalent.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1)

Use technology to graph the function $f(x) = \frac{-x^2}{x-2}$. Sketch the graph and then answer the questions that follow.

Part A. Are there any intercepts? If so, describe them.
Part B. Are there any horizontal asymptotes? If so, describe them or give their equation(s).
Part C. Are there any vertical asymptotes? If so, describe them or give their equation(s).
Part D. Are there any oblique asymptotes? If so, describe them or give their equation(s).
Part E. Describe the end behavior of this function.

Instructional Items

Instructional Item 1
Graph the function $f(x) = \frac{2}{3x} - \frac{4}{x}$, naming its domain and range.

Instructional Item 2
Use the graph of the function $f(x) = \frac{1}{x^2-x}$ to identify its end behavior, asymptotes, and intervals where the function is increasing or decreasing.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.8.3

Benchmark

MA.912.AR.8.3	Solve and graph mathematical and real-world problems that are modeled with rational functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 3: Instruction includes using rational functions to represent inverse proportional relationships.
Clarification 4: Within the Algebra 2 course, numerators and denominators are limited to linear and quadratic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.6, MA.912.AR.1.9
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.7.3
- MA.912.F.1.1, MA.912.F.1.7

Terms from the K-12 Glossary

- Domain

- End Behavior
- Equation
- Function
- Intercept
- Least Common Multiple
- Proportional Relationships
- Range (of a Relation or Function)
- Rational Expression
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.1.1 (Algebra 1)
- MA.912.AR.2.5 (Algebra 1)
- MA.912.AR.3.8 (Algebra 1)
- MA.912.F.1.1 (Algebra 1)
- MA.912.F.1.6 (Algebra 1)

Next Benchmarks

- MA.912.AR.6.6 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved and graphed mathematical and real-world problems modeled with linear functions, including interpreting key features. In Algebra 2, students will solve and graph mathematical and real-world problems modeled with rational functions, including interpreting their key features. In later courses, students will solve and graph real-world and mathematical problems where there are no limitations on the expressions in the numerator and denominator.

- Instruction should focus on a variety of real-world and mathematical contexts. When using real-world problems, students are expected to interpret the key features in context.
- When making connections to transformations of functions, use graphing software to explore rational functions applying transformations to see the effects on the graph. Encourage students to make predictions. *(MTR.4.1)*
- Real-world contexts include inverse proportional relationships such as increased speed and decreased time to travel a certain distance, increased number of workers and the time it takes to complete a task, brightness of a light and a person's distance from that light, the freshness of a vegetable over time, spending and saving, etc.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.
    - Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, $\cup$, for union.

        - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
    - Instruction includes making connections between inequality notation and interval notation.
        - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
        - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
        - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- In Algebra 2, students are introduced to new mathematical symbols. Consider having a place where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the oepn interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

Common Misconceptions or Errors

- Students may struggle to determine if a common denominator is needed to solve a rational equation.
- If students determine a common denominator is needed, they may struggle to decide what factors need to be multiplied to the numerator and the denominator while keeping the rational equation equivalent.
- Students may forget to test the solutions in the original equation to determine if it is extraneous or not. To address this misconception, consider having students list excluded values before manipulating the equation to solve.
- Students may not check the feasibility of a solution within the context of the problem.

Instructional Tasks

Instructional Task 1(MTR.3.1, MTR.7.1)

Mabel needs to finish filling her saltwater pool. The pool currently contains 100 gallons of water that have been mixed with 2 pounds of salt. Mabel will turn on the hose to pour water into the pool at 15 gallons per minute while also pouring in salt at a half pound per minute.

Part A. How could Mabel calculate the concentration of salt in her pool?
Part B. Find the concentration, in pounds per gallon, of salt in the pool after 20 minutes.
Part C. Find the concentration, in pounds per gallon, of salt in the pool after 45 minutes.
Part D. How long will it take Mabel to fill her pool if it holds 13,500 gallons of water?
Part E. If the optimal range for salt in a saltwater pool is 2,700-3,400 ppm (parts per million), will Mabel have the correct concentration once her pool is full?

Instructional Items

Instructional Item 1

Solve the equation $\frac{3x+3}{x^2+2x+1} - \frac{4}{x+1} = \frac{7}{2x}$.

Instructional Item 2

Brandon and Jamarcus would like to start a business on the weekends. They want to do lawn mowing. Brandon's family lives on a half-acre lot and it takes him 4 hours to mow the lawn. Last month when Brandon's family went on vacation, his friend Jamarcus mowed the lawn, and it took him 3 hours to finish the job. They would like to use the amount of time it would take them working together as a guideline to figure out how many lawns they could complete in a day. How long would it take Brandon and Jamarcus working together to mow the lawn?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9 *Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.*

MA.912.AR.9.2

Benchmark

MA.912.AR.9.2	Given a mathematical or real-world context, solve a system consisting of a two-variable linear equation and a non-linear equation algebraically or graphically.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.4.2
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.7.3
- MA.912.AR.8.3

Terms from the K-12 Glossary

- Equation
- Linear Equation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.5 (Algebra 1)
- MA.912.AR.9.1 (Algebra 1)
- MA.912.F.1.6 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved systems of two linear equations. In Algebra 2, students will solve a system of equations where one is linear and one is non-linear, specifically the non-linear functions of this course. In other mathematics courses, students will solve systems of non-linear equations that may be any function type.

- Instruction includes connections with MA.912.AR.9.3, MA.912.AR.9.5 and MA.912.AR.9.7 as equations, inequalities and their constraints are all related, and the connections between them should be reinforced.
- Instruction includes building on students' understanding of solving a system of two linear equations.
- Instruction includes opportunities to use graphing technology to visualize the possible solutions for a system of equations. Systems of linear and non-linear equations are not limited to one solution, infinite solutions, and no solutions, as they were with a system of two linear equations. Each type of system can be graphed for analysis of each type of solution set.
- Instruction allows students to solve using any method (substitution, elimination, or graphing) but recognizing that one method may be more efficient than another. *(MTR.3.1)*
- Include cases where students must interpret solutions to systems of equations.

Common Misconceptions or Errors

- Students may not be able to correctly complete a table of values as a strategy to compare two functions.
- Students may continue to look for only one solution, as is the case for the intersection of 2 non-parallel lines.
- Students may struggle with writing the equations when given a written description or real-world context.

Instructional Tasks

Instructional Task 1 (MTR.2.1,MTR.4.1)

Let $f(x) = x^3$ and $g(x) = 3x + 2$. For this task, divide students into groups. Assign the following activities to the group members:

Group Member A:

Find solutions of the equation $f(x) = g(x)$ by creating a table of values.

Group Member B:

Graph $f(x)$ and $g(x)$ on a coordinate plane. Determine the solution(s) of the equation $f(x) = g(x)$.

Group Member C:

Graph $f(x)$ and $g(x)$ using a graphing utility.

Group Member D:

Solve $f(x) = g(x)$ algebraically

Within the group:

Part A. Compare the solution(s).
Part B. Does the graph from Group Member C change which values Group Member A may have chosen for the table?
Part C. Determine which method was the most efficient for finding the solutions. Explain your reasoning.

Instructional Items

Instructional Item 1

Determine the solution(s) to the system of equations.
$y = x + 2$ and $y = x^2 - 4$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.3

Benchmark

MA.912.AR.9.3	Given a mathematical or real-world context, solve a system consisting of two-variable non-linear equations algebraically or graphically.

Benchmark Clarifications:

Clarification 1: Within the Algebra 2 course, non-linear equations are limited to quadratic equations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.7.3
- MA.912.AR.8.3
- MA.912.F.1.7

Terms from the K-12 Glossary

- Equation
- Linear Equation

- Quadratic Equation

Vertical Alignment

Previous Benchmarks
- MA.912.AR.2.5 (Algebra 1)
- MA.912.AR.9.1 (Algebra 1)
- MA.912.F.1.6 (Algebra 1)

Next Benchmarks
- MA.912.AR.9.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved systems of two linear equations. In Algebra 2, students will solve a system of quadratic equations. In other mathematics courses, students will solve systems of non-linear equations that may be any function type.
- Instruction includes connections with MA.912.AR.9.2, MA.912.AR.9.5 and MA.912.AR.9.7 as equations, inequalities and their constraints are all related, and the connections between them should be reinforced.
- Instruction builds on student work with solving systems of linear equations, graphing quadratic equations and solving quadratic equations.
- Instruction allows students to solve using any method (substitution, elimination or graphing) but recognizing that one method may be more efficient than another. *(MTR.3.1)*
- Instruction includes opportunities to use graphing technology to visualize the possible solutions for a system of quadratic equations. Systems of quadratic equations may have zero, one, two or infinite solutions, which builds on students' understanding of systems of linear equations.
- Include cases where students must interpret solutions to systems of equations.

Common Misconceptions or Errors

- Students may not be able to correctly complete a table of values as a strategy to compare two functions.
- Students may continue to look for only one solution, as is the case for the intersection of 2 non-parallel lines.
- Students may struggle with writing the equations when given a written description or real-world context.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.7.1)

The profit, P, in thousands of dollars, from the sale of laptops by Office Co. can be modeled by the function $P(x) = -3x^2 + 33x - 72$, where x is the price of laptops in hundreds of dollars.

Part A. What are the zeros of this function and what do they mean in context?

Part B. What is the maximum profit the company can make?

Part C. Graph Office Co.'s profit function on a coordinate plane, using an appropriate scale.

Part D. The function $P(x) = -4x^2 + 39x - 77$ models the profit, in thousands of dollars, from the sale of VR headsets by one of Office Co.'s competitors, where *x* is the price of VR headsets in hundreds of dollars. What are the zeros of this function and what do they mean in context?
Part E. Graph the competitor's profit function on the same coordinate plane.
Part F. At what price will both companies make the same profit? What will the profit be at this price?

Instructional Task 2 (MTR.5.1)
Write a system of quadratic equations that results in:

a. No solutions
b. One solution
c. Two solutions
d. Infinitely many solutions

Instructional Items

Instructional Item 1
Stewart jumps off a cliff 5 meters high into a lake. His position can be represented by the function $h(t) = -5t^2 + 1.5t + 5$, where *h* represents the Stewart's height relative to the lake's surface and *t* represents the time in seconds. At a lower height on the path to the cliff, Pete throws a ball into the air for Stewart to catch on his way down. The ball's position can be represented by the function $h(t) = -5t^2 + 9t + 2$, where *h* represents the ball's height relative to the lake's surface and *t* represents the time in seconds. At what height will Stewart meet the ball to catch it?

Instructional Item 2
Determine the solution(s), if any, of the given system of equations.

$$\begin{cases} y = -x^2 - 3x + 18 \\ y = x^2 + 4x - 12 \end{cases}$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.5

Benchmark

MA.912.AR.9.5 Graph the solution set of a system of two-variable inequalities.

Benchmark Clarifications:
Clarification 1: Within the Algebra 2 course, two-variable inequalities are limited to linear and quadratic.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.3, MA.912.AR.3.9, MA.912.AR.3.10
- MA.912.AR.4.2

Terms from the K-12 Glossary

- Linear Expression
- Quadratic Expression

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.6, MA.912.AR.2.7, MA.912.AR.2.8 (Algebra 1)
- MA.912.AR.4.2 (Algebra 1)
- MA.912.AR.9.4, MA.912.9.6 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students graphed the solution set of a system of two-variable linear inequalities. In Algebra 2, students graph the solution set of a system of two-variable linear or quadratic inequalities. In other courses, students will graph the solution set of any system of two-variable inequalities.

- Instruction includes building on students' understanding of solving a system of two linear inequalities. *(MTR.5.1)*
- For quadratic inequalities, students will graph the parabola using a solid (inclusive) or dashed (exclusive) curve, and shading "inside" or "outside" the curve to represent the solutions, based on a selected test point.
- Instruction includes determining whether the point(s) of intersection of the boundaries of the inequalities is (are) within the solution set.
  - For example, if one or both boundaries are dashed (< or >), then the point(s) of intersection is (are) not in the solution set.
- The solution set to a system of inequalities is all the coordinate pairs, (x, y) in the region where all the shading overlaps. If the shaded regions do not overlap, the system of inequalities has no solution.
- Instruction allows students to make connections between the algebraic and graphical representations of inequalities in two variables. *(MTR.2.1)*

Common Misconceptions or Errors

- Students may struggle to determine if the intersection point(s) of the inequalities are included or excluded from the solution set.
- Students may think that the inequality symbol's orientation always determines the side of the line or curve to shade.
  - For example, students may say that inequalities with a less than symbol should be shaded below the line while inequalities with a greater than symbol should be shaded above the line. To address this, provides counterexamples to this such as $3x - 2y < 15 \text{ or } -4x - 7 \geq y$. Use these counterexamples to emphasize the benefit of using a test point to confirm the direction of shading.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Add a second inequality to create a system of inequalities to produce the identified solution set.

