Name ______________________________ Date ______________

Prime Factorization

Because 3 is a factor of 24 and $3 \cdot 8 = 24$, 8 is also a factor of 24. The pair 3, 8 is called a **factor pair** of 24.

The **prime factorization** of a composite number is the number written as a product of its prime factors. You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product.

Example 1 **A classroom has 42 students. The teacher arranges the students in rows. Each row has the same number of students. How many possible arrangements are there?**

Use the factor pairs of 42 to find the number of arrangements.

$42 = 1 \cdot 42$	1 row of 42 or 42 rows of 1	$42 = 2 \cdot 21$	2 rows of 21 or 21 rows of 2
$42 = 3 \cdot 14$	3 rows of 14 or 14 rows of 3	$42 = 6 \cdot 7$	6 rows of 7 or 7 rows of 6

▶ There are 8 possible arrangements: 1 row of 42, 42 rows of 1, 2 rows of 21, 21 rows of 2, 3 rows of 14, 14 rows of 3, 6 rows of 7, or 7 rows of 6.

Example 2 **Write the prime factorization of 54.**

Choose any factor pair of 54 to begin the factor tree.

Tree 1

54

$3 \cdot 18$

$2 \cdot 9$

$3 \cdot 3$

$54 = 3 \cdot 2 \cdot 3 \cdot 3$

Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

Tree 2

54

$2 \cdot 27$

$3 \cdot 9$

$3 \cdot 3$

$54 = 2 \cdot 3 \cdot 3 \cdot 3$

▶ The prime factorization of 54 is $2 \cdot 3 \cdot 3 \cdot 3$, or $2 \cdot 3^3$.

Practice

Check your answers at BigIdeasMath.com.

List the factor pairs of the number.

1. 16 **2.** 30 **3.** 63

4. 100 **5.** 135 **6.** 275

Write the prime factorization of the number.

7. 24 **8.** 66 **9.** 50 **10.** 80

11. 98 **12.** 126 **13.** 154 **14.** 310

Find the greatest perfect square that is a factor of the number.

15. 117 **16.** 150 **17.** 539 **18.** 936

19. **EXERCISE** An exercise class has 28 participants. The instructor arranges the participants in rows. Each row has the same number of participants. How many possible arrangements are there?

Name ______________________________ Date ______________

Greatest Common Factor

Factors that are shared by two or more numbers are called **common factors.** The greatest of the common factors is called the **greatest common factor** (GCF). There are several different ways to find the GCF of two or more numbers.

Example 1 **Find the greatest common factor (GCF) of 56 and 104.**

Method 1 List the factors of each number. Then circle the common factors.

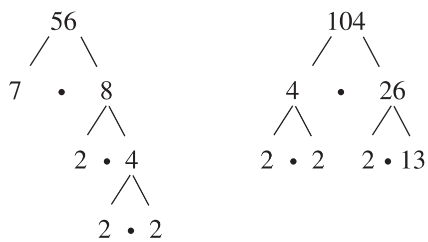
Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

Factors of 104: 1, 2, 4, 8, 13, 26, 52, 104

The common factors are 1, 2, 4, and 8. The greatest of these common factors is 8.

▶ So, the GCF of 56 and 104 is 8.

Method 2 Make a factor tree for each number.



Write the prime factorization of each number. Then circle the common prime factors. The GCF is the product of the common prime factors.

$$56 = 2 \cdot 2 \cdot 2 \cdot 7$$
$$104 = 2 \cdot 2 \cdot 2 \cdot 13$$

▶ So, the GCF of 56 and 104 is $2 \cdot 2 \cdot 2 = 8$.

Practice

Check your answers at BigIdeasMath.com.

Find the GCF of the numbers using the two methods shown above.

1. 30, 45 **2.** 12, 54 **3.** 16, 96 **4.** 42, 98

5. 27, 66 **6.** 50, 160 **7.** 21, 70 **8.** 76, 95

9. 60, 84 **10.** 60, 120, 210 **11.** 44. 64, 100 **12.** 15, 28, 70

13. Write a set of two numbers that have a GCF of 20. Explain how you found your answer.

14. Write a set of three numbers that have a GCF of 25. Explain how you found your answer.

15. **BOUQUETS** A florist is making identical bouquets using 90 white roses, 60 red roses, and 45 pink roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

16. **FABRIC** You have two pieces of fabric. One piece is 6 feet wide and the other piece is 7.5 feet wide. You want to cut both pieces into strips of equal width that are as wide as possible. How wide should you cut the strips of fabric?

Name ____________________ Date ____________

Least Common Multiple

Multiples that are shared by two or more numbers are called **common multiples.** The least of the common multiples is called the **least common multiple** (LCM). There are several different ways to find the LCM of two or more numbers.

Example 1 **Find the least common multiple (LCM) of 18 and 30.**

Method 1 List the multiples of each number. Then circle the common multiples.

Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180

Multiples of 30: 30, 60, 90, 120, 150, 180, 210

Some common multiples of 18 and 30 are 90 and 180. The least of these common multiples is 90.

▶ So, the LCM of 18 and 30 is 90.

Method 2 Make a factor tree for each number.

18 → 2 • 9, 9 → 3 • 3

30 → 5 • 6, 6 → 2 • 3

Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$18 = 2 \cdot 3 \cdot 3$ — 2 appears once in both factorizations, so circle it here. 3 appears more often here, so circle all 3s.

$30 = 2 \cdot 3 \cdot 5$ — 5 appears once. Do not circle the 2s or 3s again.

$2 \cdot 3 \cdot 3 \cdot 5 = 90$ — Find the product of the circled factors.

▶ So, the LCM of 18 is 30 is 90.

Practice

Check your answers at BigIdeasMath.com.

Find the LCM of the numbers using the two methods shown above.

1. 6, 10 **2.** 12, 16 **3.** 15, 25 **4.** 20, 50

5. 9, 24 **6.** 10, 22 **7.** 25, 35 **8.** 12, 14

9. 4, 6, 10 **10.** 6, 9, 12 **11.** 10. 18, 20 **12.** 16, 24, 30

13. SPOTLIGHTS A spotlight at a dance club flashes every 25 seconds. Another spotlight flashes every 30 seconds. Both lights just flashed. After how many minutes will both lights flash at the same time again?

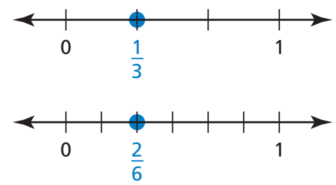
14. CLOCKS A clock chimes every 15 minutes. Another clock chimes every half hour. Both clocks just chimed at midnight. How many times will both clocks chime at the same time over the next 24 hours?

15. SUBWAYS Three subway lines arrive at a station at the same time. Line A arrives at the station every 20 minutes, Line B arrives every 24 minutes, and Line C arrives every 25 minutes. How long must you wait until all three lines arrive at the station at the same time again?

Name ______________________________ Date ______________

Equivalent Fractions and Simplifying Fractions

The number lines show the graphs of two fractions, $\frac{1}{3}$ and $\frac{2}{6}$. These fractions represent the same number. Two fractions that represent the same number are called **equivalent fractions**. To write equivalent fractions, you can multiply or divide the numerator and the denominator by the same nonzero number.



Example 1 **Write two fractions that are equivalent to $\frac{8}{12}$.**

Multiply the numerator and denominator by 2.

$$\frac{8}{12} = \frac{8 \cdot 2}{12 \cdot 2} = \frac{16}{24}$$

Divide the number and denominator by 2.

$$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

▶ Two equivalent fractions are $\frac{16}{24}$ and $\frac{4}{6}$.

A fraction is in **simplest form** when its numerator and its denominator have no common factors besides 1.

Example 2 **Write the fraction $\frac{18}{24}$ in simplest form.**

Divide the numerator and denominator by 6, the greatest common factor of 18 and 24.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

▶ $\frac{18}{24}$ in simplest form is $\frac{3}{4}$.

Practice

Check your answers at BigIdeasMath.com.

Write two fractions that are equivalent to the given fraction.

1. $\frac{4}{10}$ **2.** $\frac{3}{7}$ **3.** $\frac{10}{15}$ **4.** $\frac{16}{20}$

5. $\frac{9}{30}$ **6.** $\frac{1}{8}$ **7.** $\frac{9}{16}$ **8.** $\frac{12}{14}$

Write the fraction in simplest form.

9. $\frac{18}{27}$ **10.** $\frac{3}{18}$ **11.** $\frac{35}{50}$ **12.** $\frac{14}{32}$

13. $\frac{4}{36}$ **14.** $\frac{48}{80}$ **15.** $\frac{24}{63}$ **16.** $\frac{33}{88}$

17. $\frac{45}{100}$ **18.** $\frac{60}{150}$ **19.** $\frac{48}{96}$ **20.** $\frac{110}{170}$

21. Is the fraction $\frac{45}{61}$ in simplest form? Explain.

22. Write five fractions that each simplify to one-ninth.

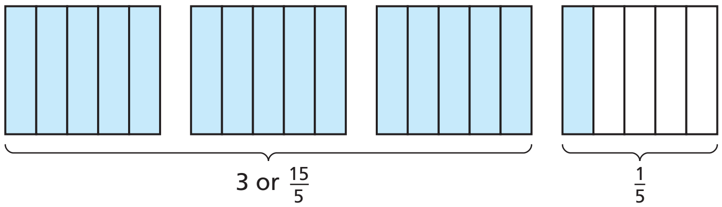
23. **SLEEP** It is recommended that 10- to 17-year old students should sleep about 9 hours each night. What fraction of the day is this? Write your answer in simplest form.

Name ______________________ Date ____________

Mixed Numbers and Improper Fractions

A **mixed number** is the sum of a whole number and a fraction. An **improper fraction** is a fraction with a numerator that is greater than or equal to the denominator.

The shaded part of the model represents the mixed number $3\frac{1}{5}$ and the improper fraction $\frac{16}{5}$.



Example 1 **Write $4\frac{5}{8}$ as an improper fraction.**

$$\begin{aligned} 4\frac{5}{8} &= 4 + \frac{5}{8} && \text{Definition of mixed number} \\ &= \frac{32}{8} + \frac{5}{8} && \text{1 whole} = \tfrac{8}{8}\text{. So, 4 wholes} = \tfrac{32}{8}. \\ &= \frac{37}{8} && \text{Add.} \end{aligned}$$

▶ $4\frac{5}{8}$ written as an improper fraction is $\frac{37}{8}$.

Example 2 **Write $\frac{19}{7}$ as a mixed number.**

$$\begin{array}{r} 2 \\ 7\overline{)19} \\ \underline{14} \\ 5 \end{array}$$

Divide the numerator, 19, by the denominator, 7. The quotient is 2.

The remainder is 5. Write the remainder as a fraction, $\frac{\text{remainder}}{\text{divisor}}$.

▶ $\frac{19}{7}$ written as a mixed number is $2\frac{5}{7}$.

Practice

Check your answers at BigIdeasMath.com.

Write the mixed number as an improper fraction.

1. $1\frac{4}{5}$ **2.** $3\frac{1}{6}$ **3.** $10\frac{7}{10}$ **4.** $2\frac{12}{13}$

5. $6\frac{5}{9}$ **6.** $4\frac{3}{20}$ **7.** $7\frac{6}{7}$ **8.** $25\frac{2}{3}$

Write the improper fraction as a mixed number.

9. $\frac{9}{2}$ **10.** $\frac{13}{5}$ **11.** $\frac{25}{3}$ **12.** $\frac{31}{9}$

13. $\frac{59}{10}$ **14.** $\frac{43}{4}$ **15.** $\frac{35}{8}$ **16.** $\frac{67}{11}$

17. Find the improper fraction with a denominator of 8 that is equivalent to $3\frac{1}{2}$.

18. Find the improper fraction with a denominator of 12 that is equivalent to $5\frac{3}{4}$.

Name ________________________________ Date ______________

Adding and Subtracting Fractions

To add or subtract two fractions with *like denominators*, write the sum or difference of the numerators over the denominator.

Adding or Subtracting Fractions with Like Denominators
$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$ $\quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$

Example 1 Find $\frac{7}{12} + \frac{1}{12}$.

$\frac{7}{12} + \frac{1}{12} = \frac{7+1}{12}$ Add the numerators.

$= \frac{8}{12}$, or $\frac{2}{3}$ Simplify.

Example 2 Find $\frac{7}{9} - \frac{2}{9}$.

$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9}$ Subtract the numerators.

$= \frac{5}{9}$ Simplify.

To add or subtract two fractions with *unlike denominators*, first write equivalent fractions with a common denominator. There are two methods you can use.

Adding or Subtracting Fractions with Unlike Denominators	
Method 1	Multiply the numerator and the denominator of each fraction by the denominator of the other fraction.
Method 2	Use the **least common denominator** (LCD). The LCD of two or more fractions is the least common multiple (LCM) of the denominators.

Example 3 Find $\frac{1}{8} + \frac{5}{6}$.

Method 1: $\frac{1}{8} + \frac{5}{6} = \frac{1 \cdot 6}{8 \cdot 6} + \frac{5 \cdot 8}{6 \cdot 8}$ Rewrite using a common denominator of $8 \cdot 6 = 48$.

$= \frac{6}{48} + \frac{40}{48}$ Multiply.

$= \frac{46}{48}$, or $\frac{23}{24}$ Simplify.

Example 4 Find $5\frac{3}{4} - 1\frac{7}{10}$.

Method 2: Rewrite the difference as $\frac{23}{4} - \frac{17}{10}$.

The LCM of 4 and 10 is 20. So, the LCD is 20.

$\frac{23}{4} - \frac{17}{10} = \frac{23 \cdot 5}{4 \cdot 5} - \frac{17 \cdot 2}{10 \cdot 2}$ Rewrite using the LCD, 20.

$= \frac{115}{20} - \frac{34}{20}$ Multiply.

$= \frac{81}{20}$, or $4\frac{1}{20}$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Evaluate.

1. $\frac{1}{14} + \frac{5}{14}$ **2.** $\frac{2}{5} + \frac{1}{5}$ **3.** $\frac{9}{10} - \frac{1}{10}$ **4.** $\frac{11}{16} - \frac{3}{16}$

5. $\frac{5}{8} + \frac{7}{8}$ **6.** $\frac{1}{6} + \frac{1}{6}$ **7.** $\frac{7}{9} + \frac{2}{3}$ **8.** $\frac{3}{5} + \frac{4}{7}$

9. $\frac{3}{4} - \frac{1}{6}$ **10.** $\frac{7}{12} - \frac{5}{9}$ **11.** $\frac{9}{10} - \frac{5}{6}$ **12.** $\frac{5}{12} + \frac{11}{16}$

13. $2\frac{3}{5} + 1\frac{2}{5}$ **14.** $4\frac{6}{7} - 2\frac{4}{7}$ **15.** $5\frac{5}{12} + 3\frac{3}{8}$

16. $8\frac{1}{3} - 3\frac{2}{11}$ **17.** $\frac{1}{2} + 3\frac{2}{9}$ **18.** $4\frac{3}{14} - \frac{1}{7}$

19. $\frac{2}{7} + \frac{3}{4} + \frac{1}{2}$ **20.** $\frac{13}{16} - \frac{1}{4} - \frac{3}{8}$ **21.** $2\frac{1}{6} - \frac{5}{9} + \frac{2}{3}$

Name ____________________ Date ____________

Multiplying and Dividing Fractions

To multiply two fractions, multiply the numerators and multiply the denominators.

Multiplying Fractions

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}, \text{ where } b, d \neq 0$$

Example 1 Find $\frac{2}{5} \bullet \frac{3}{8}$.

$\frac{2}{5} \bullet \frac{3}{8} = \frac{2 \bullet 3}{5 \bullet 8}$ Multiply the numerators. Multiply the denominators.

$= \frac{\cancel{2}^{1} \bullet 3}{8 \bullet \cancel{8}_{4}}$ Divide out common factors.

$= \frac{3}{20}$ Simplify.

Example 2 Find $5\frac{1}{2} \bullet \frac{3}{4}$.