Part A. One solution

$$\begin{cases} y \geq 3x^2 - 6x + 4 \\ ? \end{cases}$$

Part B. The greatest y-value of the solution set is 5 and the least x-value is 1.

$$\begin{cases} y \geq 3x^2 - 6x + 4 \\ ? \end{cases}$$

Part C. Compare your answers with a classmate.

Instructional Items

Instructional Item 1

Graph the solution set of the system of inequalities given.

$$\begin{cases} y \geq 4x^2 - 3x + 2 \\ y < -3x + 5 \end{cases}$$

Instructional Item 2

Graph the solution set of the system of inequalities given.

$$\begin{cases} y > x^2 - 2x + 1 \\ y < -2x^2 + 5x + 3 \end{cases}$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.7

Benchmark

MA.912.AR.9.7	Given a real-world context, represent constraints as systems of linear and non-linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.

Benchmark Clarifications:

Clarification 1: Instruction focuses on analyzing a given function that models a real-world situation and writing constraints that are represented as non-linear equations or non-linear inequalities.

Clarification 2: Within the Algebra 2 course, non-linear equations and inequalities are limited to quadratic.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.3, MA.912.AR.3.9, MA.912.AR.3.10
- MA.912.AR.4.2

Terms from the K-12 Glossary

- Equation
- Linear Equation
- Quadratic Equation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.2 (Algebra 1)
- MA.912.AR.3.4 (Algebra 1)
- MA.912.AR.5.4 (Algebra 1)
- MA.912.AR.9.6 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students represented constraints as systems of linear equations or inequalities and interpreted solutions as viable or non-viable options. In Algebra 2, students represent constraints as systems of linear and quadratic equations or inequalities and interpret solutions as viable or non-viable options. In later courses, students will solve problems involving linear programming and work with constraints within various function types.

- Instruction includes connections with MA.912.AR.9.2, MA.912.AR.9.3 and MA.912.AR.9.5 as equations, inequalities and their constraints are all related, and the connections between them should be reinforced.
- When specific contexts are modeled by linear or quadratic functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Allow for both inequalities and equations as constraints. Include cases where students must determine a valid model of a function.
    - Students often use inequalities to represent constraints throughout Algebra 1 and Algebra 2. Equations can be thought of as constraints as well. Solving a system of equations requires students to find a point or pair of points that are constrained to lie on specific lines or curves simultaneously.
- Instruction includes graphing multiple inequalities and identifying points that bound the region made by the solution set. Instruction also includes maximizing or minimizing a quantity that depends on x and y, where x and y satisfy the inequalities.
- Instruction includes the use of graphing utilities to help students visualize the solution to the system of equations/inequalities. *(MTR.7.1)*

Common Misconceptions or Errors

- Students may have difficulty translating word problems into systems of equations and inequalities.
- Students may shade inequality solutions incorrectly on a graph.
- Students may graph an incorrect boundary line or curve (dashed versus solid) due to incorrect translation of the word problem.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation.

Instructional Tasks

Instructional Task 1 (MTR.6.1, MTR.7.1)

The profit, P, in thousands of dollars, from the sale of laptops by Office Co. can be modeled by the function $P(x) = -3x^2 + 33x - 72$, where x is the price of laptops in hundreds of dollars.

Part A. The CEO of Office Co. states that their profits must be at least \$500. Represent this constraint as an inequality.
Part B. The CFO of Office Co. believes consumers will not buy a laptop for more than \$650. Represent this constraint as an inequality.
Part C. Name all other constraints that must be used based on the context of this scenario.
Part D. Determine 3 possible prices Office Co. can sell their laptops based on the requirements from the CEO and CFO. Explain your reasoning.
Part E. Is there a price that will meet the CEO's requirement, but falls outside the beliefs of the CFO?

Instructional Items

Instructional Item 1

Stewart jumps off a cliff 5 meters high into a lake. His position can be represented by the function $h(t) = -5t^2 + 1.5t + 5$, where h represents the Stewart's height in meters relative to the lake's surface and t represents the time in seconds. He must complete a flip before he reaches 1 meter from the lake's surface. It takes him at least 1 second to complete the flip. What are some possible times and heights at which he can complete the flip?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Functions

MA.912.F.1 *Understand, compare and analyze properties of functions.*

MA.912.F.1.1

Benchmark

MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, functions represented as tables are limited to linear, quadratic and exponential.
Clarification 2: Within the Algebra 1 course, functions represented as equations or graphs are limited to vertical or horizontal translations or reflections over the x-axis of the following parent functions:
$f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = \sqrt{x}$, $f(x) = \sqrt[3]{x}$, $f(x) = |x|$, $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.4
- MA.912.AR.5.4
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Equation
- Exponential function
- Function
- Linear Function
- Quadratic Function
- Refection
- Translation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.2, MA.912.AR.2.3 (Algebra 1)
- MA.912.AR.5.4 (Algebra 1)

Next Benchmarks

- MA.912.T.3 (Precalculus)
- MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students classified function types limited to simple linear, quadratic, cubic, square root, cube root, absolute value and exponential functions. In Algebra 2, students classify additional function types such as rational functions. In later courses, students will classify other function types outside the scope of this course.

- The purpose of this benchmark is to lay the groundwork for students to be able to choose appropriate functions to model real-world data.
- Instruction includes the connection of the graph to its parent function. As new function types are introduced, take time to allow students to produce a rough graph of the parent function from a table of values they develop. Lead student discussion to build connections with why these function types produce their corresponding graphs. *(MTR.4.1)*
- Instruction develops the understanding that if given a table of values, unless stated, one cannot absolutely determine the function type, but state which function the table of values could represent.
    - For example, if given the function $y = |x|$ and only positive values were given in a table, one could say that table of values could represent a linear or absolute value function.
- Instruction includes opportunities to use graphing technology to graph functions and relate them to the graphs of their parent functions and determine their key features.

Common Misconceptions or Errors

- Some students may miscalculate the first and second differences that deal with negative values, especially if they perform them mentally. In these cases, have students quickly write out the subtraction expression (i.e., $-14 - (-2)$) so they can see that they are subtracting a negative value and should convert it to adding a positive value.

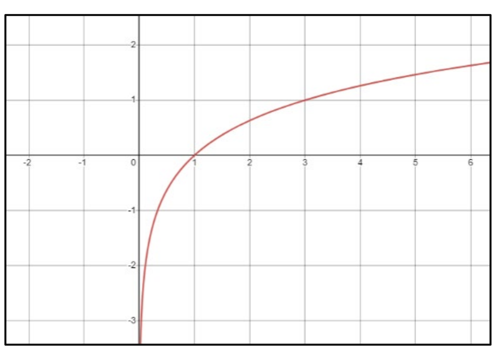
Instructional Tasks

Instructional Task 1 (MTR.3.1)

Determine the function types of the following.

Part A. $f(x) = 3^{x-2}$

Part B.



Part C. $f(x) = \sqrt{x-7}$

Part D.

x	***f(x)***

2	–4
3	–9
4	–16
5	–25
6	–36
7	–49

Part E. $f(x) = \frac{x}{x+4}$

Instructional Items

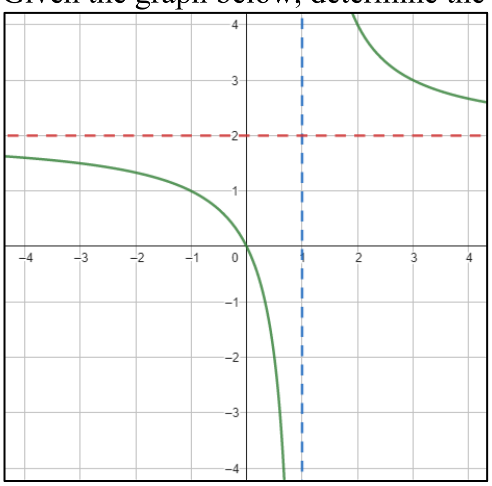
Instructional Item 1

Given the table below, determine the function type that could represent it.

X	6	8	10	12	14
Y	–1.5	0	2.5	6	10.5

Instructional Item 2

Given the graph below, determine the function type.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.7

Benchmark

MA.912.F.1.7 Compare key features of two functions each represented algebraically, graphically, in tables or written descriptions.

Benchmark Clarifications:

Clarification 1: Key features include domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior and asymptotes.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.7.2, MA.912.AR.7.3
- MA.912.AR.8.2, MA.912.AR.8.3

Terms from the K-12 Glossary

- Domain
- End Behavior
- Function
- Intercept
- Range (of a relation or function)

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.4, MA.912.2.5 (Algebra 1)
- MA.912.AR.3.7, MA.912.3.8 (Algebra 1)
- MA.912.AR.4.3 (Algebra 1)
- MA.912.AR.5.6 (Algebra 1)
- MA.912.F.1.5 (Algebra 1)
- MA.912.F.1.8 (Algebra 1)

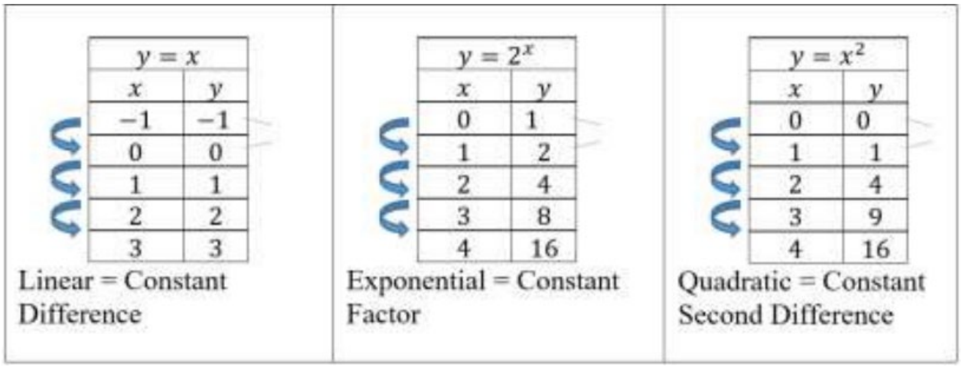
Next Benchmarks

- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students compared key features of two or more linear or nonlinear functions, with the focus of nonlinear being quadratic and exponential functions. In Algebra 2, students compare key features of two functions, including linear, quadratic, exponential, absolute value, square root, cube root, rational and logarithmic functions. In later courses, students will compare key features of any nonlinear functions represented graphically, algebraically or with written descriptions.

- Instruction includes exploring parent functions to begin comparisons of various functions and their key features.
- Instruction includes opportunities to use graphing software for exploring key features when comparing functions.
- Given a table, students will look at the change of y over the change of x, to distinguish between linear and nonlinear functions. If it's a linear function, it will have a constant slope, but if it does not have a constant slope, it will be a nonlinear function.



$y = x$	
x	y
−1	−1
0	0
1	1
2	2
3	3

Linear = Constant Difference

$y = 2^x$	
x	y
0	1
1	2
2	4
3	8
4	16

Exponential = Constant Factor

$y = x^2$	
x	y
0	0
1	1
2	4
3	9
4	16

Quadratic = Constant Second Difference

- Instruction includes connections with key features explored in benchmarks such as MA.912.AR.3.8, MA.912.AR.4.4, MA.912.AR.5.7, MA.912.AR.5.9, MA.912.AR.7.3 and MA.912.AR.8.3.

Common Misconceptions or Errors

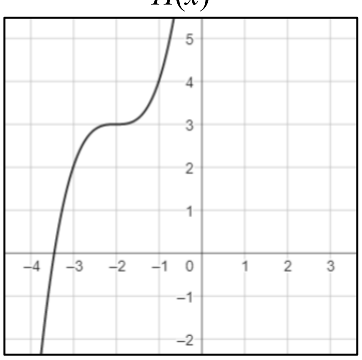
- When describing domain or range, students may assign intervals to the incorrect variable. To address this misconception, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may also miss the need for compound inequalities when describing domain or range. To address this misconception, use a graph of the function to point out areas where the function is defined.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable.
- When describing intervals where functions are increasing or decreasing, students may incorrectly include the turning point.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Three functions are given below.

$H(x)$



$G(x)$

$$y = 2\sqrt{x+1} + 3$$

$F(x)$

x	y
-2	9
-1	7
0	5
1	3
2	5
3	7

Part A. Identify a function, $F(x)$, that could be represented by the table.
Part A. Describe the domain and range of each function.
Part B. Determine the intercepts of each function.

Part C. Determine the intervals where each function is increasing, decreasing, positive or negative.
Part D. Describe the end behavior of each function.
Part E. Create a list of similarities between these key features, including which are the same for two, or all three, functions.

Instructional Items

Instructional Item 1

Describe the similarities and differences of the key features of the given functions.

$y = \sqrt{x-2} - 3 \qquad y = (x-2)^2 - 3$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.9

Benchmark

MA.912.F.1.9	Determine whether a function is even, odd or neither when represented algebraically, graphically or in a table.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.7.2, MA.912.AR.7.3
- MA.912.AR.8.2, MA.912.AR.8.3

Terms from the K-12 Glossary

- Function

Vertical Alignment

Previous Benchmarks

- MA.912.F.1.2, MA.912.1.6 (Algebra 1)

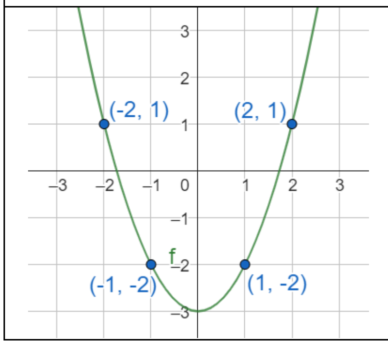
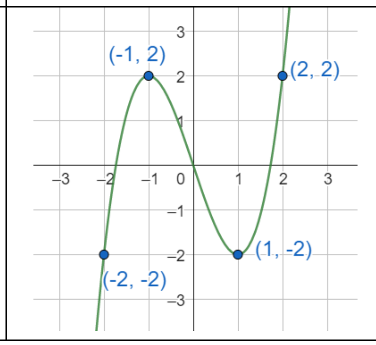
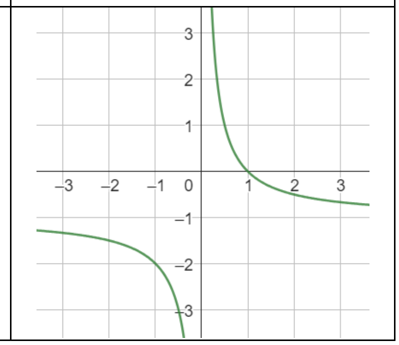
Next Benchmarks

- MA.912.AR.6.4, MA.912.AR.6.6 (Precalculus)
- MA.912.AR.7.4 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1 and Geometry, students compared linear and nonlinear functions, graphed various function types and worked with symmetry. In Algebra 2, students determine whether a function is even, odd or neither when represented algebraically, graphically or in a table. In later courses, students will use even, odd and neither when describing key features and doing comparisons of various function types.

- Instruction should connect key features of functions with determining graphically whether a function is even, odd or neither.
- Algebraically, an even function is a function such that $f(x) = f(-x)$, meaning the value of the function remains the same when the sign of the independent variable is reversed. An odd function is a function such that $-f(x) = f(-x)$, meaning the value of the function is opposite when the sign of the independent variable is reversed. If neither of these cases is true, then the function is neither even, nor odd.
- Graphically, an even function is a function whose graph is symmetrical about the y-axis. An odd function is one whose graph is symmetrical about the origin. If neither of these symmetries occurs, then the function is neither even, nor odd.

<table>
<tr><th>Even</th><th>Odd</th><th>Neither</th></tr>
<tr><td></td><td></td><td></td></tr>
</table>

- Instruction includes a variety of function types and transformations beyond the parent functions.
- Instruction includes using graphing technology to explore the symmetry of functions.

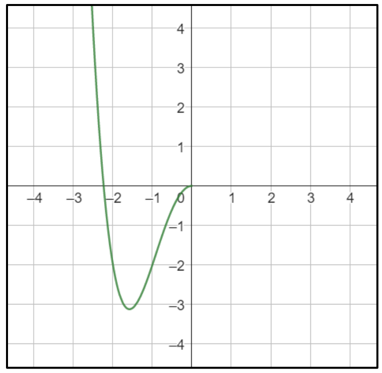
Common Misconceptions or Errors

- Students may incorrectly assume the degree of a polynomial function determines whether it is an even or odd function.
- Students may not understand what symmetry about the y-axis or symmetry about the origin means graphically.
- Students may make sign errors when substituting $-x$ into a function to determine algebraically whether it is even, odd or neither.
- Students may think even and odd refer to the end behavior.
    - For example, the function has to be even if both sides are pointing in the same direction (fallacy).

Instructional Tasks

Instructional Task 1(MTR.5.1)

Using the graph of the partial function below, complete the graph to satisfy the given requirements.



Part A. Complete the graph so the function is even. Explain your reasoning.
Part B. Complete the graph so the function is odd. Explain your reasoning.
Part C. Complete the graph so the function is neither even, nor odd. Explain your reasoning.

Instructional Items

Instructional Item 1
Determine whether each of these functions is odd, even or neither.

a. $f(x) = 3^x + 3^{-x}$
b. $h(x) = x^2 + 4x - 2$
c. $j(x) = x^3 - 4x$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2 *Identify and describe the effects of transformations on functions. Create new functions given transformations.*

MA.912.F.2.2

Benchmark

MA.912.F.2.2	Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x- or y-values or multiplying the x- or y-values by a real number.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Coordinate Plane
- Dilation
- Function Notation
- Reflection
- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.AR.2.4, MA.912.AR.2.5 (Algebra 1)
- MA.912.GR.2 (Geometry)

Next Benchmarks

- MA.912.3.2, MA.912.3.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Algebra 2, students identify effects of transformations on linear, quadratic, exponential, logarithmic, simple rational, square root, cube root, absolute value and related functions. In future courses, students will also identify transformations of trigonometric functions.

- In this benchmark, students will examine the impact of two or more transformations on the graph of a given function. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.

- Instruction includes identifying the impact of function transformations involving a combination of translations, dilations and reflections.
- When combining transformations, students will need to consider the order of the transformations.
  - Vertical Transformations affect the output value of the function.
    - Given $4f(x) - 2$, vertically shifting by 2 and then vertically stretching by 4 does not create the same graph as vertically stretching by 4 and then vertically shifting by 2. When we shift first, both the original function and the shift gets stretched, while only the original function gets stretched when we stretch first.
    - When students see an expression such as $4f(x) - 2$, we must first multiply the output value of $f(x)$ by 4, causing a vertical stretch, and then subtract 2, causing the vertical shift. Therefore, students should be reminded that multiplication comes before addition.
  - Horizontal Transformations affect the inputs of the function.
    - Given $h(x) = f(5x + 2)$, we must first think about how the inputs to this function h relate to the inputs to function f.
      - For example, if we know $f(5) = 12$, what input to h would produce that output? That means, what value of x will allow $h(x) = f(5x + 2) = 12$. We would need $5x + 2 = 5$.

Combining Vertical Transformations in form $af(x) + k$	First vertically stretch by a (and if $a < 0$, reflect across the x-axis) and then vertically shift by k
Combining Horizontal Transformations in form $f(bx + h)$	First horizontally shift by -h and then horizontally stretch by $\frac{1}{b}$ (and if $b < 0$, reflect across the y-axis)
Combining Horizontal Transformations in form $f(b(x + h))$	First horizontally stretch by $\frac{1}{b}$ (and if $b < 0$, reflect across the y-axis) and then horizontally shift by -h
Horizontal and Vertical Transformation in form $f\ (x - h) + k$ are independent	The order does not affect the transformation

Common Misconceptions or Errors

- Similar to writing functions in vertex form, students may confuse the effect of the sign of k in $f(x + k)$. To address this misconception, direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same. To address this misconception, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A function is shown.

$$f(x) = 0.23(x - 3)^2 + 5$$

Part A. What transformations occurred to the quadratic parent function $g(x) = x^2$ to produce $f(x)$?
Part B. In what order would these transformations occur?
Part C. Suppose $h(x) = f\left(\frac{1}{2}x\right) - 7$. What transformations would need to occur to transform $f(x)$ to $h(x)$?
Part D. What transformations would transform $g(x)$ to $h(x)$?

Instructional Task 2 (MTR.3.1)
Describe the transformations that map the function $p(x) = 2^x$ to each of the following.

$$f(x) = 2^x - 2$$
$$g(x) = 2^{2x} + 3$$
$$h(x) = 2^{x-3}$$
$$b(x) = 3(2)^{x+1} + 2$$

Instructional Items

Instructional Item 1
Suppose $g(x)$ is constructed by multiplying all the *y*-coordinate values for a function $f(x)$ by $\frac{1}{2}$, then adding 2 to each. Describe how the graph of $g(x)$ would change in relation to $f(x)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2.3

Benchmark

MA.912.F.2.3	Given the graph or table of $\boldsymbol{f(x)}$ and the graph or table of $\boldsymbol{f(x) + k}$, $\boldsymbol{kf(x)}$, $\boldsymbol{f(kx)}$ and $\boldsymbol{f(x + k)}$, state the type of transformation and find the value of the real number $\boldsymbol{k}$.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, functions are limited to linear, quadratic and absolute value.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Coordinate Plane
- Dilation
- Function Notation
- Origin

- Reflection
- Rigid Transformation
- Scale Factor
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.AR.2.4, MA.912.AR.2.5 (Algebra 1)
- MA.912.GR.2 (Geometry)

Next Benchmarks

- MA.912.3.2, MA.912.3.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

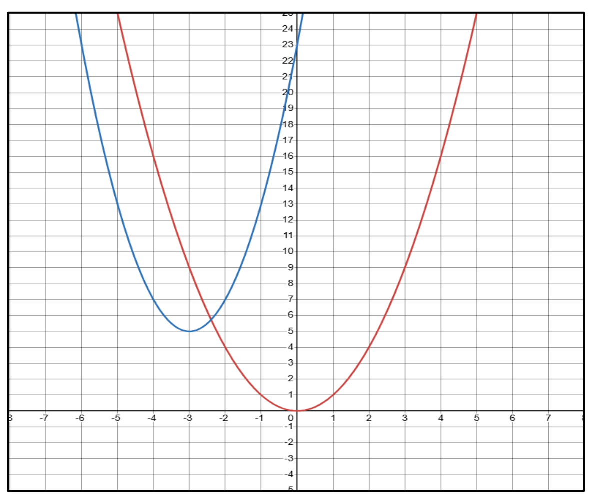
Purpose and Instructional Strategies

In Algebra 1, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Algebra 2, students determine the type of transformations on linear, quadratic, exponential, logarithmic, simple rational, square root, cube root and absolute value functions. In future courses, students will also identify transformations of trigonometric functions.

- Transformations can be either horizontal (changes to the input: x) or vertical (changes to the output: $f(x)$).
- In this benchmark, there are three different types of transformations: translations, dilations and reflections.
    - By combining single transformations, a parent function can become a more advanced function.

Example: $f(x) = x^2 \rightarrow g(x) = 2(x + 3)^2 + 5$

x	$f(x) = x^2$	$g(x) = 2(x + 3)^2 + 5$
-2	4	7
-1	1	13
0	0	23
1	1	37
2	4	55



- Using a graphing utility can help students accurately and efficiently understand how changing the value of the real numbers in the function's equation changes its graph.

- Encourage students' discussion about the effects of changing the value of the real number k in each of the expressions shown in the benchmark. Ask them to generalize their findings.

Translations	$x \to x + k$	Shifts horizontally to the left k units
	$x \to x - k$	Shifts horizontally to the right k units
	$f(x) \to f(x) + k$	Shifts vertically up k units
	$f(x) \to f(x) - k$	Shifts vertically down k units
Dilations	$x \to kx,\ x > 1$	Dilates horizontally with a scale factor of $\frac{1}{k}$ (Compress horizontally) *Important to note that if using the terms **compressing** horizontally, it would be **by a factor of** k since compressing already implies getting smaller.
	$x \to kx,\ 0 < x < 1$	Dilates horizontally with a scale factor of k (Stretch horizontally)
	$f(x) \to kf(x),\ x > 1$	Dilates vertically with a scale factor of k (Stretch vertically)
	$f(x) \to kf(x),\ 0 < x < 1$	Dilates vertically with a scale factor of $\frac{1}{k}$ (Compress vertically) *Important to note that if using the terms **compressing** vertically, it would be **by a factor of** k since compressing already implies getting smaller.
Reflections	$x \to -x$	Reflects over the y-axis
	$f(x) \to -f(x)$	Reflects over the x-axis

Common Misconceptions or Errors

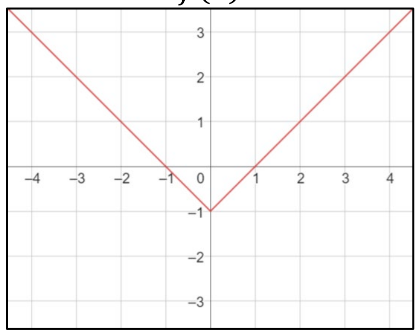
- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. To address this, encourage students to convert the graph to a second table, using the same domain as the first table, and compare.
- Similar to writing functions in vertex form, students may confuse the effect of the sign of k in $f(x + k)$. To address this misconception, direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same. To address this misconception, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A graph and table, each representing an absolute value function, are shown below. Describe and determine the value of the real number that defines the transformation from $f(x)$ to $g(x)$.