$5\frac{1}{2} \bullet \frac{3}{4} = \frac{11}{2} \bullet \frac{3}{4}$ Rewrite $5\frac{1}{2}$ as $\frac{11}{2}$.

$= \frac{11 \bullet 3}{2 \bullet 4}$ Multiply the numerators. Multiply the denominators.

$= \frac{33}{8}$, or $4\frac{1}{8}$ Simplify.

Two numbers whose product is 1 are **reciprocals**. To write the reciprocal of a number, write the number as a fraction. Then invert the fraction. Every number except 0 has a reciprocal.

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{a \bullet d}{b \bullet c}, \text{ where } b, c, d \neq 0$$

Example 3 Find $\frac{3}{7} \div \frac{5}{6}$.

$\frac{3}{7} \div \frac{5}{6} = \frac{3}{7} \bullet \frac{6}{5}$ Multiply by the reciprocal of $\frac{5}{6}$, which is $\frac{6}{5}$.

$= \frac{3 \bullet 6}{7 \bullet 5}$ Multiply.

$= \frac{18}{35}$ Simplify.

Example 4 Find $8 \div 2\frac{1}{3}$.

$8 \div 2\frac{1}{3} = 8 \div \frac{7}{3}$ Rewrite $2\frac{1}{3}$ as $\frac{7}{3}$.

$= 8 \bullet \frac{3}{7}$ Multiply by the reciprocal of $\frac{7}{3}$, which is $\frac{3}{7}$.

$= \frac{8 \bullet 3}{7}$ Multiply.

$= \frac{24}{7}$, or $3\frac{3}{7}$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Write the reciprocal of the number.

1. $\frac{3}{8}$
2. 7
3. -12
4. $-\frac{6}{5}$

Evaluate.

5. $\frac{3}{4} \bullet \frac{1}{6}$
6. $\frac{3}{10} \bullet \frac{2}{3}$
7. $\frac{4}{9} \bullet \frac{2}{9}$
8. $\frac{5}{8} \bullet \frac{7}{12}$
9. $4 \bullet \frac{3}{16}$
10. $3\frac{1}{2} \bullet \frac{6}{7}$
11. $1\frac{7}{20} \bullet 2\frac{4}{5}$
12. $\frac{1}{10} \bullet 10$
13. $\frac{1}{6} \div \frac{1}{2}$
14. $\frac{7}{8} \div \frac{7}{8}$
15. $\frac{9}{10} \div \frac{3}{5}$
16. $\frac{3}{4} \div \frac{5}{8}$
17. $18 \div \frac{2}{3}$
18. $7\frac{1}{2} \div 2\frac{1}{10}$
19. $6\frac{3}{7} \div 3$
20. $1\frac{3}{25} \div \frac{1}{5}$

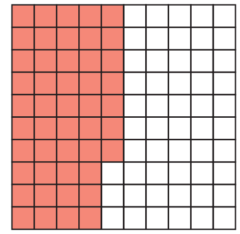
21. **AREA** Find the area of a rectangular court that is $21\frac{3}{5}$ meters long and $13\frac{3}{4}$ meters wide.

22. **CARPENTRY** How many $1\frac{1}{4}$-foot pieces can you cut from a piece of wood that is 20 feet long?

Name ______________________________________ Date ________________

Writing Fractions, Decimals, and Percents

A **percent** is a part-to-whole ratio where the whole is 100. The symbol for percent is %.



In the model, 47 of the 100 squares are shaded. You can write the shaded part of the model as a fraction, a decimal, or a percent.

Fraction: forty-seven out of one hundred, or $\frac{47}{100}$

Decimal: forty-seven hundredths, or 0.47

Percent: forty-seven percent, or 47%

Example 1 Write the percent or decimal as a fraction.

a. $86\% = \frac{86}{100} = \frac{43}{50}$ **b.** $125\% = \frac{125}{100} = \frac{5}{4}$, or $1\frac{1}{4}$ **c.** $0.2 = \frac{2}{10} = \frac{1}{5}$

Example 2 Write the percent or fraction as a decimal.

a. $19\% = 19. = 0.19$ **b.** $\frac{3}{8} = 3 \div 8 = 0.375$ **c.** $\frac{3}{20} = \frac{3 \cdot 5}{20 \cdot 5} = \frac{15}{100} = 0.15$

Example 3 Write the decimal or fraction as a percent.

a. $0.34 = 0.34 = 34\%$ **b.** $0.915 = 0.915 = 91.5\%$ **c.** $\frac{3}{2} = \frac{3 \cdot 50}{2 \cdot 50} = \frac{150}{100} = 150\%$

Practice

Check your answers at BigIdeasMath.com.

Write the percent or decimal as a fraction.

1. 0.7 **2.** 0.08 **3.** 0.79 **4.** 1.75

5. 0.125 **6.** 0.744 **7.** 25% **8.** 38%

9. 96% **10.** 1% **11.** 225% **12.** 0.5%

Write the percent or fraction as a decimal.

13. $\frac{3}{4}$ **14.** $\frac{5}{8}$ **15.** $\frac{11}{50}$ **16.** $\frac{17}{25}$

17. $\frac{31}{20}$ **18.** $\frac{101}{200}$ **19.** 10% **20.** 27%

21. 65% **22.** 100% **23.** 0.8% **24.** 350%

Write the decimal or fraction as a percent.

25. 0.35 **26.** 0.91 **27.** 0.5 **28.** 1.4

29. 0.02 **30.** 0.006 **31.** $\frac{3}{25}$ **32.** $\frac{17}{20}$

33. $\frac{7}{8}$ **34.** $\frac{1}{16}$ **35.** $\frac{11}{2}$ **36.** $\frac{23}{40}$

37. Which is greater, $\frac{5}{6}$ or 83%?

38. Which is greater, 11.1% or $0.\overline{1}$?

Name ______________________________ Date ______________

Calculating with Percents

To represent"a is p percent of w," use the **percent proportion** or the **percent equation**.

Percent Proportion	Percent Equation
part → a, whole → w: $\dfrac{a}{w} = \dfrac{p}{100}$ ← percent	part → $a = p \cdot w$ ← whole; ↑ percent in fraction or decimal form

Example 1 Answer each question.

a. What percent of 40 is 18?

percent proportion:

$$\frac{18}{40} = \frac{p}{100}$$

$$1800 = 40p$$

$$45 = p$$

percent equation:

$$18 = p \cdot 40$$

$$0.45 = p$$

▶ So, 45% of 40 is 18.

b. What number is 32% of 75?

$$\frac{a}{75} = \frac{32}{100}$$

$$100a = 2400$$

$$a = 24$$

$$a = 0.32 \cdot 75$$

$$a = 24$$

▶ So, 24 is 32% of 75.

c. 125% of what number is 80?

$$\frac{80}{w} = \frac{125}{100}$$

$$8000 = 125w$$

$$64 = w$$

$$80 = 1.25 \cdot w$$

$$64 = w$$

▶ So, 125% of 64 is 80.

A **percent of change** is the percent that a quantity changes from the original amount.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

Example 2 Find the percent of change.

a. Your number of hours worked increases from 24 hours to 42 hours.

$$\frac{42 - 24}{24} = \frac{18}{24} = 0.75$$

▶ The change is a 75% increase.

b. A price decreases from \$25.75 to \$15.50.

$$\frac{25.75 - 15.50}{25.75} = \frac{10.25}{25.75} \approx 0.40$$

▶ The change is about a 40% decrease.

Practice

Check your answers at BigIdeasMath.com.

Use the percent proportion to answer the question.

1. 80% of what number is 64?

2. What number is 15% of 130?

3. What percent of 240 is 6?

Use the percent equation to answer the question.

4. What number is 55% of 94?

5. 3% of what number is 111?

6. What percent of 72 is 64?

Find the percent of change. Round to the nearest percent, if necessary.

7. A club's membership increases from 540 members to 995 members.

8. A graduating class decreases from 482 students to 398 students.

Name ______________________________ Date ______________

Converting Measures

The **U.S. customary system** is a system of measurement that contains units for length, capacity, and weight. The **metric system** is a decimal system of measurement, based on powers of 10, that contains units for length, capacity, and mass. To convert from one unit of measure to another, multiply by one or more *conversion factors*. A **conversion factor** is a rate that equals 1 and can be written using fraction notation. A list of equivalent measures is located in the back of your textbook. There are two different conversion factors for each statement of equivalent measures.

Statement of Equivalent Measures	Conversion Factors
1 in. = 2.54 cm	$\frac{1\text{ in.}}{2.54\text{ cm}}$ and $\frac{2.54\text{ cm}}{1\text{ in.}}$

Example 1 **Convert 24 quarts to gallons.**

$$24\ \cancel{\text{qt}} \cdot \frac{1\text{ gal}}{4\ \cancel{\text{qt}}} = \frac{24 \cdot 1\text{ gal}}{4} = 6\text{ gal}$$

▶ So, 24 quarts is 6 gallons.

Example 2 **Convert 6.5 meters to feet.**

$$6.5\ \cancel{\text{m}} \cdot \frac{3.28\text{ ft}}{1\ \cancel{\text{m}}} = 6.5 \cdot 3.28\text{ ft} \approx 21.32\text{ ft}$$

▶ So, 6.5 meters is about 21.32 feet.

Example 3 **A sports car reaches a maximum speed of 210 miles per hour. What is the speed in feet per second?**

$$\frac{210\ \cancel{\text{mi}}}{1\ \cancel{\text{h}}}\left(\frac{5280\text{ ft}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\text{ sec}}\right) = \frac{210 \cdot 5280\text{ ft}}{3600\text{ sec}} = \frac{1{,}108{,}800\text{ ft}}{3600\text{ sec}} = \frac{308\text{ ft}}{1\text{ sec}}$$

▶ The speed is 308 feet per second.

Practice

Check your answers at BigIdeasMath.com.

Copy and complete the statement. Round to the nearest hundredth, if necessary.

1. 36 oz = ☐ lb **2.** 3400 mL = ☐ L **3.** 18 pt = ☐ c **4.** 7 kg = ☐ g

5. 246 ft = ☐ yd **6.** 65 cm = ☐ mm **7.** 8.5 mi = ☐ ft **8.** 8 h = ☐ sec

9. 20 cm ≈ ☐ in. **10.** 150 lb ≈ ☐ kg **11.** 2.5 km ≈ ☐ mi **12.** 4 gal ≈ ☐ L

13. $6\frac{1}{2}$ ft ≈ ☐ m **14.** 12 L ≈ ☐ qt **15.** 2500 mL ≈ ☐ c **16.** 24 oz ≈ ☐ g

17. **ENGINE COOLANT** An automobile engine holds 5.8 liters of coolant. You have 2 gallons of coolant. Do you have enough coolant to fill the engine? Explain.

18. **HUMMINGBIRD** A hummingbird's heart rate is about 1250 beats per minute. What is the heart rate in beats per second?

19. **EROSION** A shoreline is eroding at a rate of 6.8 meters per year. What is the erosion rate in feet per week?

20. **SWIMMING POOL** A swimming pool is draining at a rate of 10 fluid ounces per second. What is the drainage rate in gallons per hour?

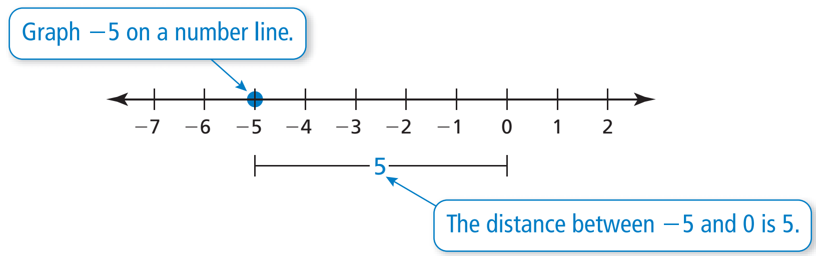
Name ______________________________ Date ______________

Operations with Integers

Adding and Subtracting Integers

The **absolute value** of an integer is the distance between the number and 0 on a number line. The absolute value of a number x is written as $|x|$.

Example 1 **Find the absolute value of -5.**



▶ So, $|-5| = 5$.

Rules for Adding and Subtracting Integers	
Adding:	To add integers with the *same* sign, add the absolute values of the integers. Then use the common sign. To add integers with *different* signs, subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.
Subtracting:	To subtract an integer, add its opposite.

Example 2 **Find (a) $-3 + (-8)$ and (b) $-9 + 6$.**

a. $-3 + (-8) = -11$ Add $|-3|$ and $|-8|$. Use the common sign.

▶ The sum is -11.

b. $-9 + 6 = -3$ $|-9| > |6|$. So, subtract $|6|$ from $|-9|$. Use the sign of -9.

▶ The sum is -3.

Example 3 **Find (a) $5 - (-12)$ and (b) $1 - 7$.**

a. $5 - (-12) = 5 + 12$ Add the opposite of -12.

$= 17$ Add.

▶ The difference is 17.

b. $1 - 7 = 1 + (-7)$ Add the opposite of 7.

$= -6$ Add.

▶ The difference is -6.

Example 4 **Simplify $|-14 - (-10)|$.**

$|-14 - (-10)| = |-14 + 10|$ Add the opposite of -10.

$= |-4|$ Add.

$= 4$ Find the absolute value.

▶ So, $|-14 - (-10)| = 4$.

Name ______________________________ Date ______________

Operations with Integers

Multiplying and Dividing Integers

Rules for Multiplying and Dividing Integers
Multiplying and Dividing: The product or quotient of two integers with the *same* sign is *positive*. The product or quotient of two integers with *different* signs is *negative*.

Example 5 **Find (a) $-7 \cdot (-1)$ and (b) $-9 \cdot 4$.**

a. $-7 \cdot (-1) = 7$ The integers have the same sign, so the product is positive.

▶ The product is 7.

b. $-9 \cdot 4 = -36$ The integers have different signs, so the product is negative.

▶ The product is -36.

Example 6 **Find (a) $18 \div (-2)$ and (b) $-25 \div (-5)$.**

a. $18 \div (-2) = -9$ The integers have different signs, so the quotient is negative.

▶ The quotient is -9.

b. $-25 \div (-5) = 5$ The integers have the same sign, so the quotient is positive.

▶ The quotient is 5.

Practice

Check your answers at BigIdeasMath.com.

Find the absolute value.

1. $|13|$ **2.** $|-8|$ **3.** $|0|$ **4.** $|-297|$

Evaluate.

5. $5 + (-11)$ **6.** $4 - 9$ **7.** $-15 + (-10)$ **8.** $9 + (-6)$

9. $0 - (-50)$ **10.** $-8 + 20$ **11.** $-11 - 11$ **12.** $-14 + 0$

13. $20 - (-21)$ **14.** $-34 - (-25)$ **15.** $-8 + (-3) + 6$ **16.** $1 + 7 - 9$

Simplify the expression.

17. $|-15 - 9|$ **18.** $|18 - (-11)|$ **19.** $|-14 + 17|$ **20.** $|-24 - (-19)|$

Evaluate.

21. $-8 \cdot 25$ **22.** $-33 \div (-3)$ **23.** $-13(-1)$ **24.** $-24 \div 4$

25. $0(-4)$ **26.** $-15(8)$ **27.** $\frac{0}{-12}$ **28.** $-1(-1)$

29. $\frac{-16}{-1}$ **30.** $240 \div (-8)$ **31.** $5 \cdot (-7) \cdot (-4)$ **32.** $12 \div (-3) \cdot 2$

33. **ELEVATION** The highest elevation in California is 14,494 feet, on Mount Whitney. The lowest elevation in California is -282 feet in Death Valley. Find the range of elevations in California.

34. **GOLF** The table shows a golfer's score for each round of a tournament. Find the golfer's total score and the golfer's mean score per round.

	Round 1	Round 2	Round 3
Score	-3	-4	$+1$

Name ____________________ Date ____________

Operations with Rational Numbers

To add, subtract, multiply, or divide rational numbers, use the same rules for signs as you used for integers.