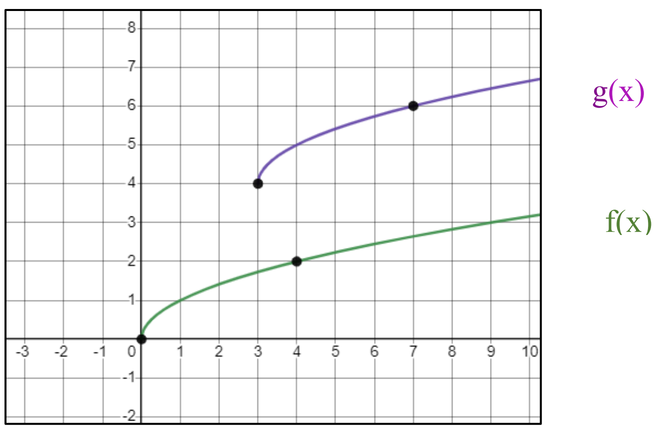
$f(x)$



x	-2	-1	0	1	2
$g(x)$	2	0	-2	0	2

Instructional Task 2 (MTR.5.1)

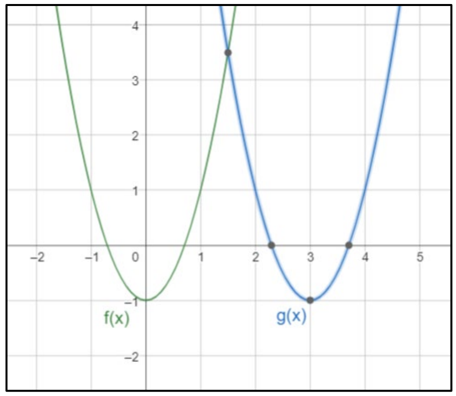
Considering the graphs of $f(x)$ and $g(x)$ below, describe a pair of transformations that can be used to transform $f(x)$ to $g(x)$ and determine the value of the real number, k, that defines each transformation. Does it matter which order these transformations are applied?



Instructional Items

Instructional Item 1

Considering the graphs of $f(x)$ and $g(x)$ below, describe the transformation and determine the value of the real number, k, that defines the transformation from $f(x)$ to $g(x)$.



Instructional Item 2
Considering the table below, describe the transformation and determine the value of the real number, k, that defines the transformation from $f(x)$ to $g(x)$.

x	$f(x)$	$g(x)$
1	-9	-8.4
2	-13.2	-12.6
3	-17.4	-16.8
4	-21.6	-21

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2.5

Benchmark

MA.912.F.2.5	Given a table, equation or graph that represents a function, create a corresponding table, equation or graph of the transformed function defined by adding a real number to the x- or y-values or multiplying the x- or y-values by a real number.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Coordinate Plane
- Dilation
- Function Notation
- Origin
- Reflection
- Rigid Transformation
- Scale Factor

- Translation

Vertical Alignment

Previous Benchmarks
- MA.8.GR.2
- MA.912.AR.2.4, MA.912.AR.2.5 (Algebra 1)
- MA.912.GR.2 (Geometry)

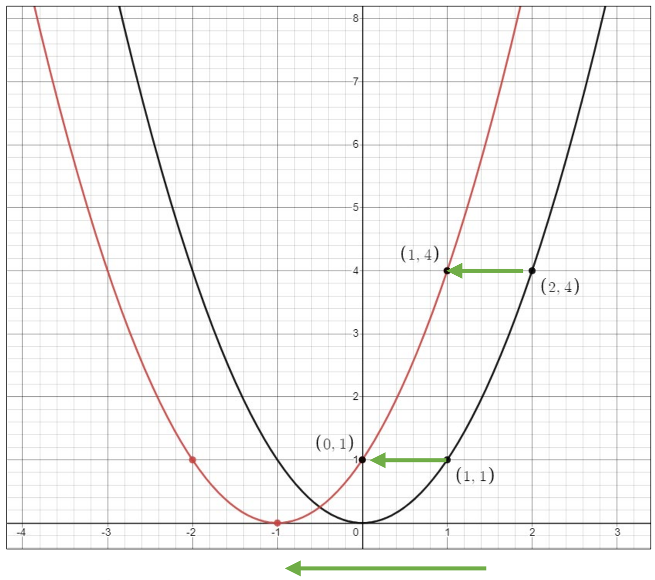
Next Benchmarks
- MA.912.3.2, MA.912.3.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Algebra 2, students identify effects of transformations on linear, quadratic, exponential, logarithmic, simple rational, square root, cube root and absolute value functions. In future courses, students will also identify transformations of trigonometric functions.

- In this benchmark, students will create a table, equation or graph of a transformed function defined by adding a real number to the x- or y- values or multiplying the x- or y- values by a real number.
- Instruction includes the use of graphic software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
- Given a function f, the transformed function $g(x) = f(x - k)$ is a horizontal shift of $f(x)$. Adding a real number, k, to all the inputs (x-values) of a function will result in shifting the output left or right depending on the sign of k. If k is positive, the graph will shift right. If k is negative, the graph will shift left.

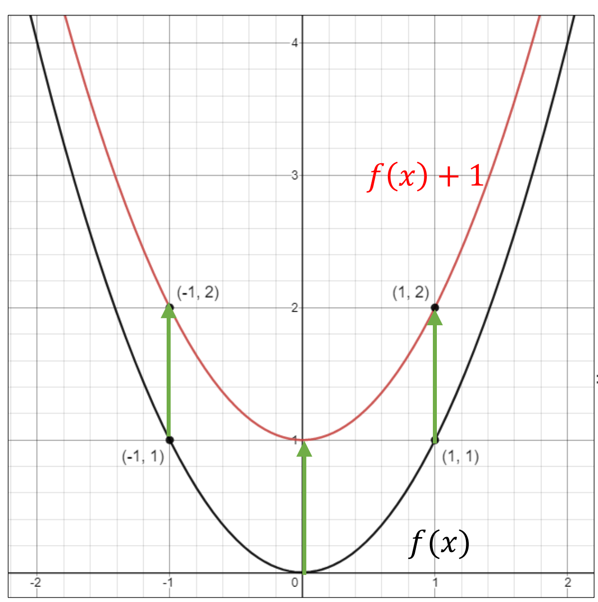
x	$\left(x, f(x)\right)$	$f(x)$	$g(x) = f(x-k); k = -1$	$\left(x, g(x)\right)$	$g(x)$
-2	(-2,4)	4	$g(-2) = f(-2+1) = f(-1) = 1$	(-2,1)	1
-1	(-1,1)	1	$g(-1) = f(-1+1) = f(0) = 0$	(-1,0)	0
0	(0,0)	0	$g(0) = f(0+1) = f(1) = 1$	(0,1)	1
1	(1,1)	1	$g(1) = f(1+1) = f(2) = 4$	(1, 4)	4
2	(2,4)	4	$g(2) = f(2+1) = f(3) = ?$	(2,?)	



$k = -1$, Shift 1 unit to the left

- Given a function f, the transformed function $g(x) = f(x) + k$ is a vertical shift of $f(x)$. Adding a real number, k, to all the outputs (y-values) of a function will result in shifting the output up or down depending on the sign of k. If k is positive, the graph will shift up, and if k is negative, the graph will shift down.

x	$(x, f(x))$	$f(x)$	$g(x) = f(x) + k;\ k = 1$	$(x, g(x))$	$g(x)$
-2	(-2,4)	4	$g(-2) = f(-2) + 1 = 4 + 1 = 5$	(-2,5)	5
-1	(-1,1)	1	$g(-1) = f(-1) + 1 = 1 + 1 = 2$	(-1,2)	2
0	(0,0)	0	$g(0) = f(0) + 1 = 0 + 1 = 1$	(0,1)	1
1	(1,1)	1	$g(1) = f(1) + 1 = 1 + 1 = 2$	(1, 2)	2
2	(2,4)	4	$g(2) = f(2) + 1 = 4 + 1 = 5$	(2,5)	5

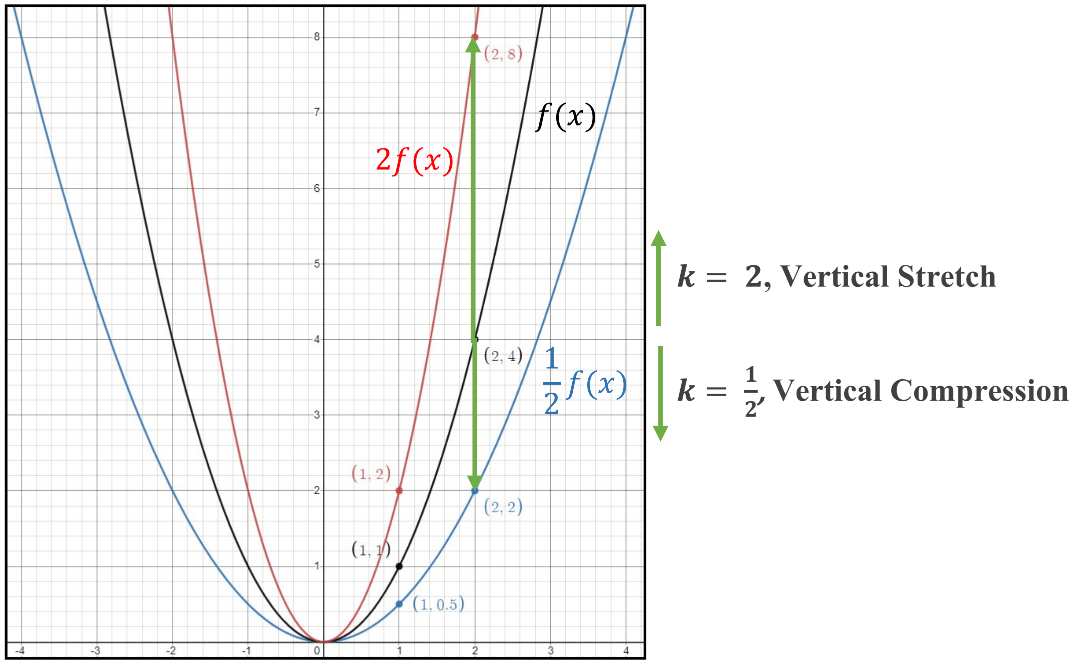


= 1, Shift 1 unit up

- Discuss with students that similar to translations of two-dimensional figures, adding a constant to either the input or output of a function changes the position of the graph, but it doesn't change the shape of the graph. *(MTR.4.1)*
- Given a function f, the transformed function $g(x) = kf(x)$ is a vertical stretch or compression of $f(x)$. Multiplying all the outputs (y-values) of a function by a real number, k, will result in a vertical stretching or compression depending on the value of k. If k is between 0 and 1 ($0 < k < 1$), the graph will be vertically compressed and if k is greater than 1 ($k > 1$), the graph will be vertically stretched.
- If k is a negative number ($k < 0$), the transformed graph will be a combination of a vertical stretch or compression, and a reflection over the x-axis. Discuss with students how multiplying all the y-values by -1 is the same as reflecting a two-dimensional figure over the x-axis.

x	$(x, f(x))$	$f(x)$	$g(x) = kf(x);\ k = 2$	$(x, g(x))$	$g(x)$
-2	(-2,4)	4	$g(-2) = 2f(-2) = 2 \cdot 4 = 8$	(-2,8)	8
-1	(-1,1)	1	$g(-1) = 2f(-1) = 2 \cdot 1 = 2$	(-1,2)	2
0	(0,0)	0	$g(0) = 2f(0) = 2 \cdot 0 = 0$	(0,1)	0
1	(1,1)	1	$g(1) = 2f(1) = 2 \cdot 1 = 2$	(1, 2)	2
2	(2,4)	4	$g(2) = 2f(2) = 2 \cdot 4 = 8$	(2,8)	8

x	$(x, f(x))$	$f(x)$	$g(x) = kf(x);\ k = \frac{1}{2}$	$(x, g(x))$	$g(x)$
-2	(-2,4)	4	$g(-2) = \frac{1}{2}f(-2) = \frac{1}{2} \cdot 4 = 2$	(-2,2)	2
-1	(-1,1)	1	$g(1) = \frac{1}{2}f(1) = \frac{1}{2} \cdot 1 = \frac{1}{2}$	$(-1, \frac{1}{2})$	$\frac{1}{2}$
0	(0,0)	0	$g(0) = \frac{1}{2}f(0) = \frac{1}{2} \cdot 0 = 0$	(0,1)	0
1	(1,1)	1	$g(-1) = \frac{1}{2}f(-1) = \frac{1}{2} \cdot 1 = \frac{1}{2}$	$(1, \frac{1}{2})$	$\frac{1}{2}$
2	(2,4)	4	$g(-2) = \frac{1}{2}f(-2) = \frac{1}{2} \cdot 4 = 2$	(2,5)	2



- Given a function f, the transformed function $g(x) = f(kx)$ is a horizontal stretch or compression of $f(x)$. Multiplying all the inputs (x-values) of a function by a real number, k, will result in a horizontal stretching or compression depending on the value of k. If k is between 0 and 1 ($0 < k < 1$), the graph will be horizontally *dilated* by a factor of $\frac{1}{k}$ or horizontally stretched by a factor of $\frac{1}{k}$. If k is greater than 1 ($k > 1$), the graph will be horizontally *dilated* by a factor of $\frac{1}{k}$ or horizontally *compressed* by a factor of k.
- If k is a negative number ($k < 0$), the transformed graph will be a combination of a horizontal stretch or compression, and a reflection over the y-axis. Discuss with students how multiplying all the x-values by -1 is the same as reflecting a two-dimensional figure over the y-axis.
- After completing the table below, discuss with students the meaning of $g(x) = f(2x)$. In this case, the output value, $g(x)$, is the same as the output value of $f(x)$ at an input that is twice the size.

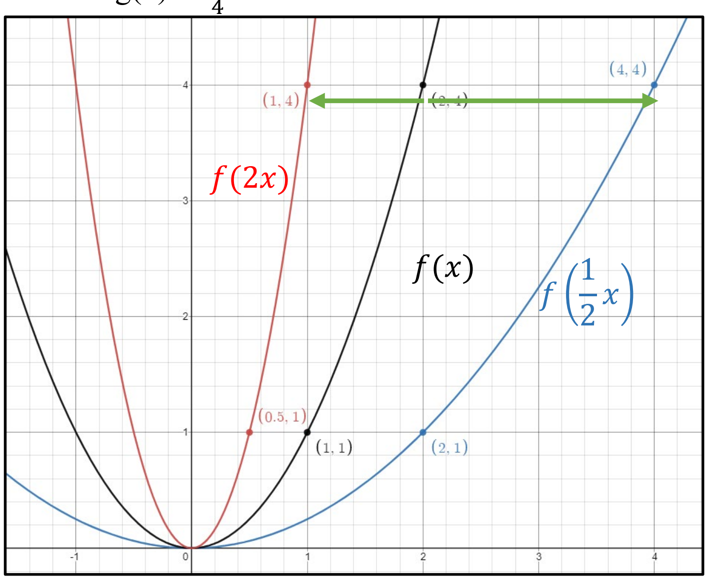
x	$(x, f(x))$	$f(x)$	$g(x) = f(kx);\ k = 2$	$(x, g(x))$	$g(x)$
-4	(-4,16)	16	$g(-4) = f(2 \cdot -4) = f(-8)$ $=?$	(-4,?)	
-2	(-2,4)	4	$g(-2) = f(2 \cdot -2) = f(-4)$ $= 16$	(-2,16)	16
-1	(-1,1)	1	$g(-1) = f(2 \cdot -1) = f(-2)$ $= 4$	(-1,4)	4
0	(0,0)	0	$g(0) = f(2 \cdot 0) = f(0) = 0$	(0,0)	0
1	(1,1)	1	$g(1) = f(2 \cdot 1) = f(2) = 4$	(1, 4)	4
2	(2,4)	4	$g(2) = f(2 \cdot 2) = f(4) = 16$	(2,16)	16
4	(4,16)	16	$g(4) = f(2 \cdot 4) = f(8) =?$	(4,?)	

  - Instruction includes connections to MA.912.F.1.1 where students can determine a function type to represent a table of values. For the above example, students should recognize the function $f(x) = x^2$, and therefore be able to complete the outputs for g(-4) and g(4) as 64.

- After completing the table below, discuss with students the meaning of $g(x) = f\left(\frac{1}{2}x\right)$. In this case, the output value, $g(x)$, is the same as the output value of $f(x)$ at an input that is half the size.

x	$(x, f(x))$	$f(x)$	$g(x) = f(kx);\ k = \frac{1}{2}$	$(x, g(x))$	$g(x)$
-4	(-4,16)	16	$g(-4) = f\left(\frac{1}{2} \cdot -4\right) = f(-2) = 4$	(-4,4)	4
-2	(-2,4)	4	$g(-2) = f\left(\frac{1}{2} \cdot -2\right) = f(-1) = 1$	(-2,1)	1
-1	(-1,1)	1	$g(-1) = f\left(\frac{1}{2} \cdot -1\right) = f\left(-\frac{1}{2}\right) =?$	(-1, ?)	
0	(0,0)	0	$g(0) = f\left(\frac{1}{2} \cdot 0\right) = f(0) = 0$	(0,0)	0
1	(1,1)	1	$g(1) = f\left(\frac{1}{2} \cdot 1\right) = f\left(\frac{1}{2}\right) =?$	(1, ?)	
2	(2,4)	4	$g(2) = f\left(\frac{1}{2} \cdot 2\right) = f(1) = 1$	(2,1)	1
4	(4,16)	16	$g(4) = f\left(\frac{1}{2} \cdot 4\right) = f(2) = 4$	(4,4)	4

- Instruction includes connections to MA.912.F.1.1 where students can determine a function type to represent a table of values. For the above example, students should recognize the function f(x) = x^2, and therefore be able to complete the outputs for g(-1) and g(1) as $\frac{1}{4}$.



$k =$ 2, Horizontal Compression

$k = \frac{1}{2}$, Horizontal Stretch

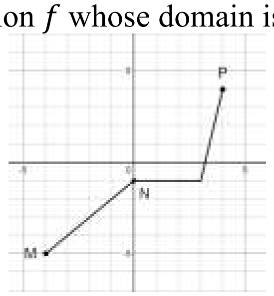
Common Misconceptions or Errors

- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. To address this, encourage students to convert the graph to a second table, using the same domain as the first table, and compare.
- Some students misinterpret how the parameters of the equation of a transformed function are affected by a horizontal translation. This may indicate that students do not understand the relationship between the graph and the equation of the function.
    - For example, a student may think that $g(x) = f(x + 1)$ is a horizontal translation to the right because of the positive addend for x. One potential teaching strategy to address this misconception would be using a graphing utility to graph the function$f(x) = (x - k)^2$ creating k as a slider, and then allowing students to explore the translation results as the value of the slider changes.
- Some students may have difficulties understanding that multiplying the input of a function by a number greater than 1 will result in a horizontal compression of the graph, instead of a stretching. To address this misconception, it is important to point out that multiplying the x-value does not change the original value of the input. Because the input is being multiplied by a number greater than 1, a smaller input in the transformed function is needed to obtain the same output from the original function. Another potential teaching strategy would be using a graphing utility to graph the function $f(x) = (kx)^2$ creating k as a slider, and then allowing students to explore the stretching/compression results as the value of the slider changes from 0 to 2.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

The figure shows the graph of a function f whose domain is the interval $-4 \leq x \leq 4$.



Part A. Sketch the graph of each transformation described below and compare it with the graph of f. Explain what you see.

a. $g(x) = f(x) + 2$
b. $h(x) = f(x + 2)$
c. $k(x) = 2f(x)$
d. $r(x) = f(2x)$

Part B: The points labeled M, N, P on the graph of f have coordinates $M = (-4, -5)$, $N = (0, -1,)$ and $P = (-4, 4)$. Complete the table below with the coordinates of the points corresponding to M, N, P on the graphs of g, h, k and r.

$f(x)$	$g(x)$	$h(x)$	$k(x)$	$r(x)$
(-4,-5)				
(0,-1)				
(-4,4)				

Instructional Items

Instructional Item 1 (MTR.3.1)
Given the function $f(x) = |x|$, graph the function $f(x)$ and the transformation $g(x) = f(x - 3)$ on the same axes. What do you notice about the x-intercepts of $g(x)$?

Instructional Item 2 (MTR.3.1)
Given the function $f(x) = \log x$, graph the function $f(x)$ and the transformation $g(x) = 3f(x)$ on the same coordinate plane. Describe the transformed function, $g(x)$, as it relates to the graph of $f(x)$.

Instructional Item 3 (MTR.3.1)
A function $f(x)$ is given. Create a table for the functions below.

a. $g(x) = f(x) + 5$
b. $h(x) = f(2x)$

x	2	4	6	8
$f(x)$	1	3	7	11

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3 *Create new functions from existing functions.*

MA.912.F.3.2

Benchmark

MA.912.F.3.2	Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function.

Benchmark Clarifications:
Clarification 1: Instruction includes representing domain restrictions with inequality notation, interval notation or set-builder notation.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.4, MA.912.AR.3.8
- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Domain
- Function
- Function Notation
- Set-Builder Notation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.1
- MA.912.AR.1.4, MA.912.AR.1.7 (Algebra 1)

Next Benchmarks

- MA.912.F.3.3 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students performed addition, subtraction, multiplication and division with polynomials. In Algebra 2, students continue to use function notation and they combine functions from different families using arithmetic operations. In later courses, students will solve mathematical and real-world problems involving functions that have been combined using arithmetic operations.

- In this benchmark, students will combine functions through addition, subtraction, multiplication, and division. This process will utilize their prior experience with polynomial arithmetic in MA.912.AR.1 from Algebra 1.
    - In mathematical contexts, combinations through addition may be represented as $(f+g)(x)$ or $f(x)+g(x)$.
    - In mathematical contexts, combinations through subtraction should be represented as $(f-g)(x)$ or $f(x)-g(x)$.
    - In mathematical contexts, combinations through multiplication should be represented as $(f \cdot g)(x)$ or $f(x) \cdot g(x)$.
    - In mathematical contexts, combinations through division should be represented as $\left(\frac{f}{g}\right)(x)$ or $\frac{f(x)}{g(x)}$where $g(x) \neq 0$. Additionally for division, it can be represented using the division symbol.
- When appropriate, the domain restrictions will be determined for the new function. Students will evaluate the output when combining two functions for a provided input.
- Explain that when functions are combined using addition, subtraction, and multiplication, the domain of the resulting function is only the inputs (x-values) that are common to the domains of the original functions. The domain of $f+g$, $f-g$ and $f \cdot g$ is the intersection of the domains of f and g.
- When combining functions by division, students will need to find value(s) in the domain that would make the denominator, $g(x)$, equal to zero. The domain will be the intersection of the domains of f and g, but with the identified value(s) taken out of the intersection.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation. In previous courses, students worked with using words, inequality notation and set-builder notation, but interval notation is new to students in this course.

- Interval notation should be written with the lower bound first, then the upper bound. A bracket signifies that the bound is included in the interval, while a parenthesis signifies that the bound is not included in the interval. An interval may have a parenthesis around one bound and a bracket around the other. Parenthesis will always go around an infinite bound since infinity does not have a value to it, so it cannot be included in the interval. Intervals can be joined with the symbol, ∪, for union.
  - If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values (0 in this case).

- In Algebra 2, students are introduced to new mathematical symbols. Consider using a graphic organizer where students can record and build on their mathematical language. An example of this is shown below.

Math Symbol	(Name) Meaning	Example
{	(Curly Bracket) The set of...	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
|	(Vertical bar) Such that, given	$\{x \mid x<0\}$ The set of all x values such that x is less than 0
∪	Union, Or	$(-1,3) \cup (4,7)$ x must be in the open interval from –1 to 3 **or** in the open interval from 4 to 7
∈	Is an element of...	M= {0, 1, 2, 3} $1 \in M$ 1 is an element of the set M
ℝ	The set of all real numbers	$x \in \mathbb{R}$ x is an element of the set of all real numbers

- Instruction of this benchmark should include problems involving a real-world contexts. *(MTR.7.1)*
- Instruction may include the use of graphs and tables to foster a deeper understanding of operations with functions.