Example 1 **Find (a) $-\frac{5}{6}+\frac{2}{3}$ and (b) $7.3-(-4.8)$.**

a. Write the fractions with the same denominator, then add.

$$-\frac{5}{6}+\frac{2}{3}=-\frac{5}{6}+\frac{4}{6}=\frac{-5+4}{6}=\frac{-1}{6}=-\frac{1}{6}$$

b. To subtract a rational number, add its opposite.

$$7.3-(-4.8)=7.3+4.8=12.1$$

The opposite of -4.8 is 4.8.

Example 2 **Find (a) $2.25 \cdot 8$, (b) $-2.25 \cdot (-8)$, and (c) $-2.25 \cdot 8$.**

a. $2.25 \cdot 8 = 18$ **b.** $-2.25 \cdot (-8) = 18$ **c.** $-2.25 \cdot 8 = -18$

Example 3 **Find $-\frac{4}{9} \div \frac{3}{4}$.**

To divide by a fraction, multiply by its reciprocal.

$$-\frac{4}{9}\div\frac{3}{4}=-\frac{4}{9}\cdot\frac{4}{3}=-\frac{4\cdot 4}{9\cdot 3}=-\frac{16}{27}$$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Practice

Check your answers at BigIdeasMath.com.

Add, subtract, multiply, or divide.

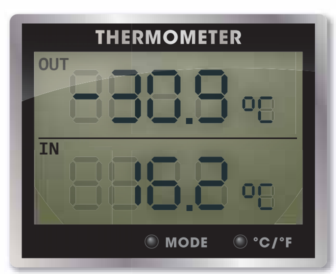
1. $-7.5+3.8$ **2.** $-18.3+(-6.7)$ **3.** $0.6-0.85$ **4.** $6.13-(-2.82)$

5. $-6 \cdot 4.75$ **6.** $-3.2 \cdot (-4.8)$ **7.** $-1.8 \div (-9)$ **8.** $3.6 \div (-1.5)$

9. $-\frac{1}{6}+\frac{5}{6}$ **10.** $-\frac{7}{10}+\left(-\frac{3}{5}\right)$ **11.** $\frac{4}{9}-\frac{2}{3}$ **12.** $-\frac{5}{6}-\frac{1}{4}$

13. $-\frac{3}{2}\cdot\left(-\frac{1}{8}\right)$ **14.** $-\frac{3}{4}\cdot\frac{7}{12}$ **15.** $\frac{5}{8}\div\left(-\frac{1}{4}\right)$ **16.** $-\frac{4}{7}\div\frac{2}{5}$

17. **TEMPERATURE** The temperature at midnight is shown. The outside temperature decreases 2.3°C over the next two hours. What is the outside temperature at 2 A.M.?



18. **SNOWFALL** In January, a city's snowfall was $\frac{5}{8}$ foot below the historical average. In February, the snowfall was $\frac{3}{4}$ foot above the historical average. Was the city's snowfall in the two-month period above or below the historical average? By how much?

Name ______________________________ Date ______________

Square Roots

A **square root** of a number is a number that, when multiplied by itself, equals the given number. Every positive number has a positive and a negative square root. A **perfect square** is a number with integers as its square roots.

Example 1 **Find the two square roots of 64.**

$8 \cdot 8 = 64$ and $-8 \cdot (-8) = 64$

▶ So, the square roots of 64 are 8 and -8.

The symbol $\sqrt{}$ is called a **radical sign**. It is used to represent a square root. The number under the radical sign is called the **radicand**.

Example 2 **Find the square root(s).**

a. $\sqrt{49}$

▶ Because $7^2 = 49$, $\sqrt{49} = \sqrt{7^2} = 7$.

b. $-\sqrt{\frac{1}{4}}$

▶ Because $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $-\sqrt{\frac{1}{4}} = -\sqrt{\left(\frac{1}{2}\right)^2} = -\frac{1}{2}$.

c. $\pm\sqrt{1.21}$

▶ Because $1.1^2 = 1.21$, $\pm\sqrt{1.21} = \pm\sqrt{1.1^2} = \pm 1.1$.

Example 3 **Evaluate $3\sqrt{144} - 10$.**

$3\sqrt{144} - 10 = 3(12) - 10$	Evaluate the square root.
$= 36 - 10$	Multiply.
$= 26$	Subtract.

Practice

Check your answers at BigIdeasMath.com.

Find the two square roots of the number.

1. 9 **2.** 100 **3.** 169 **4.** 400

Find the square root(s).

5. $\sqrt{4}$ **6.** $-\sqrt{81}$ **7.** $\pm\sqrt{900}$ **8.** $\pm\sqrt{\frac{1}{36}}$

9. $\sqrt{\frac{4}{9}}$ **10.** $-\sqrt{\frac{36}{25}}$ **11.** $\sqrt{2.25}$ **12.** $\pm\sqrt{0.01}$

Evaluate the expression.

13. $\sqrt{10 + 6}$ **14.** $4 - 2\sqrt{9}$ **15.** $12 - \sqrt{\frac{98}{2}}$ **16.** $4\left(2\sqrt{25} + 3\right)$

17. **PERIMETER** What is the perimeter of a square with an area of 900 square feet?

18. **DIAMETER** What is the diameter of a circle with an are of 100π square yards?

Name ________________________ Date ____________

Properties of Square Roots

A **radical expression** is an expression that contains a radical. A radical expression involving square roots is in **simplest form** when these three conditions are met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the properties below to simplify radical expressions involving square roots.

Product Property of Square Roots	Quotient Property of Square Roots
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \ge 0$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \ge 0$ and $b > 0$

Example 1 **Simplify (a) $\sqrt{75}$ and (b) $\sqrt{\frac{13}{25}}$.**

a. $\sqrt{75} = \sqrt{25 \cdot 3}$ — Factor using the greatest perfect square factor.

$= \sqrt{25} \cdot \sqrt{3}$ — Product Property of Square Roots

$= 5\sqrt{3}$ — Simplify.

b. $\sqrt{\frac{13}{25}} = \frac{\sqrt{13}}{\sqrt{25}}$ — Quotient Property of Square Roots

$= \frac{\sqrt{13}}{5}$ — Simplify.

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

Example 2 **Simplify $\frac{10}{\sqrt{7}}$ by rationalizing the denominator.**

$\frac{10}{\sqrt{7}} = \frac{10}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ — Multiply by $\frac{\sqrt{7}}{\sqrt{7}}$.

$= \frac{10\sqrt{7}}{\sqrt{49}}$ — Product Property of Square Roots

$= \frac{10\sqrt{7}}{7}$ — Simplify.

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $\sqrt{12}$ **2.** $-\sqrt{45}$ **3.** $\sqrt{500}$ **4.** $\sqrt{112}$

5. $\sqrt{\frac{3}{4}}$ **6.** $\sqrt{\frac{10}{49}}$ **7.** $-\sqrt{\frac{8}{25}}$ **8.** $\sqrt{\frac{48}{81}}$

9. $\frac{3}{\sqrt{5}}$ **10.** $-\frac{14}{\sqrt{10}}$ **11.** $\sqrt{\frac{3}{8}}$ **12.** $\sqrt{\frac{7}{32}}$

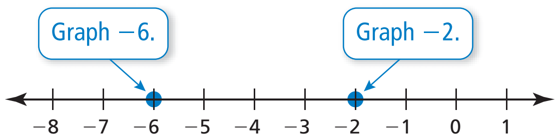
Name ______________________________ Date ______________

Comparing and Ordering Real Numbers

There are several ways to compare real numbers. One way is to write the numbers as decimals and use a number line.

Example 1 **Complete the statement with <, >, or =.**

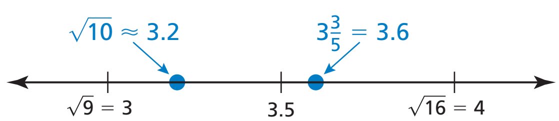
a. -2 ▭ -6



▶ -2 is to the right of -6. So, $-2 > -6$.

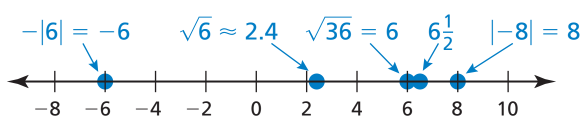
b. $\sqrt{10}$ ▭ $3\frac{3}{5}$

Estimate $\sqrt{10}$ to the nearest tenth. Then graph the numbers on a number line.



▶ $3\frac{3}{5}$ is to the right of $\sqrt{10}$. So, $\sqrt{10} < 3\frac{3}{5}$.

Example 2 **Order the values from least to greatest: $\sqrt{36}$, $|-8|$, $\sqrt{6}$, $6\frac{1}{2}$, $-|6|$.**



▶ So, the order from least to greatest is $-|6|$, $\sqrt{6}$, $\sqrt{36}$, $6\frac{1}{2}$, and $|-8|$.

Practice

Check your answers at BigIdeasMath.com.

Complete the statement with <, >, or =.

1. -4 ▭ -1

2. 0 ▭ -10

3. -12 ▭ $|-13|$

4. 7 ▭ $|-7|$

5. $\sqrt{14}$ ▭ 3.75

6. $2\frac{1}{4}$ ▭ $2.\overline{3}$

7. $-\sqrt{15}$ ▭ -4

8. π ▭ $3\frac{1}{10}$

Order the values from least to greatest.

9. $3, -|-2|, |-2|, |0|, -1$

10. $-12, -|14|, 10, |-15|, -9$

11. $\pi, 3.14, \sqrt{7}, 2\frac{3}{4}, \sqrt{4}$

12. $2\pi, 5.1\overline{6}, 5\frac{1}{8}, \sqrt{25}, 5.25$

13. $-|-1|, 0.11, 0, -|11|, 1.1, |-1|$

14. $|-4^3|, |-9 \cdot 7|, 60, \sqrt{64}$

Name ______________________________ Date ______________

Operations with Complex Numbers

A **complex number** written in standard form is a number $a + bi$, where a and b are real numbers. The number a is the real part, and the number bi is the imaginary part. To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Example 1 **Add or subtract. Write the answer in standard form.**

a. $(6 + 3i) + (2 - 5i)$

b. $(13 + 4i) - (8 + 5i)$

a. $(6 + 3i) + (2 - 5i) = (6 + 2) + [3 + (-5)]i$ — Definition of complex addition

$= 8 - 2i$ — Write in standard form.

b. $(13 + 4i) - (8 + 5i) = (13 - 8) + (4 - 5)i$ — Definition of complex subtraction

$= 5 - i$ — Write in standard form.

To multiply two complex numbers, use the Distributive Property, or the FOIL Method, just as you do when multiplying real numbers or algebraic expressions.

Example 2 **Multiply. Write the answer in standard form.**

a. $3i(2 + 9i)$

b. $(4 - 2i)(11 + 8i)$

a. $3i(2 + 9i) = 6i + 27i^2$ — Distributive Property

$= 6i + 27(-1)$ — Use $i^2 = -1$.

$= -27 + 6i$ — Write in standard form.

b. $(4 - 2i)(11 + 8i) = 44 + 32i - 22i - 16i^2$ — Multiply using FOIL.

$= 44 + 10i - 16(-1)$ — Simplify and use $i^2 = -1$.

$= 44 + 10i + 16$ — Simplify.

$= 60 + 10i$ — Write in standard form..

Practice

Check your answers at BigIdeasMath.com.

Perform the operation. Write the answer in standard form.

1. $(6 - i) + (9 + 5i)$

2. $(7 + 3i) + (11 + 2i)$

3. $(12 + 4i) - (2 - 15i)$

4. $(3 - 7i) - (3 + 5i)$

5. $7 - (2 - 3i) + 6i$

6. $-16 + (3 + 4i) - 4i$

7. $3i(6 - 5i)$

8. $-2i(8 + 2i)$

9. $(-5 + i)(8 - 6i)$

10. $(3 - 6i)(-1 + 7i)$

11. $(2 + 5i)(2 - 5i)$

12. $(-3 - i)(-3 + i)$

13. $(4 + i)^2$

14. $(5 - 9i)^2$

Name ______________________ Date ______________

Evaluating Algebraic Expressions

An **algebraic expression** is an expression that may contain numbers, operations, and one or more symbols. A symbol that represents one or more numbers is called a **variable**. To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

Example 1 **Evaluate each expression when $x = 3$.**

a. $5x + 7$

$5x + 7 = 5(3) + 7$ — Substitute 3 for x.

$= 15 + 7$ — Multiply.

$= 22$ — Add.

b. $14 - x^2$

$14 - x^2 = 14 - 3^2$ — Substitute 3 for x.

$= 14 - 9$ — Evaluate power.

$= 5$ — Subtract.

c. $2x^2 - 8x + 4$

$2x^2 - 8x + 4 = 2(3)^2 - 8(3) + 4$ — Substitute 3 for x.

$= 2(9) - 8(3) + 4$ — Evaluate power.

$= 18 - 24 + 4$ — Multiply.

$= -2$ — Simplify.

Example 2 **Evaluate each expression when $x = -2$ and $y = 6$.**

a. $7x - 5y$

$7x - 5y = 7(-2) - 5(6)$

$= -14 - 30$

$= -44$

b. $x^2 - 2xy + y^2$

$x^2 - 2xy + y^2 = (-2)^2 - 2(-2)(6) + 6^2$

$= 4 - 2(-2)(6) + 36$

$= 4 - (-24) + 36$

$= 64$

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression when $x = 2$ and $y = -3$.

1. $3x + 10$

2. $14 - 2y$

3. $5 - y^2$

4. $4x^2 + 9$

5. $y^2 + 8y - 4$

6. $-3x^2 - x + 7$

7. $0.75x - 4x - 1.5$

8. $3(y + 8 - 4y)$

9. $2x + 3y$

10. $6y - 5x$

11. $4x^2 + 3y$

12. $x^2 - y^2$

13. $y - x + y^2$

14. $x^2y^2 + xy$

15. $\dfrac{x + y}{y - x}$

16. $\dfrac{2x + y}{xy}$

Copy and complete the table.

17.

x	0	1	2	3	4
$3x - 2$					

18.

x	-2	-1	0	1	2
$-4x + 1$					

19. **MONEY** You earn $8x + 7y$ dollars for working x hours at a restaurant and y hours at a bus station. How much do you earn for working 12 hours at the restaurant and 16 hours at the bus station?

Name ______________________________ Date ______________

Simplifying Algebraic Expressions

Parts of an algebraic expression are called *terms*. **Like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

An algebraic expression is in **simplest form** when it has no like terms and no parentheses. To *combine* like terms that have variables, use the Distributive Property to add or subtract the coefficients.

Example 1 **Simplify $8y + 7y$.**

$8y + 7y = (8 + 7)y$ — Distributive Property

$= 15y$ — Add coefficients.

Example 2 **Simplify $2(x + 5) - 3(x - 2)$.**

$2(x + 5) - 3(x - 2) = 2(x) + 2(5) - 3(x) - 3(-2)$ — Distributive Property

$= 2x + 10 - 3x + 6$ — Multiply.

$= 2x - 3x + 10 + 6$ — Group like terms.

$= -x + 16$ — Combine like terms.

Example 3 **Simplify $xy + 3y - 2x + 5y - 3xy$.**

$xy + 3y - 2x + 5y - 3xy = xy - 3xy + 3y + 5y - 2x$ — Group like terms.

$= -2xy + 8y - 2x$ — Combine like terms.

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $7x + 15x$

2. $8y - 14y$

3. $7d + 9 - 5d$

4. $3w + 2(2 - 3w) + 2$

5. $(x + 3) + (3x - 7)$

6. $(5k + 6) + (4k - 8)$

7. $(-7n + 6) + (5n + 15)$

8. $(9z + 12) - (6z + 8)$

9. $(8b + 1) - (-10b - 5)$

10. $s(8 - 2t) + 3t(4 - 2s) + 5t$

11. $qr + 2q^2 - 3qr - r^2 - 6q^2$

12. $g^3(h - 4g) - h(3 - 2g^3)$

13. **EARNINGS** The original price of a model car is d dollars. You use a coupon and buy the kit for $(d - 10)$ dollars. You assemble the model car and sell it for $(2d - 20)$ dollars. Write an expression that represents your earnings. Interpret the expression.