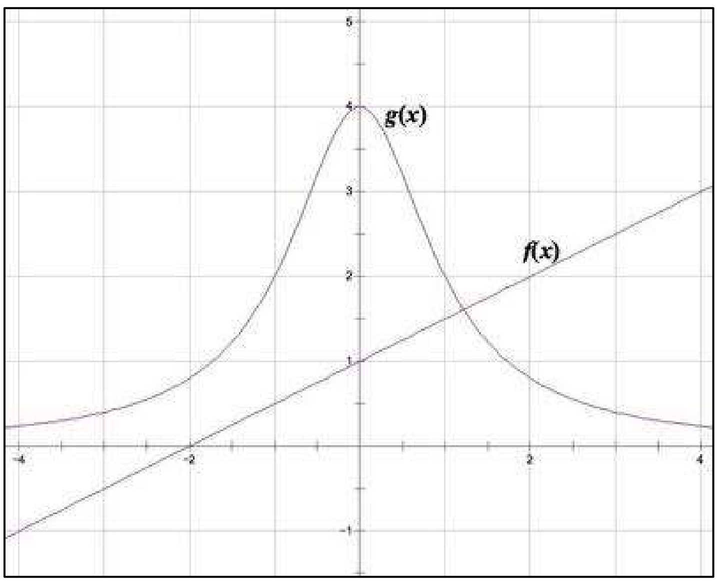
Common Misconceptions or Errors

- When multiplying and dividing functions, students may struggle working with exponents. To address this misconception, remind students of the multiplication and division properties of exponents, including negative exponents.
- When finding the domain of the new function, students may incorrectly find the union of the domains instead of the intersection. To address this misconception, emphasize the need to find what the domains have in common.
- When finding the domain of $\frac{f}{g}(x)$, students may forget to identify the restrictions on the domain. To address this error, remind students that they always need to find the values that make $g(x) = 0$, and exclude them from the domain of $\frac{f}{g}(x)$.
- Students may forget that when subtracting a function, they need to include parentheses around the function being subtracted and distribute the –1 properly.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

Using the graphs below, sketch a graph of the function $s(x) = f(x) + g(x)$ and state its domain and range.



Instructional Task 2 (MTR.7.1)

A large business is evaluating their budget and trying to determine their product price setting. For this task, R = revenue, p = price and E = expense. The expense function is $E = -3500p + 235{,}000$ and the revenue function is $R = 450p^2 + 20{,}000p$.

Part A. Graph each of the functions on the same coordinate plane.
Part B. Determine where the two functions intersect.
Part C. Describe the meaning of the intersection related to the revenue.
Part D. Describe the meaning of the area above and below the linear equation as it relates to the quadratic equation in this context.

Instructional Items

Instructional Item 1

Given $f(x) = x^2 + 2x + 4$ and $g(x) = -2x + 5$, find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\frac{f(x)}{g(x)}$. Determine the domain for each combination.

Instructional Item 2

Given $f(x) = \sqrt{2 - x}$ and $g(x) = \ln x$, find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\frac{f(x)}{g(x)}$. Determine the domain for each combination.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.4

Benchmark

MA.912.F.3.4	Represent the composition of two functions algebraically or in a table. Determine the domain and range of the composite function.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.1
- MA.912.F.2.2, MA.912.F.2.3, MA.912.2.5

Terms from the K-12 Glossary

- Domain
- Function
- Function Notation

Vertical Alignment

Previous Benchmarks

- MA.912.F.2.1 (Algebra 1)

Next Benchmarks

- MA.912.F.3.5 (Precalculus)
- MA.912.C.2.3, MA.912.C.2.4 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students worked with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Algebra 2, students continue to use function notation with other function types and combine functions, including compositions of functions. In later courses, students will solve mathematical and real-world problems involving composite functions.

- Instruction includes defining function composition and demonstrating composing functions with several examples. Explain that function composition is an operation that can be performed on two functions so the output of one function becomes the input of another, to form a new function. The resulting function is known as a composite function.

- Introduce the notation for composition of functions as $(f \circ g)(x) = f(g(x))$ and explain that we read the left-hand side as "f composed with g at x," and the right-hand side as "f of g of x."
- Emphasize the difference between $f(g(x))$ and $g(f(x))$ and explain that function composition is not commutative. Explain that it is important to follow the order of operations when evaluating a composite function, evaluating the inner function first.
- Instruction includes determining the domain and range of the composite function $f(g(x))$. Explain that the domain of the composite function is all inputs x, such that x is in the domain of g and $g(x)$ is in the domain of f.
- Instruction includes recognizing the connection between transformations of functions and composition of functions.
    - For example, the transformation $f(x) \rightarrow f(3x - 2)$ can be thought of as composing the function, $f(x)$ with the function $g(x) = 3x - 2$.

Common Misconceptions or Errors

- Students sometimes believe that variables represent just a fixed number, so they struggle understanding that a variable can also represent a function.
- Students who struggle evaluating functions for numerical inputs will find it more difficult to evaluate inputs that are functions.
- Students may not understand the function notation for composition thinking that $f(g(x))$ means to multiply function g by function f.
- Students confuse the composition $(f \circ g)$ with the product $(f \cdot g)$. Emphasize that the composition means to "evaluate g at x, then evaluate f at the result $g(x)$."
- Students may incorrectly believe $(f \circ g)$ is the same as $(g \circ f)$.
- When finding the domain of a composite function, students may find the union or the intersection. To address this misconception, emphasize the need to find what values of x of the domain of the inner function produce outputs that are in the domain of the outer function.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Your parents purchased a new reclining chair for the living room. The chair will be delivered to your home. The cost of the chair is d dollars, the tax rate is 6.5%, and the delivery fee is $50.

Part A. Write a function $f(d)$ for the cost of the chair including the delivery fee, but not the taxes. State the domain.

Part B. Write another function $g(d)$ for the cost of the chair including taxes, but not the delivery fee. State the domain.

Part C. Write the function $f(g(d))$ and interpret its meaning. State the domain of $f(g(d))$.

Part D. Write the function $g(f(d))$ and interpret its meaning. State the domain of $g(f(d))$.

Part E. Which results in a lower cost to you, $f(g(d))$ or $g(f(d))$? Explain why.

Instructional Task 2 (MTR.7.1)
Let f be the function that assigns to a temperature in degrees Celsius its equivalent in degrees Fahrenheit. Let g be the function that assigns to a temperature in degrees Kelvin its equivalent in degrees Celsius.

Part A. Explain what x and $f(g(x))$ represent in terms of temperatures, or explain why there is no reasonable representation.

Part B. Explain what x and $g(f(x))$ represent in terms of temperatures, or explain why there is no reasonable representation.

Part C. Given that $f(x) = \frac{9}{5}x + 32$ and $g(x) = x - 273$, find an expression for $f(g(x))$.

Part D. Find an expression for the function h which assigns to a temperature in degrees Fahrenheit its equivalent in degrees Kelvin.

Instructional Items

Instructional Item 1
Given $f(x) = 3x^2 + x + 4$ and $g(x) = x + 4$.
a) Determine $f \circ g$.
b) Determine $g \circ f$.
c) Determine the domain of each composite function.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.6

Benchmark

MA.912.F.3.6	Determine whether an inverse function exists by analyzing tables, graphs and equations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.7.2, MA.912.AR.7.3
- MA.912.F.1.6

Terms from the K-12 Glossary

- Inverse Functions

Vertical Alignment

Previous Benchmarks

- MA.912.F.1 (Algebra 1)

Next Benchmarks

- MA.912.F.3.8, MA.912.F.3.9 (Precalculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students analyzed linear, quadratic, exponential and absolute value functions. In Algebra 2, students extend their knowledge of analyzing functions to determine whether an inverse function exists by analyzing tables, graphs and equations. *(MTR.2.1)* In later courses, students will produce an invertible function from a non-invertible function and solve mathematical and real-world problems involving inverse functions.

- Students will need to use what they have learned about key features and characteristics and patterns that are evident with each function type to determine whether the functions they see represented in graphs, tables and/or equations are functions that are invertible. *(MTR.5.1)*
- When dealing with a graph, students can use the horizontal line test and its connection to the vertical line test, to determine if an inverse to a given function exists. The horizontal line test allows a student to check if a function is one-to-one, meaning there is only one x-value for each y-value. To use the horizontal line test, students can consider all horizontal lines and check to see if any intersect the graph of the function in at more than one point. If it does not, then the given function should have an inverse that is also a function.
- Instruction includes discussing what adjustments could be made to a function's domain so that an inverse function would exist. *(MTR.5.1)*
- Instruction includes using compositions of functions to determine whether two functions are inverses of each other. *(MTR.2.1)*

Common Misconceptions or Errors

- Students may confuse the horizontal line test, which is used to determine if a function has an inverse function, with the vertical line test, which determines if a graph represents a function.
- Students may not understand how to transpose the input and output values in a table to determine the inverse of a function.
- Students may not understand that it is possible for a function to have an inverse that may not be a function.
- Students may struggle with determining how to restrict the domain of a function in order to make its inverse a function.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)
Create a function that has an inverse function. Explain using a table, graph and equation how you know that an inverse function exists.

Instructional Task 2 (MTR.5.1)
Create a function for which an inverse function does not exist. Explain what about that function makes it not invertible. Explain what adjustments could be made to the function to make it invertible.

Instructional Task 3 (MTR.2.1, MTR.4.1)
Let $f(x) = 3x^2 - 14x + 7$.

Part A. Create a table for this function showing at least 5 input values. Does your table determine whether this function has an inverse function? Explain.
Part B. Draw a graph for this function. Does your graph determine whether this function has an inverse function? Explain.
Part C. Can you think of another way to determine if this function has an inverse function?

Instructional Items

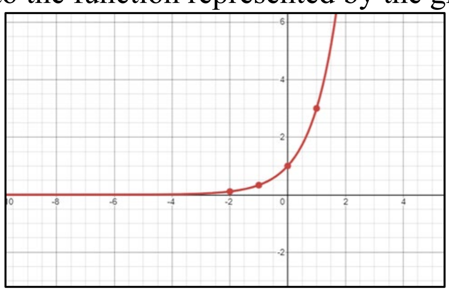
Instructional Item 1

Determine whether an inverse function to the function represented in the table exists. Explain.

x	$f(x)$
-2	-5
-1	-7
0	-9
1	-7
2	-5

Instructional Item 2

Determine whether an inverse to the function represented by the graph exists. Explain.



Instructional Item 3

Determine whether an inverse to the function represented with the equation exists. Explain.

$$f(x) = \ln x$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.7

Benchmark

MA.912.F.3.7 Represent the inverse of a function algebraically, graphically or in a table. Use composition of functions to verify that one function is the inverse of the other.

Benchmark Clarifications:
Clarification 1: Instruction includes the understanding that a logarithmic function is the inverse of an exponential function.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.7.2, MA.912.AR.7.3
- MA.912. F.1.7

Terms from the K-12 Glossary

- Inverse Functions

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.3

Next Benchmarks

- MA.912.F.3.8, MA.912.F.3.9 (Precalculus)
- MA.912.C.2.6 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In eighth grade, students use square roots and cube roots to solve equations involving x^2 and x^3, anticipating the use of inverse functions to solve for x. In Algebra 1, students identified and interpreted key features for linear, quadratic, exponential and absolute value functions. In Algebra 2, students use those skills around analyzing tables, graphs and equations to represent an inverse to a function given to them. *(MTR.2.1, MTR.5.1)*

- Instruction includes noticing patterns such as transposing of the input and output values in a table to create an inverse of a function.
- Instruction includes the connection between inverse functions and inverse operations.
- Instruction includes the understanding that an inverse of a function does not always represent a function. Students should be able to know how to look at two functions in any representation and determine if they are inverse functions.
- Instruction includes the connection to MA.912.F.3.6 to help students be able to represent inverses to given functions, if they exist, algebraically, graphically or in a table. If students are not able to determine whether an inverse of a given function exists by examining the features given in the function, they may struggle to be able to represent the inverse of the given function in various representations. In this benchmark, students are also expected to use composition of functions to verify that one function is the inverse of the other.
- Students need to show understanding that certain function types are inverses of each other such as quadratic and square root, or exponential and logarithmic functions. Students should also demonstrate an understanding that a linear function will have a linear inverse unless the linear function is constant. *(MTR.5.1)*
- Instruction includes giving students a function represented in a table, graph or equation and asking them to determine if the function is invertible (able to produce an inverse function) or non-invertible (not able to produce an inverse function). Once they have determined if the inverse function is able to be produced, then students should be

expected to represent that inverse in multiple ways. In a table, students should notice that the input and output values transpose with each other. In a graph, students should notice that the points of the graph reflect over the line $y = x$.
- Students should be able to use compositions of functions to verify that two functions are inverses of each other. When they compose the functions, they should demonstrate an understanding that if they end up with anything other than x as their solution to the compositions of functions then those functions are not inverses of each other.
- Instruction may include asking the students to use another representation (table or graph) to look at the two functions to justify their response as to whether the functions are inverse of each other. If they can see graphically that two functions are inverses of each other but algebraically they do not get x when composing the functions, it will prompt the students to go back and check their work. Likewise, if they see graphically or in a table that the two functions are not inverses of each other, but their composition of functions does produce an x they will be prompted to go back and check their work. *(MTR 6.1)*
- Instruction includes opportunities for students to explore the idea that when functions are inverses of each other, they can compose the functions both ways [$f(g(x))$ and $g(f(x))$] and still get "x" as the solution. While compositions are not commutative, in this special case, compositions can be done in either order.