Name ______________________________ Date ______________

Powers and Exponents

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base exponent

$$\underbrace{\left(\frac{2}{5}\right)^3}_{\text{power}} = \underbrace{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}}_{\frac{2}{5} \text{ is used as factor } 3 \text{ times}}$$

Example 1 **Write each product using exponents.**

a. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because -9 is used as a factor 5 times, its exponent is 5.

▶ So, $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$.

b. $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because π is used as a factor 2 times, its exponent is 2. Because h is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$.

Example 2 **Evaluate each expression.**

a. $(-5)^4$

$(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$ Write as repeated multiplication.

$= 625$ Simplify.

b. -5^4

$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5)$ Write as repeated multiplication.

$= -625$ Simplify.

Practice

Check your answers at BigIdeasMath.com.

Write the product using exponents.

1. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

2. $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$

3. $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$

4. $2.5 \cdot 2.5 \cdot b \cdot b \cdot b \cdot b$

5. $(-n) \cdot (-n) \cdot (-n) \cdot (-n)$

6. $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

Evaluate the expression.

7. 10^4

8. -15^2

9. $\left(\frac{3}{4}\right)^3$

10. $\left(-\frac{1}{2}\right)^5$

11. **VOLUME** Write an expression involving a power that represents the volume (in cubic centimeters) of the die shown. Then find the volume.

$1\frac{3}{5}$ cm

Name ______________________________ Date ______________

The Distributive Property

To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses, then evaluate.

Distributive Property	
With addition: $5(7 + 3) = 5(7) + 5(3)$	$a(b + c) = a(b) + a(c)$
With subtraction: $5(7 - 3) = 5(7) - 5(3)$	$a(b - c) = a(b) - a(c)$

Example 2 **Simplify each expression.**

a. $6(x + 9)$

$$6(x + 9) = 6(x) + 6(9)$$
$$= 6x + 54$$

b. $10(12 + z + 7)$

$$10(12 + z + 7) = 10(12) + 10(z) + 10(7)$$
$$= 120 + 10z + 70$$
$$= 10z + 190$$

c. $16(8w - 3)$

$$16(8w - 3) = 16(8w) - 16(3)$$
$$= 128w - 48$$

d. $5(4m - 3n - 1)$

$$5(4m - 3n - 1) = 5(4m) - 5(3n) - 5(1)$$
$$= 20m - 15n - 5$$

Practice

Check your answers at BigIdeasMath.com.

Evaluate.

1. $25(7 + 11)$ **2.** $4(13 - 5)$ **3.** $9(16 + 7 - 8)$ **4.** $-4(10 - 9 - 6)$

Simplify the expression.

5. $4(y + 7)$ **6.** $-2(z + 5)$ **7.** $5(b - 11)$ **8.** $-8(d - 1)$

9. $12(4a + 13)$ **10.** $9(20 + 17m)$ **11.** $11(2k - 11)$ **12.** $-7(-2n - 9)$

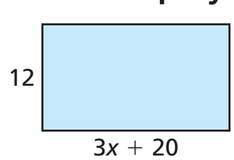
13. $3(x + 4 + 9)$ **14.** $6(25 + 6z + 10)$ **15.** $8(p - 6 - 5)$ **16.** $-10(4 + v - 1)$

17. $7(2x + 7 + 9y)$ **18.** $-4(4r - s + 17)$ **19.** $-3(-12 - 3d - 8)$ **20.** $2 - 6(2n - 9)$

21. $1.5(6c + 10d + 3)$ **22.** $\frac{3}{4}\left(q + \frac{1}{6} + \frac{7}{8}\right)$ **23.** $-2.4(5h - 10 + 4)$ **24.** $0.5(2.6x + 5.8)$

Write and simplify an expression for the area of the rectangle.

7.



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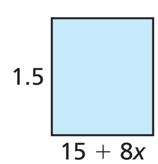
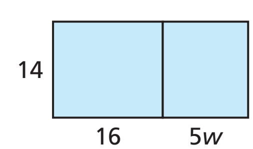


9.



Name ____________________ Date ____________

Order of Operations

To evaluate numerical expressions, use a set of rules called the **order of operations**.

Order of Operations
1. Perform operations in **P**arentheses.
2. Evaluate numbers with **E**xponents.
3. **M**ultiply or **D**ivide from left to right.
4. **A**dd or **S**ubtract from left to right.

Example 1 **Evaluate each expression.**

a. $20 - 5 \cdot 6$

$20 - 5 \cdot 6 = 20 - 30$ Multiply 5 and 6.

$= -10$ Subtract 30 from 20.

b. $12 \cdot 3 + 4^2 \div 8$

$12 \cdot 3 + 4^2 \div 8 = 12 \cdot 3 + 16 \div 8$ Evaluate 4^2.

$= 36 + 16 \div 8$ Multiply 12 and 3.

$= 36 + 2$ Divide 16 by 8.

$= 38$ Add 36 and 2.

c. $7(5 - 3) + 6^2 \div (-3)$

$7(5 - 3) + 6^2 \div (-3) = 7(2) + 6^2 \div (-3)$ Perform operation in parentheses.

$= 7(2) + 36 \div (-3)$ Evaluate 6^2.

$= 14 + 36 \div (-3)$ Multiply 7 and 2.

$= 14 + (-12)$ Divide 36 by -3.

$= 2$ Add 14 and -12.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $8 + 2 \cdot 5$

2. $40 \div 8 - 7$

3. $5 \cdot 4^2 \div 8$

4. $1 - 7 + 5^2$

5. $\dfrac{3 - (-9)}{-10 + 6}$

6. $\dfrac{2 + 4}{1 - 5} - 1$

7. $(12 - 8)^2 \div 2^5$

8. $18 + 9^2 - 7 \cdot (-3)$

9. $32 \div 8 + 2 \cdot 8^2$

10. $6 \div (7 \div 28)$

11. $36 \div \left(1 - |2 - 7|\right)$

12. $(-2)^2 \cdot 5 - 7(9 - 5)$

13. $4(3 + 8) - 8^2 \div 32$

14. $10(3 - 6)^3 + 41$

15. $(2 - 5)^2 - (4 \cdot 5^2)$

16. **RESTAURANT** There are 82 people in a restaurant. Four groups of 3 leave and then five groups of 2 enter. Evaluate the expression $82 - 4(3) + 5(2)$ to find how many people are in the restaurant.

Name ______________________________ Date ______________

Properties of Exponents

Product of Powers	Power of a Product	Power of a Power	
$a^m \cdot a^n = a^{m+n}$ Add exponents.	$(ab)^m = a^m b^m$ Find the power of each factor.	$(a^m)^n = a^{mn}$ Multiply exponents.	
Quotient of Powers	**Power of a Quotient**	**Negative Exponent**	**Zero Exponent**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ Subtract exponents.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ Find the power of the numerator and the power of the denominator.	$a^{-n} = \frac{1}{a^n}, a \neq 0$	$a^0 = 1, a \neq 0$

Example 1 Evaluate (a) 4.9^0 and (b) $(-3)^{-4}$.

a. $4.9^0 = 1$ Definition of zero exponent

b. $(-3)^{-4} = \frac{1}{(-3)^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

Example 2 Simplify each expression. Write your answer using only positive exponents.

a. $2^3 \cdot 2^4 = 2^7 = 128$

b. $\frac{5^9}{5^6} = 5^{9-6} = 5^3 = 125$

c. $\frac{12y^0}{x^{-7}} = 12y^0x^7 = 12x^7$

d. $\frac{x^6 \cdot x^2}{x^5} = \frac{x^{6+2}}{x^5} = x^{8-5} = x^3$

e. $(z^4)^2 = z^{4 \cdot 2} = z^8$

f. $(6mn)^3 = 6^3 \cdot m^3 \cdot n^3 = 216m^3n^3$

g. $\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4} = \frac{y^4}{81}$

h. $\frac{10x^6y^{-2}}{5x^3y} = \frac{10}{5}x^{(6-3)}y^{(-2-1)} = 2x^3y^{-3} = \frac{2x^3}{y^3}$

Practice

Check your answers at BigIdeasMath.com.

Evaluate the expression.

1. $(-9)^0$ **2.** -8^{-1} **3.** 4^{-3} **4.** $\frac{-5^0}{3^{-2}}$

Simplify the expression. Write your answer using only positive exponents.

5. $2^9 \cdot 2^{-6}$ **6.** $-\frac{10^8}{10^{12}}$ **7.** $y \cdot y^{-5}$ **8.** $\frac{x^7}{x^{-7}}$

9. $-5x^7 \cdot x^{-11} \cdot 2x^4$ **10.** $\frac{x^{-2}}{5z^0}$ **11.** $(w^2)^{-3}$ **12.** $(8xy)^2$

13. $3x^5 \cdot (-2x)^4$ **14.** $(-5m^2n^{-1})^3$ **15.** $\frac{z^8}{z^{-2} \cdot z^9}$ **16.** $\frac{(x^5)^3}{x^6}$

17. $\left(\frac{3x}{2}\right)^3$ **18.** $\left(\frac{6x^4}{5y}\right)^{-2}$ **19.** $\frac{xy^{-2}}{x^4y^{-3}}$ **20.** $\frac{8xy}{6x^5yz^{-2}}$

21. **METRIC SYSTEM** There are 10^6 micrometers in a meter and 10^3 meters in a kilometer. How many micrometers are there in 10^6 kilometers?

Name ______________________________ Date ______________

Scientific Notation

A number is written in **scientific notation** when it is represented as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

$c \times 10^n$ $\qquad$ $1 \le c < 10$ and n is an integer

When a number is written in scientific notation, you can write the number in standard form using the absolute value of the exponent n. When n is negative, move the decimal point $|n|$ places to the left. When n is positive, move the decimal point $|n|$ places to the right.

Example 1 **Write (a) 7.4×10^5 and (b) 3.96×10^{-4} in standard form.**

a. $7.4 \times 10^5 = \underbrace{740{,}000}_{5}$ $\qquad$ Move decimal point $|5| = 5$ places to the right.

b. $3.96 \times 10^{-4} = \underbrace{0.000396}_{4}$ $\qquad$ Move decimal point $|-4| = 4$ places to the left.

When a number is written in standard form, you can write the number in scientific notation using the following steps.

Step 1 Move the decimal point so it is located to the right of the leading nonzero digit.

Step 2 Count the number n of places you moved the decimal point. The exponent of the power of 10 is n when you move the decimal point to the left and $-n$ when you move the decimal point to the right.

Example 2 **Write each number in scientific notation.**

a. 4,025,000,000

$\underbrace{4{,}025{,}000{,}000}_{9} = 4.025 \times 10^9$ $\qquad$ Move decimal point 9 places to the left. The exponent is 9.

b. 0.00000591

$\underbrace{0.00000591}_{6} = 5.91 \times 10^{-6}$ $\qquad$ Move decimal point 6 places to the right. The exponent is -6.

Practice

Check your answers at BigIdeasMath.com.

Write the number in standard form.

1. 2×10^4 $\qquad$ **2.** 8.4×10^1 $\qquad$ **3.** 7×10^{-3} $\qquad$ **4.** 5.05×10^{-1}

5. 1.8×10^{-7} $\qquad$ **6.** 6.29×10^{-5} $\qquad$ **7.** 5.591×10^0 $\qquad$ **8.** 3.0504×10^9

Write the number in scientific notation.

9. 400 $\qquad$ **10.** 72,000 $\qquad$ **11.** 0.8 $\qquad$ **12.** 0.00019

13. 100,500,000 $\qquad$ **14.** 324,900 $\qquad$ **15.** 0.000002621 $\qquad$ **16.** 0.05008

17. **NEURONS** A study found that the average human brain has 8.6×10^{10} neurons. Write this number in standard form. The cell bodies of neurons have widths between 0.000004 meter and 0.0001 meter. Write this range in scientific notation.

Name ______________________________ Date ______________

Polynomials

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents. The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Example 1 **Find the degree of (a) $7x^2$ and (b) $-\frac{2}{3}xy^4$.**

a. The exponent of x is 2.

▶ So, the degree of the monomial is 2.

b. The exponent of x is 1, and the exponent of y is 4.

▶ So, the degree of the monomial is $1 + 4$, or 5.

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.

degree

leading coefficient → $\underbrace{4x^2 + 8x - 9}_{\text{trinomial}}$ ← constant term

Example 2 **Write (a) $-24x^3$, (b) $8y - 1 + 10y^2$, and (c) $5z + 9z^4$ in standard form. Identify the degree and leading coefficient of each polynomial. Then classify each polynomial by the number of terms.**

	Standard Form	Degree	Leading Coefficient	Type of Polynomial
a.	$-24x^3$	3	-24	monomial
b.	$10y^2 + 8y - 1$	2	10	trinomial
c.	$9z^4 + 5z$	4	9	binomial

Practice

Check your answers at BigIdeasMath.com.

Find the degree of the monomial.

1. $-8x$ **2.** $\frac{1}{2}y^5$ **3.** 12.8 **4.** 6^2

5. x^3z^2 **6.** $-3mn$ **7.** $8q^2r^4s$ **8.** $10g^5h^7j^2$

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

9. $8y^5$ **10.** $2 + x^2 - 9x$

11. $2z^2 - 7z^3$ **12.** $-\frac{2}{5}w^7$

13. $5t^2 - t^3 + 6t^4$ **14.** $-s - 10s^8$

Name ______________________________ Date ______________

Adding and Subtracting Polynomials

To add polynomials, add like terms. You can use a vertical or a horizontal format.

Example 1 **Find each sum.**

a. $(5x^2 + 3x - 7) + (x^2 + 2)$

Use a vertical format. Align like terms vertically and add.

$$\begin{array}{rrrr} & 5x^2 & + 3x & - 7 \\ + & x^2 & & + 2 \\ \hline & 6x^2 & + 3x & - 5 \end{array}$$

b. $(-4x^3 - x + 1) + (2x^2 + 8x - 9)$

Use a horizontal format. Group like terms and simplify.

$$\begin{aligned}(-4x^3 - x + 1) + (2x^2 + 8x - 9) &= (-4x^3 + 2x^3) + (-x + 8x) + (1 - 9) \\ &= -2x^3 + 7x - 8\end{aligned}$$

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by -1.

Example 2 **Find each difference.**

a. $(6x^3 - 2x - 5) - (-x^3 + 3x^2 + 4)$

Use a vertical format. Align like terms vertically and subtract.

$$\begin{array}{rrrr} & 6x^2 & - 2x & - 5 \\ - & (-x^2 + 3x^2 & & + 4 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{rrrrr} & 6x^3 & & - 2x & - 5 \\ + & x^3 & - 3x^2 & & - 4 \\ \hline & 7x^3 & - 3x^2 & - 2x & - 9 \end{array}$$

b. $(5x^2 + 7x - 3) - (4x^2 - \ + 2x - 1)$

Use a horizontal format. Group like terms and simplify.

$$\begin{aligned}(5x^2 + 7x - 3) - (4x^2 - \ + 2x - 1) &= 5x^2 + 7x - 3 - 4x^2 + 2x + 1 \\ &= (5x^2 - 4x^2) + (7x + 2x) + (-3 + 1) \\ &= x^2 + 9x - 2\end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Find the sum or difference.

1. $(-8x + 2) + (-10x - 7)$

2. $(x^3 + 9x) - (4x^3 - x)$

3. $(x^2 - 2x - 6) + (6x^2 + x + 8)$

4. $(2x^3 + 5x^2 - x) + (x^3 - 10x^2 + 5x)$

5. $(-7x^3 - x^2 + 10) - (3x^2 + 2x - 2)$

6. $(x^3 + 8x + 3) - (-x^3 + 2x^2 - 5)$

7. $(x^2 + 4x + 1) + (-x^2 - 1)$

8. $(3x^3 - 2x^2) - (5x^3 - x^2 - x)$

9. $(2x^3 - 5) - (-8x^2 - 5x)$

10. $(-x^2 - 4) + (x^3 - 4x^2)$

Name ______________________________ Date ______________

Multiplying Polynomials

To multiply two polynomials, multiply each term of the first polynomial by each term of the second polynomial. You can use a vertical or a horizontal format.