Common Misconceptions or Errors

- Students may think that they should get a numerical value (most commonly zero or 1) for their output value when composing the two functions for the functions to be inverses, rather than getting the output value x.
- Students may not understand how the graphs of inverse functions will reflect over the line $y = x$. Students may not realize that it must reflect over the line $y = x$ rather than just be a reflection over any line.
- Students may not understand how to look for patterns in the table that will justify that a function is an inverse of their given function.

Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.7.1)
The table below shows the number of households in the U.S. in the years.

Year	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**
Households (in thousands)	97,107	98,990	99,627	101,018	102,528	103,874	104,705

Part A. Find a function, h, for a line of best fit, which models the number of households in the U.S. (in thousands) as a function of the year, t.
Part B. Represent the inverse of the linear function you wrote in Part A in a table, graph, and expression.

Instructional Task 2 (MTR.5.1, MTR.4.1)
Let f be the function defined by $f(x) = 10^x$ and g be the function $g(x)$ defined by $f(x) = \log_{10} x$.

Part A. Sketch the graph of $y = f(g(x))$. Explain your reasoning.

Part B. Sketch the graph of $y = g(f(x))$. Explain your reasoning.

Part C. Let f and g be any two inverse functions. For which values of x does $f(g(x)) = x$? For which values of x does $g(f(x)) = x$?

Instructional Task 3 (MTR.2.1, MTR.6.1)
Find an inverse to the function $f(x) = x^2 + 3$ by restricting its domain.

Part A. Represent the inverse in an equation/expression, a table and a graph.
Part B. Verify that the inverse you found was an inverse algebraically using compositions of functions.
Part C. Is there another way to restrict the domain to obtain an inverse function? If so, repeat parts A and B with this restriction.

Instructional Items

Instructional Item 1
Which of these is **true** for the inverse of the function $f(x) = (x + 1)^2$ on the domain $x \leq -1$?

a. $f^{-1}(x) = \sqrt{x} - 1$
b. $f^{-1}(x) = -\sqrt{x} - 1$
c. $f^{-1}(x) = \sqrt{x - 1}$
d. $f^{-1}(x) = -\sqrt{x - 1}$

Instructional Item 2
Find the inverse of the function $f(x) = 3^x$.

Instructional Item 3
Verify that the following functions are inverses of each other using compositions of functions.

$$f(x) = 2^x$$
$$g(x) = \log_2 x$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Financial Literacy

MA.912.FL.3 *Describe the advantages and disadvantages of short-term and long-term purchases.*

MA.912.FL.3.1

Benchmark

MA.912.FL.3.1 Compare simple, compound and continuously compounded interest over time.

Benchmark Clarifications:
Clarification 1: Instruction includes taking into consideration the annual percentage rate (APR) when comparing simple and compound interest.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1
- MA.912.AR.5.4, MA.912.AR.5.5, MA.912.AR.5.7

Terms from the K-12 Glossary

- Exponential Function
- Linear Function
- Simple Interest

Vertical Alignment

Previous Benchmarks
- MA.7.AR.3.1
- MA.912.NSO.1.1, 1.2 (Algebra 1)
- MA.912.AR.1.2 (Algebra 1)

Next Benchmarks
- MA.912.C.1.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved problems involving simple and compound interest, using arithmetic operations and graphing. In Algebra 2, students will compare simple, compound and continuously compounded interest, using algebraic operations as well as graphing. Students will build on this knowledge to analyze financial investments in Mathematics for Data and Financial Literacy. In Calculus, students will learn why the expression e^{rt} is used for continuous compounding.

- In this benchmark, students will describe the difference between simple and compound interest. Instruction should feature a variety of real-world contexts related to money and business.
- Instruction compares the differences between simple and compound interest.
  - The simple interest formula, $I = prt$, calculates **only the interest** earned over

time. Each year's interest is calculated from the initial principal, not the total value of the investment for that point in time.
    - The simple interest amount formula, $A = P(1 + rt)$, calculates the **total value** of an investment over time.
    - The compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, calculates the **total value** of an investment over time when interest is compounded. Each month/year's interest is calculated from the total value of the investment for that point in time.
    - The continuously compounded interest formula, $A = Pe^{rt}$, calculates the **total value** of an investment over time when interest is compounded continuously. The interest is calculated continuously from the total value of the investment for that point in time.
- Instruction includes the connection to linear and exponential functions MA.912.FL.3.4.
- Instruction includes comparison of simple, compound and continuously compounded interest using various representations, such as graphs, tables, equations and written description. Students will be guided to interpret key features of linear and exponential relationships in terms of context to connect to simple and compound interest problems.
- The actual amount of interest paid in a year, annual percentage yield (APY), will depend on whether the interest is compounded and how often it is compounded. See the formulas above.
- Instruction includes the use of tables, graphs and logarithms.

Common Misconceptions or Errors

- For simple interest scenarios, students may forget to add the principal to the interest they calculate using the simple interest formula when calculating the total amount of an investment over time.
- For compound interest scenarios, students may forget to subtract the principal from the total amount they calculate using a compound interest formula when calculating the total interest earned over time.
- In the formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, students may confuse the compounding period variable n, with the time variable, t.
- When forming compound interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

Instructional Tasks

Instructional Task 1 (MTR.5.1, MTR.7.1)
A principal investment of \$5,000 had an interest rate of 3%.
Complete the table below.

Years Invested	Simple Interest	Compound Interest (Annually)	Compound Interest (Quarterly)	Compound Interest (Monthly)	Compound Interest (Continuously)
5					
10					
15					

20					
25					

Instructional Items

Instructional Item 1
To the nearest cent, how much more does an investment of $3,500 earn over 10 years, compounded continuously at 3.5%, than a $3,500 investment over 10 years, compounded quarterly at 3.5%?

Instructional Item 2
Which is the better financial investment: 3.85% compounded weekly or 3.74% compounded Monthly? Justify your reasoning.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.2

Benchmark

MA.912.FL.3.2 Solve real-world problems involving simple, compound and continuously compounded interest.

Example: Find the amount of money on deposit at the end of 5 years if you started with $500 and it was compounded quarterly at 6% interest per year.
Example: Joe won $25,000 on a lottery scratch-off ticket. How many years will it take at 6% interest compounded yearly for his money to double?

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, interest is limited to simple and compound.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.1.1
- MA.912.AR.5.4, MA.912.AR.5.5, MA.912.AR.5.7

Terms from the K-12 Glossary

- Exponential Function
- Linear Function
- Simple Interest

Vertical Alignment

Previous Benchmarks
- MA.7.AR.3.1
- MA.912.NSO.1.1, MA.912.NSO.1.2 (Algebra 1)
- MA.912.AR.1.2 (Algebra 1)

Next Benchmarks
- MA.912.C.1.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students solved problems involving simple and compound interest, using arithmetic operations and graphing. In Algebra 2, this work extends to include continuously compounded interest and the use of logarithms to solve for time. Students will build on this knowledge to analyze financial investments in Mathematics for Data and Financial Literacy. In Calculus, students will learn why the expression e^{rt} is used for continuous compounding.

- Instruction compares the differences between simple and compound interest.
  - The simple interest formula, $I = prt$, calculates **only the interest** earned over time. Each year's interest is calculated from the initial principal, not the total value of the investment for that point in time.
  - The simple interest amount formula, $A = P(1 + rt)$, calculates the **total value** of an investment over time.
  - The compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, calculates the **total value** of an investment over time for interest that is compounded. Each month/year's interest is calculated from the total value of the investment for that point in time.
  - The continuously compounded interest formula, $A = Pe^{rt}$, calculates the **total value** of an investment over time for interest that is compounded continuously. Each month/year's interest is calculated from the total value of the investment for that point in time.
- Instruction includes the use of tables, graphs and logarithms.
- Compound interest problems presented for this benchmark may require students to generate equivalent expressions to identify and interpret certain parts of the context.
  - For example, Jason deposits \$850 in an account that earns an annual percentage rate (APR) of 4.8%. The interest is compounded monthly, and Jason wants to determine the total amount of interest he will earn in one year. With the information given, derive that the value of the account is equal to $850\left(1 + \frac{.048}{12}\right)^{12t}$. The expression can be rewritten as $850(1.004)^{12t}$ leading to $850(1.049)^{t}$ to find that the total amount of interest in one year would be approximately 4.9% of his initial investment.

Common Misconceptions or Errors

- For simple interest scenarios, students may forget to add the principal to the interest they calculate using the simple interest formula when calculating the total amount of an investment over time.
- For compound interest scenarios, students may forget to subtract the principal from the total amount they calculate using a compound interest formula when calculating the total interest earned over time.

- In the formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, students may confuse the compounding period variable n, with the time variable, t.
- When forming compound interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Four students make $500 investments for a period of 20 years. Jasmine earns simple interest at a rate of 13%. Brayden's investment earns 6.8%, compounded monthly. Dionne's investment earns 7%, compounded semi-annually. Arthur's investment earns 6.5%, compounded continuously.

Part A. Predict which student's investment will have the greatest value in 20 years.
Part B. Predict which student's investment will have the least value in 20 years.
Part C. Use a spreadsheet program and graphing technology to explore the value of these investments annually from year 1 to year 20.
Part D. Which investment would be best at year 16?
Part E. Choose which investment you would want. Explain your reasoning.

Instructional Task 2 (MTR.4.1, MTR.7.1)

Gibbs signs up for a new airline credit card that has a 24% annual interest rate. If he charges $300 to his credit card in January when he first gets it and only makes the minimum monthly payment of $30 after that, interest on his balance will compound daily. Assume Gibbs does not accrue any other charges to his card.

Part A. Describe what it means to have interest compound daily.
Part B. After six months, how many days is the interest compounding?
Part C. If Gibbs continues to only pay the minimum monthly payment, how long would it take for him to pay off his credit card?

Instructional Items

Instructional Item 1

Beatrice deposits $525 in an account that pays 4.3% simple annual interest. If she keeps the money in the account for 12 years, how much interest will she earn?

Instructional Item 2

Kennedy deposits $525 in an account that pays 4.3% compounded continuously. If she keeps the money in the account for 12 years, how much interest will she earn?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.4

Benchmark

MA.912.FL.3.4	Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, exponential growth is limited to compound interest.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1
- MA.912.AR.5.4, MA.912.AR.5.5, MA.912.AR.5.7

Terms from the K-12 Glossary

- Exponential Function
- Linear Function
- Simple Interest

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.1
- MA.912.NSO.1.1 (Algebra 1)
- MA.912.AR.1.2 (Algebra 1)
- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.5 (Algebra 1)

Next Benchmarks

- MA.912.C.1.8 (Calculus)
- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students explained the relationship between simple interest and linear growth and the relationship between compound interest and exponential growth. In Algebra 2, students extend this to include continuously compounded interest. Students will build on this knowledge to analyze financial investments in Mathematics for Data and Financial Literacy. In Calculus, students will learn why the expression e^{rt} is used for continuous compounding.

- In MA.912.FL.3.2, students became familiar with simple and compound interest and how to use the formulas for each to solve real-world problems. In this benchmark, students will make connections between simple interest and linear growth and between compound interest and exponential growth. To help students discover this relationship, consider guiding them to form a table.
  - For example, Kianna and Samantha both receive $1,000 cash from graduation gifts from family and friends. They each decide to invest their money in an investment account. Kianna's investment earns 10% in *simple* interest.

Samantha's investment earns 10% interest *compounded* annually. Guide students to create the interest formulas below and use them to create the table below to compare the growth of their investments over time.

- Kianna's Interest Earned would be represented by $I = 1000 \cdot 0.1 \cdot t$.
- Kianna's Total Value would be represented by $A = 1000(1 + 0.1t)$.
- Samantha's Interest Earned would be represented by $I = 1000(1 + 0.1)^t - 1000$.
- Samantha's Total Value would be represented by $A = 1000(1 + 0.1)^t$.