Example 1 **Find (a) $5x(x^2 - 2x - 4)$ and (b) $(x - 1)(x + 3)$.**

a. Use a horizontal format. Distribute $5x$ to each term of $x^2 - 2x - 4$.

$$5x(x^2 - 2x - 4) = 5x(x^2) + 5x(-2x) + 5x(-4)$$
$$= 5x^3 - 10x^2 - 20x$$

b. Use a vertical format.

$$\begin{array}{rrl} & x - 1 & \\ \times & x + 3 & \text{Align like terms vertically.} \\ \hline & 3x - 3 & \text{Distributive Property: Multiply } 3(x - 1). \\ x^2 - & x & \text{Distributive Property: Multiply } x(x - 1). \\ \hline x^2 + & 2x - 3 & \text{Combine like terms.} \end{array}$$

In Example 1(b), you multiplied two binomials. Another way to multiply two binomials is to use the Distributive Property systemically. Find the sum of the products of the ***First*** terms, the ***Outer*** terms, the ***Inner*** terms, and the ***Last*** terms of the binomials. This is called the **FOIL Method.**

You can also use the patterns shown to find the square of a binomial, or the product of the sum and difference of two terms.

Square of a Binomial Pattern	Sum and Difference Pattern
$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$

Example 2 **Find (a) $(x + 4)(2x + 1)$, (b) $(3x + 5)^2$, and (c) $(x + 2)(x - 2)$.**

a. Use the FOIL Method.

$$(x + 4x)(2x + 1) = \overset{\text{F}}{2x^2} + \overset{\text{O}}{x} + \overset{\text{I}}{8x} + \overset{\text{L}}{4}$$
$$= 2x^2 + 9x + 4$$

b. $(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2$
$= 9x^2 + 30x + 25$

c. $(x + 2)(x - 2) = x^2 - 2^2$
$= x^2 - 4$

Practice

Check your answers at BigIdeasMath.com.

Find the product.

1. $4x(x + 7)$

2. $2x(3x - 8)$

3. $8x(x^2 + 5x + 4)$

4. $5x(7 - x - 3x^2)$

5. $(x + 1)(x + 4)$

6. $(x - 3)(x - 5)$

7. $(2x + 7)(3x - 1)$

8. $(5 - 3x)(4 - x)$

9. $(x - 1)(9 - x)$

10. $(x + 6)(x - 6)$

11. $(x + 5)^2$

12. $(2x - 1)^2$

13. $(2 - 5x)^2$

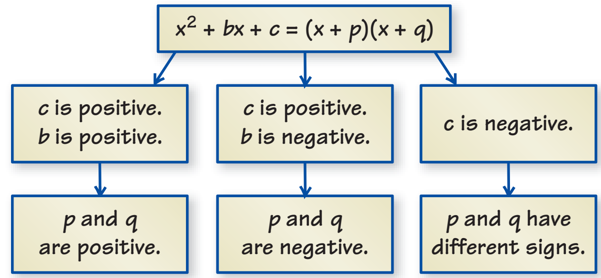
14. $(8x + 3)^2$

15. $(2x + 3)(3x - 3)$

Name ______________________________ Date ______________

Factoring Polynomials

Writing a polynomial as a product of factors is called *factoring*. To factor $x^2 + bx + c$ as $(x + p)(x + q)$, find p and q such that $p + q = b$ and $pq = c$. The diagram shows the relationships between the signs of b and c and the signs of p and q.



To factor $ax^2 + bx + c$, where $a \neq 1$, look for the GCF of the terms of the polynomial, and then factor further if possible. You can also use special product patterns to factor polynomials.

Perfect Square Trinomial Pattern	Difference of Two Squares Pattern
$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$a^2 - b^2 = (a + b)(a - b)$

Example 1 **Factor each polynomial.**

a. $2x^2 + 6x$ **b.** $4x^2 - 25$ **c.** $x^2 + 9x + 18$ **d.** $4x^2 - 21x + 5$

a. The GCF of 2 and 6 is 2. The GCF of x^2 and x is x. So, the greatest common monomial factor of the terms is $2x$.

▶ So, $2x^2 + 6x = 2x(x + 3)$.

b. Use the difference of two squares pattern.

$$4x^2 - 25 = (2x)^2 - 5^2$$
$$= (2x + 5)(2x - 5)$$

c. Notice that $a = 1$, $b = 9$, and $c = 18$. Because b and c are positive, p and q are positive. Find two positive integer factors of 18 whose sum is 9.

▶ So, $x^2 + 9x + 18 = (x + 3)(x + 6)$.

Factors of 18	1, 18	2, 9	3, 6
Sum of factors	19	11	9

The values of p and q are 3 and 6.

d. Notice that $a = 4$, $b = -21$, $c = 5$, and there is no GCF. Because b is negative and c is positive, both factors of c must be negative.

▶ So, $4x^2 - 21x + 5 = (x - 5)(4x - 1)$.

Factors of 4	Factors of 5	Possible factorization	Middle term	
1, 4	$-1, -5$	$(x - 1)(4x - 5)$	$-9x$	✗
1, 4	$-5, -1$	$(x - 5)(4x - 1)$	$-21x$	✓
2, 2	$-1, -5$	$(2x - 1)(2x - 5)$	$-12x$	✗

Practice

Check your answers at BigIdeasMath.com.

Factor the polynomial.

1. $8x - 2$ **2.** $10x^2 + 5x$ **3.** $25x - 10y$ **4.** $x^2 - 7x + 12$

5. $x^2 - x - 20$ **6.** $3x^2 + 6x - 24$ **7.** $4x^2 + 9x + 5$ **8.** $-18x^2 - 6x + 4$

9. $x^2 - 9$ **10.** $8x^2 - 50$ **11.** $x^2 + 14x + 49$ **12.** $3x^2 - 12x + 12$

Name ________________________________ Date ______________

Properties of Equality

Addition Property of Equality

Words When you add the same number to each side of an equation, the two sides remain equal.

Numbers $6 + 4 = 6 + 4$

$10 = 10$

Algebra $x - 5 + 5 = 3 + 5$

$x = 8$

Subtraction Property of Equality

Words When you subtract the same number from each side of an equation, the two sides remain equal.

Numbers $7 - 2 = 7 - 2$

$5 = 5$

Algebra $y + 3 - 3 = 1 - 3$

$y = -2$

Multiplication Property of Equality

Words When you multiply each side of an equation by the same nonzero number, the two sides remain equal.

Numbers $\frac{6}{3} \cdot 3 = 2 \cdot 3$

$6 = 6$

Algebra $\frac{z}{3} \cdot 3 = 2 \cdot 3$

$z = 6$

Division Property of Equality

Words When you divide each side of an equation by the same nonzero number, the two sides remain equal.

Numbers $6 \cdot 2 \div 2 = 12 \div 2$

$6 = 6$

Algebra $\frac{2w}{2} = \frac{12}{2}$

$w = 6$

Example 1 **Solve each equation. Tell which algebraic property of equality you used.**

a. $c - 3 = -2$

$c - 3 + 3 = -2 + 3$ Addition Property of Equality

$c = 1$ Simplify.

▶ The solution is $c = 1$. The property is the Addition Property of Equality.

b. $\frac{d}{5} = 7$

$\frac{d}{5} \cdot 5 = 7 \cdot 5$ Multiplication Property of Equality

$d = 35$ Simplify.

▶ The solution is $d = 35$. The property is the Multiplication Property of Equality.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation. Tell which algebraic property of equality you used.

1. $h - 6 = 2$

2. $\frac{j}{3} = 9$

3. $k + 8 = -9$

4. $4m = 12$

5. $n + 2 = 6$

6. $\frac{p}{6} = -2$

7. $q - 3 = -8$

8. $8r = 48$

9. $s + 9 = 5$

10. $6t = 48$

11. $w + 3 = 29$

12. $\frac{z}{7} = 7$

Name ______________________________________ Date ________________

Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation and simplify.

Example 1 **Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.**

a.

$$\begin{aligned} 5x - 1 &= -2x + 6 & \\ 5(1) - 1 &\stackrel{?}{=} -2(1) + 6 & \text{Substitute.} \\ 4 &= 4 \ ✓ & \text{Simplify.} \end{aligned}$$

So, $x = 1$ is a solution.

b.

$$\begin{aligned} 5x - 1 &= -2x + 6 & \\ 5(-2) - 1 &\stackrel{?}{=} -2(-2) + 6 & \text{Substitute.} \\ -11 &\neq 10 \ ✗ & \text{Simplify.} \end{aligned}$$

So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 **Solve each equation. Check your solution.**

a.

$$\begin{aligned} 4x - 3 &= 13 & \\ 4x - 3 + 3 &= 13 + 3 & \text{Add 3.} \\ 4x &= 16 & \text{Simplify.} \\ \frac{4x}{4} &= \frac{16}{4} & \text{Divide by 4.} \\ x &= 4 & \text{Simplify.} \end{aligned}$$

Check

$$\begin{aligned} 4x - 3 &= 13 \\ 4(4) - 3 &\stackrel{?}{=} 13 \\ 13 &= 13 \ ✓ \end{aligned}$$

b.

$$\begin{aligned} 2(y - 8) &= y + 6 & \\ 2y - 16 &= y + 6 & \text{Distributive Property} \\ 2y - y - 16 &= y - y + 6 & \text{Subtract } y. \\ y - 16 &= 6 & \text{Simplify.} \\ y - 16 + 16 &= 6 + 16 & \text{Add 16.} \\ y &= 22 & \text{Simplify.} \end{aligned}$$

Check

$$\begin{aligned} 2(y - 8) &= y + 6 \\ 2(22 - 8) &\stackrel{?}{=} 22 + 6 \\ 28 &= 28 \ ✓ \end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Determine whether (a) $x = -1$ or (b) $x = 3$ is a solution of the equation.

1. $5x + 7 = 2$ **2.** $-4x + 8 = -4$ **3.** $2x - 1 = 3x - 4$

Solve the equation. Check your solution.

4. $x - 9 = 24$ **5.** $n + 14 = 0$ **6.** $-16 = 4y$

7. $-\frac{5}{6}t = -15$ **8.** $81 = 46 - x$ **9.** $4x + 5 = 1$

10. $x + 5 = 11x$ **11.** $9(y - 3) = 45$ **12.** $6 = 7k + 8 - k$

13. $6n + 3 = -4n + 7$ **14.** $2c + 5 = 3(c - 8)$ **15.** $18m + 3(2m + 8) = 0$

16. $\frac{w - 6}{5} = 8$ **17.** $\frac{15 + h}{3} = 10$ **18.** $\frac{8 - 3x}{5} = x$

19. $(8r + 6) + (4r - 1) = 14$ **20.** $\frac{2}{3}y - 3 = 9$ **21.** $\frac{1}{2}x - \frac{3}{10} = \frac{5}{2}x + \frac{7}{10}$

22. **MONEY** You have a total of \$3.25 in change made up of 25 pennies, 6 nickels, 2 dimes, and x quarters. How many quarters do you have?

Name ______________________________ Date ______________

Solving Absolute Value Equations

An **absolute value equation** is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad or \quad ax + b = -c.$$

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number that is not negative.

To solve $|ax + b| = |cx + d|$, solve the related linear equations

$$ax + b = cx + d \quad or \quad ax + b = -(cx + d).$$

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

Example 1 **Solve $|x - 7| = 8$.**

Write the two related linear equations for $|x - 7| = 8$. Then solve.

$$\begin{array}{rcl} x - 7 &=& 8 \\ \underline{+7} && \underline{+7} \\ x &=& 15 \end{array} \qquad or \qquad \begin{array}{rcl} x - 7 &=& -8 \\ \underline{+7} && \underline{+7} \\ x &=& -1 \end{array}$$

▶ The solutions are $x = 15$ and $x = -1$.

Check

$$|x - 7| = 8$$
$$|15 - 7| \stackrel{?}{=} 8$$
$$|8| \stackrel{?}{=} |8|$$
$$8 = 8 \checkmark$$

$$|x - 7| = 8$$
$$|-1 - 7| \stackrel{?}{=} 8$$
$$|-8| \stackrel{?}{=} |-8|$$
$$8 = 8 \checkmark$$

Example 2 **Solve $|x + 3| = |x + 9|$.**

By equating the expression $x + 3$ and the opposite of $x + 9$, you obtain

$x + 3 = -(x + 9)$	Write related linear equation.
$x + 3 = -x - 9$	Distributive Property
$2x + 3 = -9$	Add x to each side.
$2x = -12$	Subtract 3 from each side.
$x = -6.$	Divide each side by 2.

However, by equating the expressions $x + 3$ and $x + 9$, you obtain

$x + 3 = x + 9$	Write related linear equation.
$x = x + 6$	Subtract 3 from each side.
$0 = 6$ ✗	Subtract x from each side.

which is a false statement. So, the original equation has only one solution.

▶ The solution is $x = -6$.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation. Check your solutions.

1. $|x - 3| = 6$

2. $|2x - 1| = 9$

3. $|x - 5| = |x + 7|$

4. $|x + 2| = |x + 8|$

5. $|x - 3| = |x - 5|$

6. $|x + 2| = |2x + 1|$

Name ______________________ Date ______________

Solving Quadratic Equations

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. You can solve quadratic equations by factoring, graphing, using square roots, completing the square, or using the Quadratic Formula.

Example 1 **Solve $x^2 - 2x - 3 = 0$ by factoring.**

$$x^2 - 2x - 3 = 0$$
$$(x + 1)(x - 3) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \quad \text{or} \quad x = 3$$

▶ The solutions are $x = -1$ and $x = 3$.

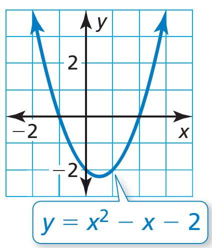
Example 2 **Solve $5x^2 = 45$ using square roots.**

$$5x^2 = 45$$
$$x^2 = 9$$
$$x = \pm\sqrt{9}$$
$$x = \pm 3$$

▶ The solutions are $x = 3$ and $x = -3$.

Example 3 **Solve $x^2 - x - 2 = 0$ by graphing.**

Graph the related function $y = x^2 - x - 2$.



The x-intercepts are -1 and 2.

▶ So, the solutions are $x = -1$ and $x = 2$.

Example 4 **Solve $2x^2 + 12x - 4 = 0$ by completing the square.**

$$2x^2 + 12x - 4 = 0$$
$$2x^2 + 12x = 4$$
$$x^2 + 6x = 2$$
$$x^2 + 6x + 3^2 = 2 + 3^2$$
$$(x + 3)^2 = 11$$
$$x + 3 = \pm\sqrt{11}$$
$$x = -3 \pm \sqrt{11}$$

▶ The solutions are $x = -3 + \sqrt{11} \approx 0.32$ and $x = -3 - \sqrt{11} \approx -6.32$.

Example 5 **Solve $2x^2 - 6x + 4 = 0$ using the Quadratic Formula.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(4)}}{2(2)} = \frac{6 \pm \sqrt{4}}{4} = \frac{6 \pm 2}{4}$$

▶ So, the solutions are $x = \frac{6 + 2}{4} = 2$ and $x = \frac{6 - 2}{4} = 1$.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation using any method. Explain your choice of method.

1. $x^2 + x - 12 = 0$

2. $3x^2 = 48$

3. $x^2 - 10x + 20 = 0$

4. $2x^2 + 8x - 2 = 0$

5. $3x^2 - 7x + 4 = 0$

6. $2x^2 + 3x - 5 = 0$

7. **PHYSICS** You launch a model rocket. The equation $h = -16t^2 + 40t + 2$ models the rocket's height h (in feet) after t seconds. How much time does it take for the rocket to reach the ground?

Name ______________________________ Date ______________

Solving Radical Equations

A **radical equation** is an equation that contains a radical expression with a variable in the radicand. To solve a radical equation involving a square root, first use properties of equality to isolate the radical on one side of the equation. Then use the following property to eliminate the radical and solve for the variable.

Squaring Each Side of an Equation
Words If two expressions are equal, then their squares are also equal.
Algebra If $a = b$, then $a^2 = b^2$.

Squaring each side of an equation can sometimes introduce an extraneous solution.

Example 1 **Solve $x = \sqrt{x + 2}$. Check your solutions.**

$x = \sqrt{x+2}$	Write the equation.
$x^2 = \left(\sqrt{x+2}\right)^2$	Square each side of the equation.
$x^2 = x + 2$	Simplify.
$x^2 - x - 2 = 0$	Subtract x and 2 from each side.
$(x-2)(x+1) = 0$	Factor.
$x - 2 = 0$ or $x + 1 = 0$	Zero-Product Property
$x = 2$ or $x = -1$	Solve for x.

Check Check each solution in the original equation.

$2 \stackrel{?}{=} \sqrt{2+2}$	Substitute for x.	$-1 \stackrel{?}{=} \sqrt{-1+2}$
$2 \stackrel{?}{=} \sqrt{4}$	Simplify.	$-1 \stackrel{?}{=} \sqrt{1}$
$2 = 2$ ✓	Simplify.	$-1 \neq 1$ ✗

▶ Because $x = -1$ does not satisfy the original equation, it is an extraneous solution. The only solution is $x = 2$.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation. Check your solution(s).

1. $4\sqrt{x} - 8 = 0$

2. $2\sqrt{x} - 6 = 0$

3. $2\sqrt{x-6} - 3 = 5$

4. $\sqrt{6x} + 6 = 0$

5. $\sqrt{5x} + 1 = 0$

6. $x = \sqrt{x + 12}$

7. $\sqrt{2x+8} = x$

8. $\sqrt{2x+3} = \sqrt{x+2}$

9. $\sqrt{-3x-4} = \sqrt{2x+11}$

10. **PENDULUM** The period P (in seconds) of a pendulum is given by the function $P = 2\pi\sqrt{\frac{L}{32}}$, where L is the pendulum length (in feet). What is the length of a pendulum that has a period of 3 seconds?

Name ______________________________ Date ______________

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

Example 1 **Solve each literal equation for y.**

a. $3x + 5y = 45$

$3x - 3x + 5y = 45 - 3x$ — Subtract $3x$ from each side.

$5y = 45 - 3x$ — Simplify.

$\frac{5y}{5} = \frac{45 - 3x}{5}$ — Divide each side by 5.

$y = 9 - \frac{3}{5}x$ — Simplify.

▶ The rewritten literal equation is $y = 9 - \frac{3}{5}x$.

b. $2xy + 5y = 7$

$y(2x + 5) = 7$ — Distributive Property

$\frac{y(2x + 5)}{2x + 5} = \frac{7}{2x + 5}$ — Divide each side by $2x + 5$.

$y = \frac{7}{2x + 5}$ — Simplify.

▶ The rewritten literal equation is $y = \frac{7}{2x + 5}$.

c. $2x = \frac{3 + y}{y}$

$2x \cdot y = \frac{3 + y}{y} \cdot y$ — Multiply each side by y.

$2xy = 3 + y$ — Simplify.

$2xy - y = 3 + y - y$ — Subtract y from each side.

$2xy - y = 3$ — Simplify.

$y(2x - 1) = 3$ — Distributive Property

$\frac{y(2x - 1)}{2x - 1} = \frac{3}{2x - 1}$ — Divide each side by $2x - 1$.

$y = \frac{3}{2x - 1}$ — Simplify.

▶ The rewritten literal equation is $y = \frac{3}{2x - 1}$.

Practice

Check your answers at BigIdeasMath.com.

Solve the literal equation for y.

1. $x + 3y = 9$

2. $4x - 2y = 16$

3. $2x + 7y = 5$

4. $2x + 3y = 6$

5. $5x - 4y = 10$

6. $x - 2y = 8$

7. $2xy - 6 = 8x$

8. $4x = 9y + xy$

9. $4yz = 3y - 8x$

10. $2xy = 3z + 4y$

11. $\frac{2 + 7y}{y} = x$

12. $3x = \frac{5 + y}{y}$

Name ______________________________ Date ______________

Solving Linear Inequalities

Addition Property of Inequality

When you add the same number to each side of an inequality, the inequality remains true.

Subtraction Property of Inequality

When you subtract the same number from each side of an inequality, the inequality remains true.

Multiplication and Division Properties of Inequality (Case 1)

When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Multiplication and Division Properties of Inequality (Case 2)

When you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

To solve an inequality, isolate the variable.

Example 1 **Solve each inequality. Graph the solution.**

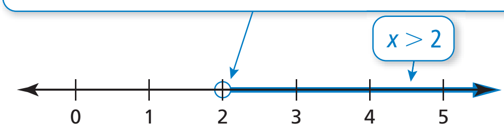
a. $x + 1 > 3$

$\underline{-1} \quad \underline{-1}$ Subtract 1 from each side.

$x > 2$ Simplify.

▶ The solution is $x > 2$.

Use an open circle because $x = 2$ is *not* a solution.



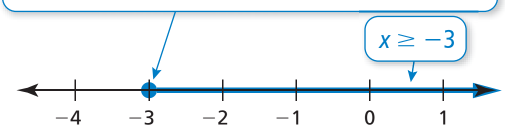
b. $-3x \leq 9$

$\dfrac{-3x}{-3} \geq \dfrac{9}{-3}$ Divide each side by -3. Reverse the inequality symbol.

$x \geq -3$ Simplify.

▶ The solution is $x \geq -3$.

Use a closed circle because $x = -3$ is a solution.



c. $-25 \geq 9y + 2$

$\underline{-2} \quad \underline{-2}$ Subtract 2 from each side.

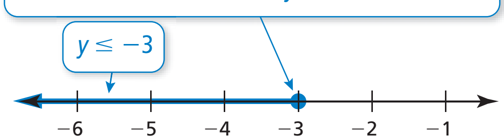
$-27 \geq 9y$ Simplify.

$\dfrac{-27}{9} \geq \dfrac{9y}{9}$ Divide each side by 9.

$-3 \geq y$ Simplify.

▶ The solution is $y \leq -3$.

Use a closed circle because $y = -3$ is a solution.



d. $-\dfrac{z}{2} + 6 > 10$

$\underline{-6} \quad \underline{-6}$ Subtract 6 from each side.

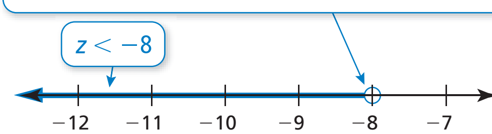
$-\dfrac{z}{2} > 4$ Simplify.

$-\dfrac{z}{2} \cdot (-2) < 4 \cdot (-2)$ Multiply each side by -2. Reverse the inequality symbol.

$z < -8$ Simplify.

▶ The solution is $z < -8$.

Use an open circle because $z = -8$ is *not* a solution.



Practice

Check your answers at BigIdeasMath.com.

Solve the inequality. Graph the solution.

1. $x + 2 > 7$

2. $y - 5 \leq -8$

3. $\dfrac{t}{-3} > -1$

4. $\dfrac{2s}{5} \leq 6$

5. $-2q + 1 \geq 15$

6. $3z - 4 < -1$

Name ______________________________ Date ______________

Solving Compound Inequalities

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

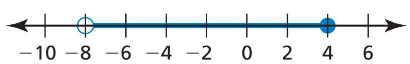
Example 1 **Write each sentence as an inequality. Graph each inequality.**

a. A number x is greater than -8 and less than or equal to 4.

b. A number y is at most 0 or at least 2.

a. A $\underbrace{\text{number } x \text{ is greater than } -8}_{x > -8}$ $\underbrace{\text{and}}_{\textit{and}}$ $\underbrace{\text{less than or equal to } 4}_{x \le 4}$.

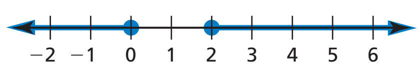
▶ An inequality is $-8 < x \le 4$.



Graph the intersection of the graphs of $x > -8$ and $x \le 4$.

b. A $\underbrace{\text{number } y \text{ is at most } 0}_{y \le 0}$ $\underbrace{\text{or}}_{\textit{or}}$ $\underbrace{\text{at least } 2}_{y \ge 2}$.

▶ An inequality is $y \le 0$ *or* $y \ge 2$.



Graph the union of the graphs of $y \le 0$ and $y \ge 2$.

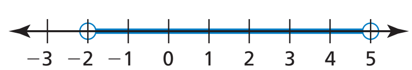
You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

Example 2 **Solve $-4 < x - 2 < 3$. Graph the solution.**

Separate the compound inequality into two inequalities, then solve.

$-4 < x - 2$	*and*	$x - 2 < 3$	Write two inequalities.
$\underline{+2} \quad \underline{+2}$		$\underline{+2} \quad \underline{+2}$	Add 2 to each side.
$-2 < x$	*and*	$x < 5$	Simplify.

▶ The solution is $-2 < x < 5$.



Practice

Check your answers at BigIdeasMath.com.

Write the sentence as an inequality. Graph the inequality.

1. A number d is more than 0 and less than 10.
2. A number a is fewer than -6 or no less than -3.

Solve the inequality. Graph the solution.

3. $5 \le m + 4 < 10$
4. $-3 < 2k - 5 < 7$
5. $4c + 3 \le -5$ *or* $c - 8 > -1$
6. $2p + 1 < -7$ *or* $3 - 2p \le -1$

Name ______________________ Date ____________

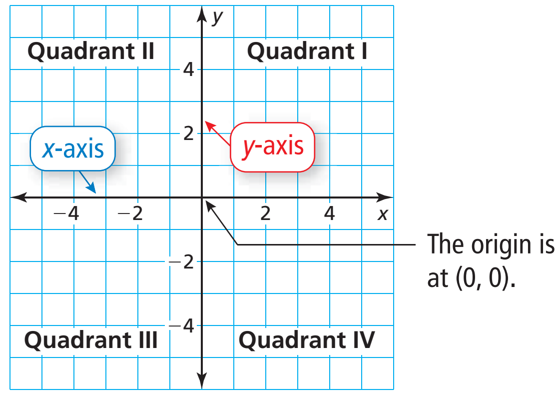
The Coordinate Plane

A **coordinate plane** is formed by the intersection of a horizontal number line and a vertical number line. The number lines intersect at the **origin** and separate the coordinate plane into four regions called **quadrants**.

An **ordered pair** is used to locate a point in a coordinate plane.

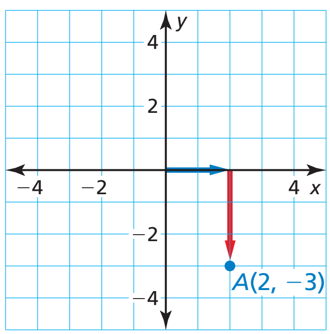
ordered pair: (4, −2)

x-coordinate *y*-coordinate

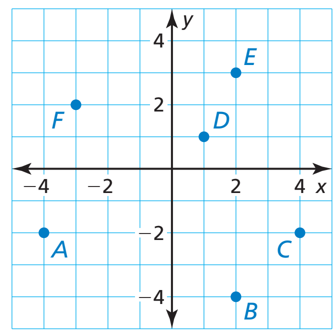


Example 1 **Plot the point *A*(2, −3) in a coordinate plane. Describe the location of the point.**

Start at the origin. Move 2 units right and 3 units down. Then plot the point. The point is in Quadrant IV.



Example 2 **What ordered pair corresponds to point *A*?**



Point A is 4 units to the left of the origin and 2 units down. So, the x-coordinate is −4 and the y-coordinate is −2.

▶ The ordered pair (−4, −2) corresponds to point A.

Practice

Check your answers at BigIdeasMath.com.

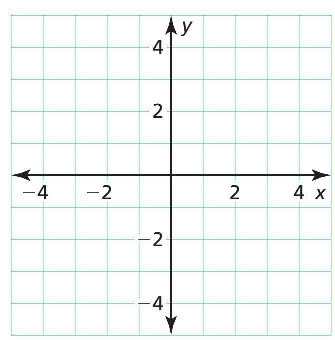
Plot the ordered pair in a coordinate plane. Describe the location of the point.

1. $A(1, 3)$ **2.** $B(-2, 2)$

3. $C(2, -4)$ **4.** $D(1, -1)$

5. $E(-4, -2.5)$ **6.** $F(-3, 0)$

7. $G(0, 1)$ **8.** $H\left(4, \frac{1}{2}\right)$



Use the graph in Example 2 to answer the questions.

9. What ordered pair corresponds to point C?

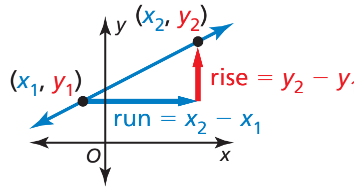
10. What ordered pair corresponds to point F?

Name ______________________________ Date ______________

Slope of a Line

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line. If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2), then the slope m is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$



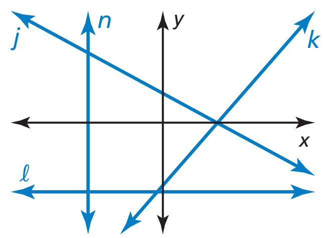
Slopes of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

Undefined slope: vertical, as in line n



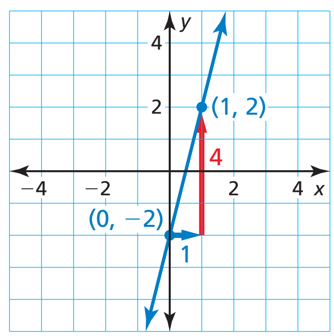
Example 1 **Find the slope of the line shown.**

Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (1, 2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$ Write formula for slope.

$$= \frac{2 - (-2)}{1 - 0}$$ Substitute.

$$= 4$$ Simplify.

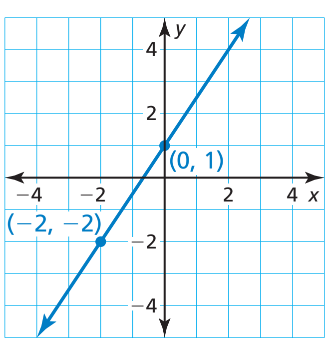


Practice

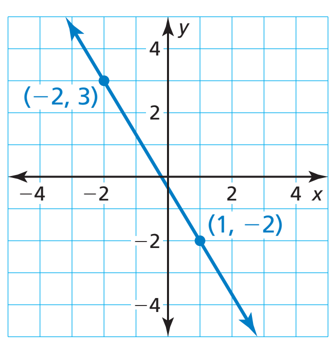
Check your answers at BigIdeasMath.com.

Find the slope of the line.

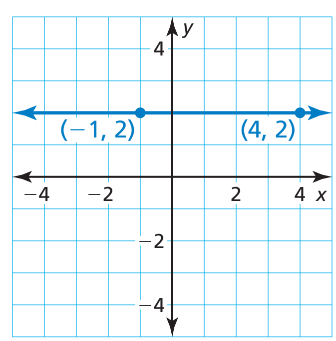
1.



2.



3.



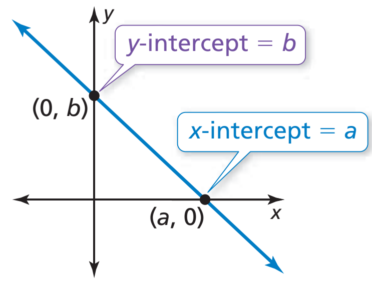
Name ______________________ Date ______________

Slope-Intercept Form

The **x-intercept** of a line is the x-coordinate of the point where the line crosses the x-axis. It occurs when $y = 0$.

The **y-intercept** of a line is the y-coordinate of the point where the line crosses the y-axis. It occurs when $x = 0$.

A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.



$$y = mx + b$$

slope (m) — y-intercept (b)

Example 1 **Identify the slope and the y-intercept of the graph of each linear equation.**

a. $y = -3x - 8$

$y = -3x + (-8)$ Write in slope-intercept form.