Years Invested	Kianna's Interest Earned ($)	Total Value of Kianna's Investment ($)	Samantha's Interest Earned ($)	Total Value of Samantha's Investment ($)
1	100	1,100	100	1,100
2	200	1,200	210	1,210
3	300	1,300	331	1,331
4	400	1,400	464.10	1,464.10
5	500	1,500	610.51	1,610.51
10	1,000	2,000	1,593.74	2,593.74
15	1,500	2,500	3,177.25	4,177.25
20	2,000	3,000	5,727.50	6,727.50
30	3,000	4,000	16,449.40	17,449.40
50	5,000	6,000	116,390.90	117,390.90

- Once completed, ask students what relationships they observe in the behavior of Kianna's versus Samantha's investment. Students should quickly discover Kianna's investment exhibits linear growth while Samantha's shows exponential growth.
- Solidify this understanding by having students graph the two functions that represent the total value of the two investments.
- Once students make this discovery, begin a conversation with them about which type of interest would be more advantageous for any investments. Take this opportunity to make connection to MA.912.F.1.7 (i.e., Suppose Kianna starts with $10,000 and invests it at 10% simple interest, and Samantha starts with $1,000 and invests it at 10% compounded annually. How long will it be before Samantha's account has more in it that Kianna's?).
  - Remember the expectation for this benchmark is for students to explain *why* these relationships occur. Be sure to discuss the equations formed and that the variation of years is used as a factor in the simple interest formula and as an exponent in the compound interest formula.

Common Misconceptions or Errors

- For simple interest scenarios, students may forget to add the principal to the interest they calculate using the simple interest formula when calculating the total amount of an investment over time.
- For compound interest scenarios, students may forget to subtract the principal from the total amount they calculate using a compound interest formula when calculating the total

interest earned over time.
- In the formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, students may confuse the compounding period variable n, with the time variable, t.
- When forming compound interest equations, students may forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)
Phoenix invests in a savings account that applies simple interest.

Part A. How will her investment grow, linearly or exponentially? Justify your answer.
Part B. If Phoenix invests \$725 and earns an annual rate of 4.2%, write an equation that would represent the total amount she would have at the end of each year.
Part C. How long will it take for her initial investment to double?
Part D. If instead the savings account had interest at the same rate but was compounded annually, how much money would she have after the amount of time found in Part C?

Instructional Items

Instructional Item 1
Trevarius invests in a savings account that applies compound interest annually. What interest rate would be required in order to double the initial savings in 15 years?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Data Analysis and Probability

MA.912.DP.2 *Solve problems involving univariate and bivariate numerical data.*

MA.912.DP.2.8

Benchmark

MA.912.DP.2.8	Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:
Clarification 1: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.3.4, MA.912.AR.3.8
- MA.912.F.1.1

Terms from the K-12 Glossary

- Bivariate Data
- Cluster (Data)
- Line of Fit
- Quadratic Function
- Scatterplot
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3
- MA.912.DP.1.1, MA.912.DP.1.2, MA.912.DP.1.3 (Algebra 1)
- MA.912.DP.2.4, MA.912.DP.2.6 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students worked with bivariate numerical data that suggested a linear association and used the model to make predictions. In Algebra 2, students work with bivariate numerical data that suggests a quadratic association and interpret the intercepts or the vertex of the model as it relates to a real-world context. In future courses students will evaluate what statistical methods are most appropriate.

- In this benchmark, students are fitting a quadratic function to numerical bivariate data, interpreting the intercepts and vertex based on the context and using that quadratic function to make predictions about values that correspond to parts of the graph that lie beyond or within the domain of the scatter plot.

- Instruction includes determining from a scatter plot whether a quadratic model seems appropriate.
  - In many cases, the context of the problem will suggest that a quadratic model is appropriate.
  - If this is not the case, students may visually look for a parabolic shape in the scatter plot.
    - For example, if the scatter plot roughly forms a U shape or upside-down U shape similar to the graph of a quadratic function, then a quadratic model may be appropriate. In cases where the scatter plot seems to have a vertex on the y-axis and only positive values of x are plotted, then the U shape will not be apparent and a more sophisticated analysis beyond the scope of this benchmark may be required.
  - Another method for checking whether a quadratic model is an appropriate fit for a scatter plot, is to calculate the second differences of the data points and determine whether they are roughly constant. This method works best when the x values are equally spaced.
- Instruction makes connection to relevant, real-world applications. *(MTR.7.1)*
- Instruction includes the use of technology for students to understand the difference between a curve of fit and a curve of best fit.
- During instruction it is important to distinguish the difference between a “curve of fit” and the “curve of best fit.”
  - A “curve of fit” is used when students are visually investigating numerical bivariate data that appears to have a non-linear relationship and can sketch a curve (using a writing instrument) that appears to “fit” the data. Using this “curve of fit” students can estimate its vertex and intercepts and interpret that information in the context of the data.
  - The “curve of best fit” is used when the data is further analyzed using regression calculations (the process of minimizing the squared distances from the individual data values to the curve), often done with the assistance of technology.

Common Misconceptions or Errors

- Students may mistake the model to have a linear association.
- Students may not know how to sketch a curve of fit.
- Students may confuse the two variables when interpreting the data as it relates to the context.
- Students may have trouble interpreting the vertex and intercepts in relation to the context of the problem.
- Students may not know the difference between interpolation (predictions with the domain of a data set) and extrapolation (predictions beyond the domain of a data set).
- Students may need assistance working with spreadsheets, calculators or other types of technology as it relates to data.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)
A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of a car with a specific make and model. The results are shown in the table below.

x	y
15	21.5
25	26.8
35	29.2
45	35
50	28.7
55	26.2
65	22.3
75	20

Part A. Create a scatterplot of the data.
Part B. Use second differences to determine whether a quadratic model seems to be appropriate. For simplicity, the point where $x = 50$ can be ignored.
Part C. Sketch a quadratic curve of fit for the data. Interpret the vertex in the context of the data.
Part D. Is it meaningful to give an interpretation of the intercepts of the data.
Part E. Using your quadratic curve of fit, estimate the mileage when the car is traveling at a speed of 60 miles per hour.
Part F. Use technology to find a quadratic model that best fits the data.
Part G. Use the model to predict the mileage of the car when the car is traveling at a speed of 60 miles per hour. How close was your prediction to the model's prediction?

Instructional Task 2 (MTR.3.1, MTR.7.1)
A kicker on a high school football team practices kicking field goals. The coach takes a video of one of his kicks. The video takes one frame every tenth of a second and the height of the ball in each frame can be approximately measured. The table shows these heights during 1.5 seconds of the video.

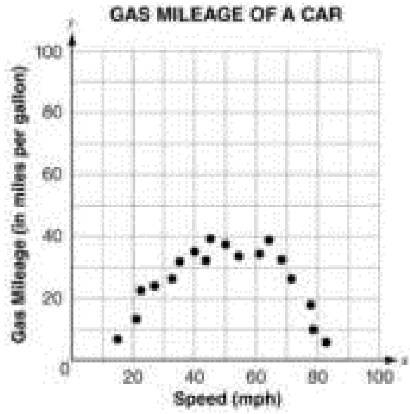
t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
h(t)	0	5.3	10.4	15.1	19.4	23.5	27.2	30.7	33.8	36.5	39	41.1	43.0	44.5	45.6	46.5

Part A. Create a scatter plot of the data.
Part B. Does a quadratic model seem appropriate for this data? Explain.
Part C. Sketch a quadratic curve of fit for the data. Interpret the vertex in the context of the data.
Part D. Determine the time, t, at which the ball was kicked and determine how far the ball will go before it hits the ground.
Part E. If the goal post is 40 yards from the kicker and the height of the crossbar is 10 feet, will the kicker make the field goal if the direction of the kick was accurate?

Instructional Items

Instructional Item 1

The graph below represents the gas mileage, in miles per gallon, of a car at different speeds.



If x denotes the speed and y denotes the mileage per gallon, which equation best represents the relationship between the speed of the car and its fuel efficiency?

a. $y = 0.025x^2 - 2.5x - 25$
b. $y = 0.025x^2 + 2.5x + 25$
c. $y = -0.025x^2 + 2.5x + 25$
d. $y = -0.025x^2 + 2.5x - 25$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.2.9

Benchmark

MA.912.DP.2.9 Fit an exponential function to bivariate numerical data that suggests an exponential association. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:

Clarification 1: Instruction focuses on determining whether an exponential model is appropriate by taking the logarithm of the dependent variable using spreadsheets and other technology.
Clarification 2: Instruction includes determining whether the transformed scatterplot has an appropriate line of best fit, and interpreting the y-intercept and slope of the line of best fit.
Clarification 3: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.4, MA.912.AR.5.5, MA.912.AR.5.7
- MA.912.NSO.1.6
- MA.912.F.1.1

Terms from the K-12 Glossary

- Bivariate Data
- Exponential Function
- Line of Fit
- Scatterplot
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3
- MA.912.DP.1.1, MA.912.DP.1.2, MA.912.DP.1.3 (Algebra 1)
- MA.912.DP.2.4, MA.912.DP.2.6 (Algebra 1)

Next Benchmarks

- Due to multiple pathways in high school, there may be other next benchmarks depending on the student.

Purpose and Instructional Strategies

In Algebra 1, students work with bivariate numerical data that suggest a linear association and use the model to make predictions. In Algebra 2, students extend this work to bivariate numerical data that suggest an exponential association and use a model for this data as it relates to a real-world context. In future courses, students will evaluate what statistical methods are most appropriate.

- Instruction focuses on determining whether an exponential model is appropriate by taking the logarithm of the dependent variable using spreadsheets, calculators or other technology. If the variables are x and y, and $\log(y)$ is plotted on the vertical axis with x being plotted on the horizontal axis, and if the resulting display suggests a linear association, then an exponential association is suggested for the relationship between the original independent and dependent variables.
- Instruction includes interpreting the y-intercept and the slope of the transformed function.
  - Transforming to a linear model using logarithms: start with an exponential model $y = a(b)^x$, where a is the initial amount and b is the growth factor.

    $$\log(y) = \log(a(b)^x)$$
    $$\log(y) = \log a + \log b^x$$
    $$\log(y) = \log a + x \log b$$

    Transformed model: $\log(y) = x \log b + \log a$, with $\log b$ representing the rate of change and $\log a$ representing the initial value.
- Instruction makes the connection to relevant, real-world applications. *(MTR.7.1)*
- Instruction includes using both logarithms, base 10 (log) and natural logarithms (ln), to transform data.
- Instruction includes the use of technology for students to understand and compare the difference between a curve of fit and a curve of best fit.
- During instruction, it is important to distinguish the difference between a “line of fit” and the “line of best fit” when looking at the **transformed** data.

- A "line of fit" is used when students are visually investigating numerical bivariate data that appear to have a linear relationship and can sketch a line that appears to "fit" the data. Using this "line of fit" students can estimate the slope and y-intercept of the **transformed scatterplot** and use that information to interpret the context of the data.
- The "line of best fit" is used when the **transformed** data is further analyzed using regression calculations (the process of minimizing the squared distances from the individual data values to the line), often done with the assistance of technology.

- Instruction addresses making predictions within the range of the data using interpolation and making predictions outside the range of the data using extrapolation. Issues with knowing how the relationship continues outside the range of the data and why extrapolation can lead to illogical results should be communicated. *(MTR.6.1)*

Common Misconceptions or Errors

- Students may not know how to sketch a line of fit.
  - For example, they may always go through the first and last points of data.
- Students may confuse the two variables (independent and dependent) when interpreting the data as related to the context.
- Students may have difficulty interpreting the slope and the y-intercept of the regression equation generated from the transformed data.
- Students may have trouble transforming the exponential relationship to a linear one using logarithms.
- Students may not know the difference between interpolation (predictions within the range of a data set) and extrapolation (predictions beyond the range of a data set).
- Students may need assistance working with spreadsheets, calculators or other types of technology as it relates to data.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Pharmaceutical company A is developing a model to determine the length of time it takes a person's body to absorb a new drug being tested. The drug is administered to the subject (800mg) and a blood sample is taken from the subject every three hours. The table below shows the results.

Drug Absorption	
Hours Since Drug Administered	Amount of Drug in Body (mg)
0	800
3	630
6	410
9	342
12	225
15	165
18	104
21	73
24	49
27	38
30	27

Part A. Is an exponential relationship appropriate to model the data? Explain.
Part B. Create a scatter plot to model the data by transforming the data to a linear model.
Part C. Sketch a line of fit for the transformed model.
Part D. Interpret the slope and y-intercept of the transformed model in terms of the context of this situation.
Part E. If a drug is not detectable when there is less than 0.005 mg in a person's bloodstream, estimate how long it would take for the drug to not be detected.

Instructional Items

Instructional Item 1

The table below gives air pressures in kPa at selected altitudes measured in km.

x (km)	0	1	2	3	4	5
y (kPa)	99	89	80	73	65	60

Part A. Use technology to create the model that best fits the data.
Part B. Use the model from Part A to estimate the air pressure in kPa 10 km above sea level.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*