▶ The slope is -3, and the y-intercept is -8.

b. $y - 4 = \frac{1}{3}x$

$y = \frac{1}{3}x + 4$ Add 4 to each side.

▶ The slope is $\frac{1}{3}$, and the y-intercept is 4.

Example 2 **Find the x-intercept and the y-intercept of the graph of $2x + y = 4$.**

To find the x-intercept, substitute 0 for y and solve for x.

$$2x + y = 4$$
$$2x + (0) = 4$$
$$x = 2$$

To find the y-intercept, substitute 0 for x and solve for y.

$$2x + y = 4$$
$$2(0) + y = 4$$
$$y = 4$$

▶ The x-intercept is 2, and the y-intercept is 4.

Practice

Check your answers at BigIdeasMath.com.

Identify the slope and the y-intercept of the graph of the linear equation.

1. $y = 4x + 7$

2. $y = -\frac{1}{3}x + 8$

3. $y = \frac{1}{9}x - 6$

4. $y + 9 = -5x$

5. $y - 2x = -6$

6. $7 + y = -\frac{2}{3}x$

Find the x-intercept and the y-intercept of the graph of the equation.

7. $y = 2x$

8. $y = x + 8$

9. $y = 3x + 6$

10. $3x + y = 9$

11. $2x + 3y = 12$

12. $2x - 5y = 10$

13. **SHOPPING** The amount of money you spend on x books and y movies is given by the equation $8x + 12y = 96$. Find the intercepts of the graph of the equation. What do these values represent?

Name ______________________________ Date ______________

Writing Linear Equations

Given a point on a line and the slope of the line, you can write an equation of the line.

Example 1 **Write an equation in slope-intercept form of the line that passes through the point (−5, 6) and has a slope of $\frac{3}{5}$.**

$y = mx + b$	Write the slope-intercept form.
$6 = \frac{3}{5}(-5) + b$	Substitute $\frac{3}{5}$ for m, −5 for x, and 6 for y.
$6 = -3 + b$	Simplify.
$9 = b$	Solve for b.

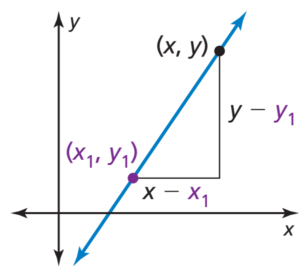
▶ So, the equation is $y = \frac{3}{5}x + 9$.

A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

slope

$$y - y_1 = m(x - x_1)$$

passes through (x_1, y_1)



Example 2 **Write an equation in point-slope form of the line that passes through the point (−8, 3) and has a slope of $\frac{3}{4}$.**

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 3 = \frac{3}{4}[x - (-8)]$	Substitute $\frac{3}{4}$ for m, −8 for x_1, and 3 for y_1.
$y - 3 = \frac{3}{4}(x + 8)$	Simplify.

▶ So, the equation is $y - 3 = \frac{3}{4}(x + 8)$.

Practice

Check your answers at BigIdeasMath.com.

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope.

1. $(1, 3)$; $m = 2$

2. $(4, 2)$; $m = 3$

3. $(-2, 3)$; $m = \frac{1}{2}$

4. $(6, -5)$; $m = \frac{2}{3}$

5. $(4, -2)$; $m = -\frac{1}{4}$

6. $(-7, -3)$; $m = -\frac{2}{7}$

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

7. $(1, 1)$; $m = 5$

8. $(-3, 4)$; $m = 2$

9. $(6, -3)$; $m = \frac{3}{2}$

10. $(5, 7)$; $m = \frac{2}{5}$

11. $(-4, 5)$; $m = -\frac{3}{4}$

12. $(-2, -3)$; $m = -\frac{3}{8}$

Name ______________________ Date ______________

Solving Systems of Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown at the right. A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

$x + 2y = 5$	Equation 1
$x - y = -1$	Equation 2

Example 1 **Solve the system of linear equations above by (a) graphing, (b) substitution, and (c) elimination.**

a. Graph each equation. The graphs appear to intersect at (1, 2). Check this point.

Equation 1 $\quad x + 2y = 5$

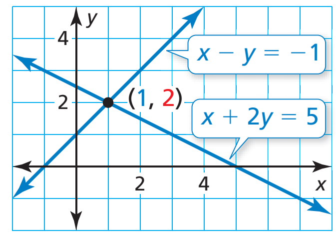
$$1 + 2(2) \stackrel{?}{=} 5$$

$$5 = 5 \checkmark$$

Equation 2 $\quad x - y = -1$

$$1 - 2 \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$



▶ The solution is (1, 2).

b. Solve for x in Equation 2.

$$x - y = -1$$
$$x = y - 1$$

Substitute $y - 1$ for x in Equation 1 and solve for y.

$$x + 2y = 5$$
$$(y - 1) + 2y = 5$$
$$y = 2$$

Substitute 2 for y in Equation 2 and solve for x.

$$x - y = -1$$
$$x - 2 = -1$$
$$x = 1$$

▶ The solution is (1, 2).

c. Multiply Equation 2 by 2. Then add the equations and solve the resulting equation.

$$\begin{aligned} x + 2y = 5 &\Rightarrow \quad x + 2y = 5 \\ x - y = -1 &\Rightarrow \quad \underline{2x - 2y = -2} \\ & \qquad\quad 3x = 3 \\ & \qquad\quad x = 1 \end{aligned}$$

Substitute 1 for x in Equation 2 and solve for y.

$$x - y = -1$$
$$1 - y = -1$$
$$2 = y$$

▶ The solution is (1, 2).

Practice

Check your answers at BigIdeasMath.com.

Solve the system of linear equations by graphing.

1. $y = x - 3$; $y = -x + 1$

2. $-x + 2y = -1$; $x + y = 4$

3. $2x + y = 5$; $4x - 2y = 6$

4. $9x - 3y = 3$; $3x + y = 1$

Solve the system of linear equations by substitution.

5. $y = 1 - x$; $-2x + y = 4$

6. $x = y + 3$; $5x - y = 7$

7. $3x - y = 5$; $2x - y = -3$

8. $x - 2y = -3$; $7x - 2y = 15$

Solve the system of linear equations by elimination.

9. $-2x + 2y = -2$; $2x + y = 5$

10. $x - 4y = -3$; $4x + y = 5$

11. $x + 5y = -2$; $5x + y = 14$

12. $2x + 3y = 5$; $4x + 6y = -10$

Name ______________________ Date ______________

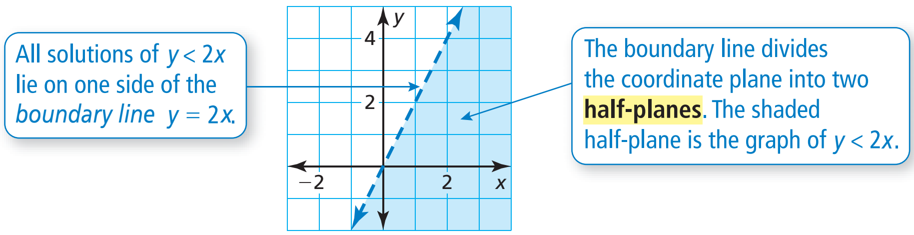
Linear Inequalities in Two Variables

A **linear inequality in two variables**, x and y, can be written as

$$ax + by < c \qquad ax + by \le c \qquad ax + by > c \qquad ax + by > c$$

where a, b, and c are real numbers. A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true.

The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.



Graphing a Linear Inequality in Two Variables	
Step 1	Graph the boundary line for the inequality. Use a dashed line for < or >. Use a solid line for ≤ or ≥.
Step 2	Test a point that is not on the boundary line to determine whether it is a solution of the inequality.
Step 3	When the test point is a solution, shade the half-plane that contains the point. When the test point is *not* a solution, shade the half-plane that does *not* contain the point.

Example 1 **Graph $-x + 2y < 4$ in a coordinate plane.**

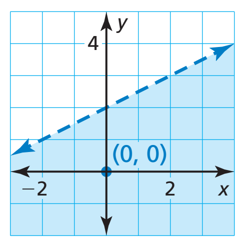
Step 1 Graph $-x + 2y = 4$, or $y = \frac{1}{2}x + 2$. Use a dashed line because the inequality symbol is <.

Step 2 Test (0, 0).

$-x + 2y < 4$ Write the inequality.

$-(0) + 2(0) \overset{?}{<} 4$ Substitute.

$0 < 4$ ✓ Simplify.

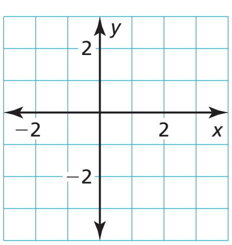


Step 3 Because (0, 0) is a solution, shade the half-plane that contains (0, 0).

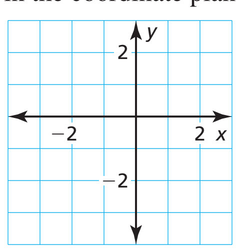
Practice

Check your answers at BigIdeasMath.com.

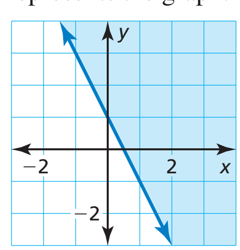
1. Graph $x + y > -1$ in the coordinate plane.



2. Graph $x - 2y \le 2$ in the coordinate plane.



3. Write an inequality that represents the graph.



Name ______________________________ Date ______________

Graphing Linear Equations

Example 1 **Graph $x + 3y = -3$ using intercepts.**

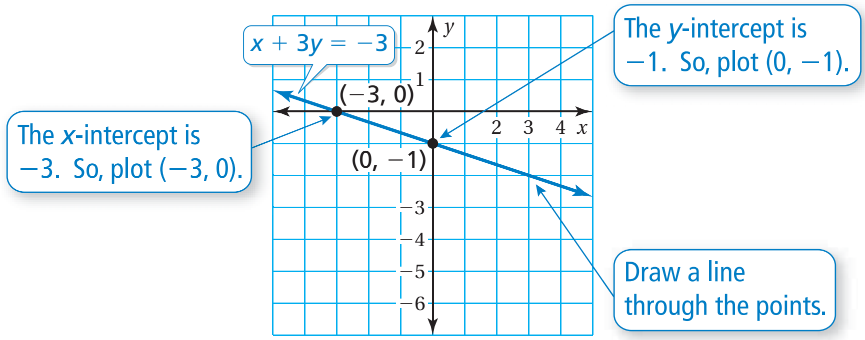
Step 1 To find the x-intercept, substitute 0 for y.

$$x + 3y = -3$$
$$x + 3(0) = -3$$
$$x = -3$$

To find the y-intercept, substitute 0 for x.

$$x + 3y = -3$$
$$0 + 3y = -3$$
$$y = -1$$

Step 2 Graph the equation.

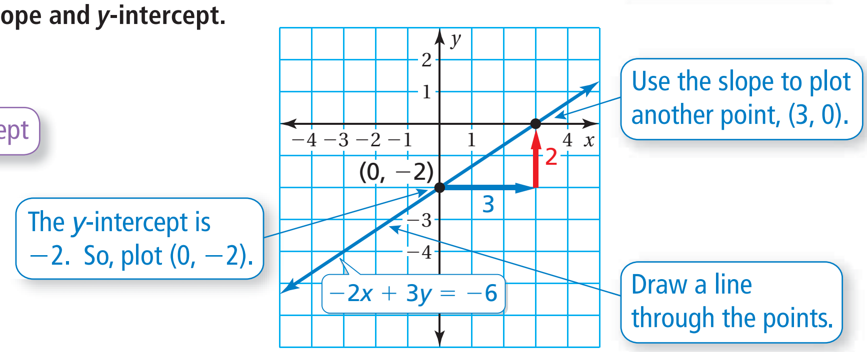


Example 2 **Graph $y = \frac{2}{3}x - 2$ using the slope and y-intercept.**

$$y = \frac{2}{3}x + (-2)$$

slope

y-intercept



Practice

Check your answers at BigIdeasMath.com.

Graph the linear equation using intercepts.

1. $x - 4y = -8$

2. $-18x + 9y = 72$

3. $2x - 3y = 12$

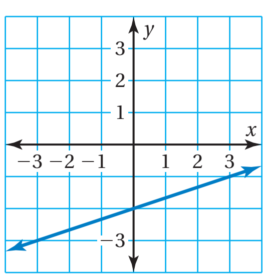
Match the equation with its graph. Identify the slope and y-intercept.

4. $y = 2x + 1$

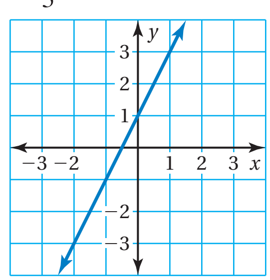
5. $y = \frac{1}{3}x - 2$

6. $y = -\frac{2}{3}x + 1$

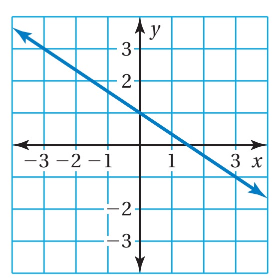
A.



B.



C.

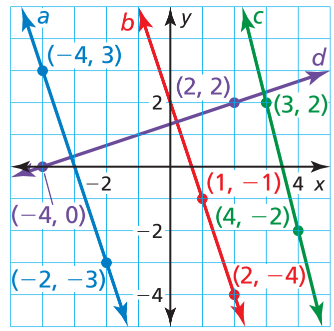


Name ______________________________ Date ______________

Parallel and Perpendicular Lines

Parallel lines are coplanar lines that do not intersect. Nonvertical parallel lines have the same slope. Two lines that intersect to form a right angle are **perpendicular lines**. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Example 1 **Determine which of the lines are parallel and which are perpendicular.**



Find the slope of each line.

Line a: $m = \dfrac{3-(-3)}{-4-(-2)} = -3$

Line b: $m = \dfrac{-1-(-4)}{1-2} = -3$

Line c: $m = \dfrac{2-(-2)}{3-4} = -4$

Line d: $m = \dfrac{2-0}{2-(-4)} = \dfrac{1}{3}$

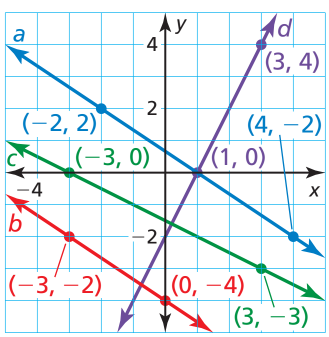
▶ Because lines a and b have the same slope, lines a and b are parallel. Because $\frac{1}{3}(-3) = -1$, lines a and d are perpendicular and lines b and d are perpendicular.

Practice

Check your answers at BigIdeasMath.com.

Determine which of the lines are parallel and which are perpendicular.

1.



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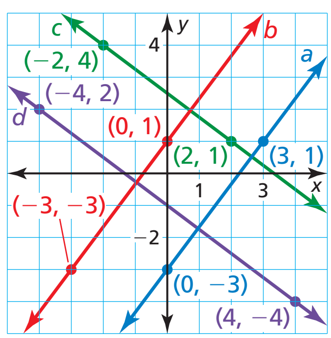
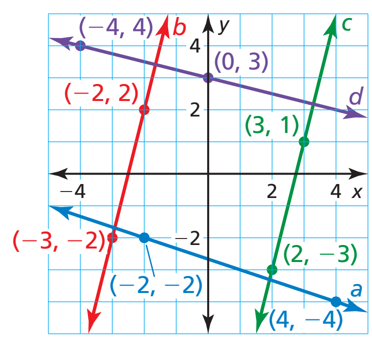


3.



4. **GEOMETRY** The vertices of a quadrilateral are $A(-5, 3)$, $B(2, 2)$, $C(4, -3)$, and $D(-2, -2)$. Is the quadrilateral a parallelogram? Explain your reasoning.

5. **GEOMETRY** The vertices of a parallelogram are $J(-5, 0)$, $K(1, 4)$, $L(3, 1)$, and $M(-3, -3)$. Is the parallelogram a rectangle? Explain your reasoning.

Name ______________________________ Date ______________

Equations of Perpendicular Lines

You can use the slope-intercept form or the point-slope form to write equations of perpendicular lines.

Example 1 **Write an equation of the line passing through $(-3, 8)$ that is perpendicular to the line $y = -3x + 4$.**

Step 1 Find the slope of the perpendicular line. The graph of the given equation has a slope of -3. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line passing through $(-3, 8)$ is $\frac{1}{3}$.

Step 2 Use the slope $m = \frac{1}{3}$ and the slope-intercept form to write an equation of the perpendicular line passing through $(-3, 8)$.

$y = mx + b$	Write the slope-intercept form.
$8 = \frac{1}{3}(-3) + b$	Substitute $\frac{1}{3}$ for m, -3 for x, and 8 for y.
$9 = b$	Solve for b.

So, an equation of the perpendicular line is $y = \frac{1}{3}x + 9$.

Example 2 **Write an equation of the line passing through $(1, -2)$ that is perpendicular to the line $2x - 3y = -9$.**

Step 1 Find the slope of the perpendicular line.

$2x - 3y = -9$	Write original equation.
$y = \frac{2}{3}x + 3$	Solve for y.

The graph of the given equation has a slope of $\frac{2}{3}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line passing through $(1, -2)$ is $-\frac{3}{2}$.

Step 2 Find the slope $m = -\frac{3}{2}$ and the point-slope form to write an equation of the perpendicular line passing through $(1, -2)$.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - (-2) = -\frac{3}{2}(x - 1)$	Substitute $-\frac{3}{2}$ for m, 1 for x_1, and -2 for y_1.
$y + 2 = -\frac{3}{2}(x - 1)$	Simplify.

So, an equation of the perpendicular line is $y + 2 = -\frac{3}{2}(x - 1)$.

Practice

Check your answers at BigIdeasMath.com.

Write an equation of the line passing through point P that is perpendicular to the line.

1. $P(-4, 5)$; $y = -4x + 2$

2. $P(6, 2)$; $y = -\frac{1}{3}x + 1$

3. $P(-3, 7)$; $2x + y = -5$

4. $P(4, -5)$; $y + 2 = \frac{4}{3}(x - 5)$

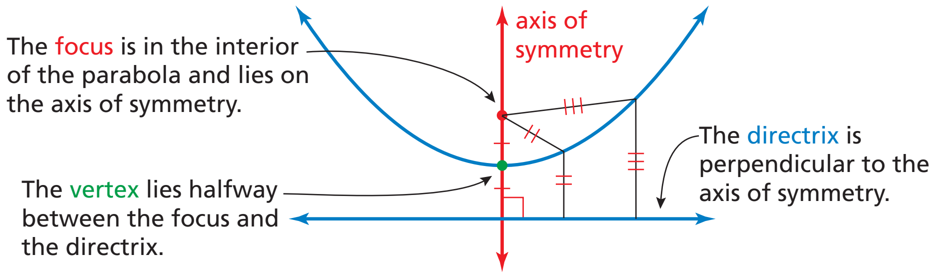
5. $P(1, 0)$; $y = 8$

6. $P(-6, -1)$; $x = 3$

Name ______________________ Date ______________

Focus of a Parabola

A parabola can be defined as the set of all points (x, y) in a plane that are equidistant from a fixed point called the **focus** and a fixed line called the **directrix**.



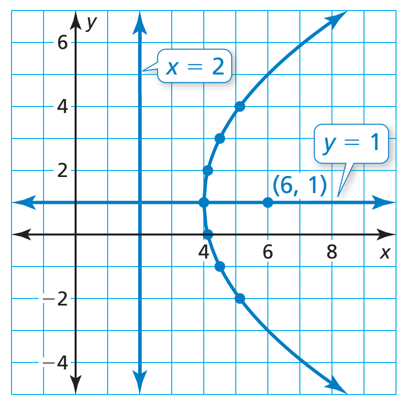
The standard form of the equation of a parabola with vertex at (h, k) is as follows.

Equation	Focus	Directrix	Axis of Symmetry	Behavior
$y = \frac{1}{4p}(x - h)^2 + k$	$(h, k + p)$	$y = k - p$	Vertical $x = h$	Opens up when $p > 0$ Opens down when $p < 0$
$x = \frac{1}{4p}(y - k)^2 + h$	$(h + p, k)$	$x = h - p$	Horizontal $y = k$	Opens right when $p > 0$ Opens left when $p < 0$

Example 1 **Identify the vertex, focus, directrix, and axis of symmetry of $x = \frac{1}{8}(y - 1)^2 + 4$. Then graph the equation.**

The equation has the form $x = \frac{1}{4p}(y - k)^2 + h$, where $p = 2$, $h = 4$, and $k = 1$. The vertex is (h, k), or $(4, 1)$. The focus is $(h + p, k)$, or $(6, 1)$. The directrix is $x = h - p$, or $x = 2$. The axis of symmetry is $y = k$, or $y = 1$. Use a table of values to graph the equation. Notice that it is easier to substitute y-values and solve for x.

y	−2	−1	0	1	2	3	4
x	5.125	4.5	4.125	4	4.125	4.5	5.125



Practice

Check your answers at BigIdeasMath.com.

Identify the vertex, focus, directrix, and axis of symmetry of the parabola. Then graph the equation.

1. $y = -\frac{1}{24}(x + 6)^2 - 4$

2. $x = -\frac{1}{4}(y + 5)^2 - 1$

3. $y = \frac{1}{6}x^2 - 3$

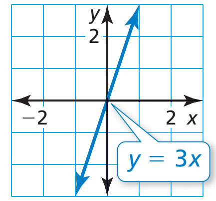
4. $x = \frac{1}{4}(y - 2)^2 + 2$

Name ______________________________ Date ______________

Direct Variation

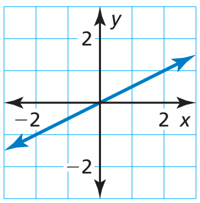
Two quantities x and y show **direct variation** when $y = kx$, where k is a number and $k \neq 0$. The number k is called the **constant of proportionality**.

The graph of $y = kx$ is a line with a slope of k that passes through the origin. So, two quantities that show direct variation are in a proportional relationship. For instance, in the graph at the right, x and y show direct variation.



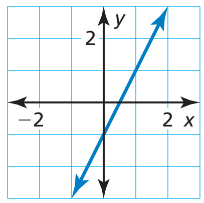
Example 1 **Tell whether x and y show direct variation. Explain your reasoning.**

a.



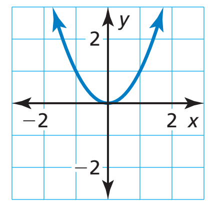
▶ The line passes through the origin. So, x and y show direct variation.

b.



▶ The line does *not* pass through the origin. So, x and y do *not* show direct variation.

c.



▶ The graph is *not* a line. So, x and y do *not* show direct variation.

Example 2 **Tell whether x and y are in a proportional relationship. Explain your reasoning.**

a. $y + 2 = 3x$

$y = 3x - 2$ Solve for y.

▶ The equation *cannot* be written in the form $y = kx$. So, x and y are *not* in a proportional relationship.

b. $\frac{1}{3}y = x$

$y = 3x$ Solve for y.

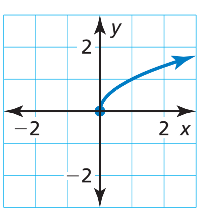
▶ The equation can be written in the form $y = kx$. So, x and y are in a proportional relationship.

Practice

Check your answers at BigIdeasMath.com.

Tell whether x and y show direct variation. Explain your reasoning.

1.



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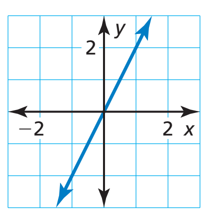

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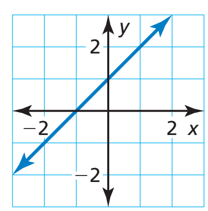

Tell whether x and y are in a proportional relationship. Explain your reasoning.

4. $y + 1 = 2x$

5. $2y = x$

6. $y = \frac{5}{x}$

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Name ______________________ Date ______________

Ratios and Proportions

A **proportion** is an equation stating that two ratios are equivalent. Two quantities that form a proportion are **proportional.**

$$\frac{3}{4} = \frac{6}{8}$$

The proportion is read "3 is to 4 as 6 is to 8."

Example 1 **Tell whether the ratios form a proportion.**

a. $\frac{4}{12}, \frac{6}{18}$

b. $\frac{27}{18}, \frac{30}{21}$

a. Compare the ratios in simplest form.

$$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

$$\frac{6}{18} = \frac{6 \div 6}{18 \div 6} = \frac{1}{3}$$

The ratios are equivalent.

So, $\frac{4}{12}$ and $\frac{6}{18}$ form a proportion.

b. Compare the ratios in simplest form.

$$\frac{27}{18} = \frac{27 \div 9}{18 \div 9} = \frac{3}{2}$$

$$\frac{30}{21} = \frac{30 \div 3}{21 \div 3} = \frac{10}{7}$$

The ratios are *not* equivalent.

So, $\frac{27}{18}$ and $\frac{30}{21}$ do *not* form a proportion.

Practice

Check your answers at BigIdeasMath.com.

Tell whether the ratios form a proportion.

1. $\frac{2}{5}, \frac{3}{15}$

2. $\frac{6}{8}, \frac{15}{20}$

3. $\frac{4}{10}, \frac{2}{6}$

4. $\frac{9}{12}, \frac{21}{28}$

5. $\frac{6}{24}, \frac{7}{28}$

6. $\frac{6}{15}, \frac{9}{36}$

7. $\frac{72}{10}, \frac{36}{8}$

8. $\frac{38}{14}, \frac{57}{21}$

9. $\frac{30}{25}, \frac{16}{12}$

10. $\frac{45}{27}, \frac{75}{45}$

11. $\frac{64}{36}, \frac{56}{38}$

12. $\frac{72}{32}, \frac{63}{28}$

13. **FITNESS** You can do 62 push-ups in 2 minutes. Your friend can do 93 push-ups in 3 minutes. Do these rates form a proportion? Explain.

14. **KAYAKS** You and your friend rent kayaks. Are the rates for renting a kayak proportional? Explain your reasoning.

	Cost	Hours
You	\$23	2
Friend	\$30	3

Name ______________________ Date ______________

Solving Proportions

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. To solve proportions, use the Cross Products Property.

Cross Products Property	
Words The cross products of a proportion are equal.	
Numbers	**Algebra**
$\frac{2}{3} = \frac{4}{6}$	$\frac{a}{b} = \frac{c}{d}$
$2 \cdot 6 = 3 \cdot 4$	$ad = bc$, where $b \neq 0$ and $d \neq 0$

Example 1 **Solve each proportion.**

a. $\frac{x}{6} = \frac{5}{2}$

$x \cdot 2 = 6 \cdot 5$ — Cross Products Property

$2x = 30$ — Multiply.

$x = 15$ — Divide.

▶ The solution is 15.

b. $\frac{8}{y} = \frac{4}{9}$

$8 \cdot 9 = y \cdot 4$

$72 = 4y$

$18 = y$

▶ The solution is 18.

Practice

Check your answers at BigIdeasMath.com.

Solve the proportion.

1. $\frac{1}{3} = \frac{x}{6}$

2. $\frac{2}{5} = \frac{y}{10}$

3. $\frac{z}{9} = \frac{2}{3}$

4. $\frac{2}{7} = \frac{j}{14}$

5. $\frac{4}{9} = \frac{k}{36}$

6. $\frac{m}{24} = \frac{3}{8}$

7. $\frac{11}{3} = \frac{p}{6}$

8. $\frac{n}{54} = \frac{8}{3}$

9. $\frac{14}{a} = \frac{7}{2}$

10. $\frac{15}{b} = \frac{3}{5}$

11. $\frac{21}{2} = \frac{42}{d}$

12. $\frac{9}{16} = \frac{27}{g}$

13. $\frac{21}{r} = \frac{7}{5}$

14. $\frac{25}{q} = \frac{5}{2}$

15. $\frac{9}{8} = \frac{36}{s}$

16. $\frac{4}{15} = \frac{20}{t}$

17. $\frac{x}{2.4} = \frac{3.1}{1.2}$

18. $\frac{4.8}{1.5} = \frac{m}{4.5}$

19. $\frac{3.3}{y} = \frac{1.1}{1.6}$

20. $\frac{2.8}{5.4} = \frac{1.4}{c}$

21. **PENCILS** Thirty-six pencils are packaged in 6 boxes. How many pencils are packaged in 10 boxes?

22. **TICKETS** Two tickets cost \$15. How much does it cost to buy seven tickets?

23. **SALADS** Three salads cost \$6.50. How much does it cost to buy six salads?

24. **FIELD TRIP** There are 108 students on a field trip. The ratio of girls to boys is 5 to 4. How many are girls?

Name ______________________ Date ____________

Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the *x*-coordinates are inputs and the *y*-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

Example 1 **Determine whether each relation is a function. Explain.**

a. $(-1, 3), (0, 3), (1, 3), (2, 1), (3, 1)$

Every input has exactly one output.

▶ So, the relation is a function.

b. $(5, 1), (9, 8), (7, 5), (5, 4), (6, 3)$

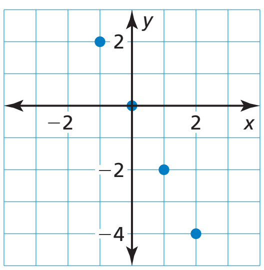
The input 5 has two outputs, 1 and 4.

▶ So, the relation is *not* a function.

The **domain** of a function is the set of all possible input values. The **range** of a function is the set of all possible output values.

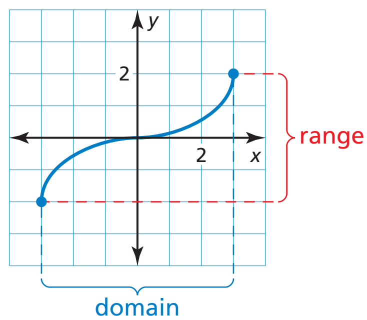
Example 2 **Find the domain and range of the function represented by the graph.**

a.



▶ The domain is -1, 0, 1, and 2.
The range is -4, -2, 0, and 2.

b.



▶ The domain is $-3 \le x \le 3$.
The range is $-2 \le y \le 2$.

Practice

Check your answers at BigIdeasMath.com.

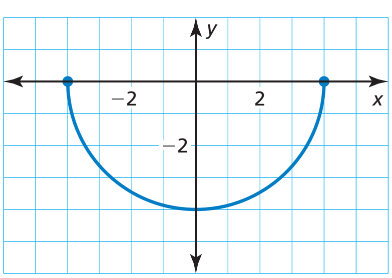
Determine whether the relation is a function. Explain.

1. $(2, -5), (3, -1), (4, 2), (5, -5), (6, 7)$

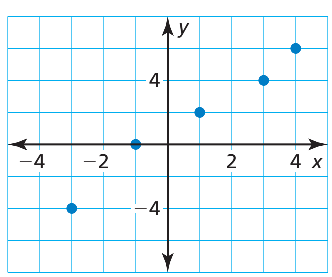
2. $(8, 5), (6, 0), (4, -7), (2, -4), (4, 7)$

Find the domain and range of the function represented by the graph.

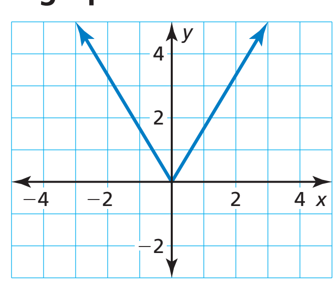
3.



4.



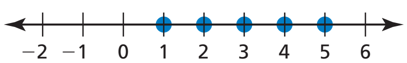
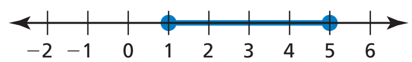
5.



Name ______________________________ Date ______________

Graphing Linear Functions

A **linear function** is a function whose graph is a nonvertical line. A linear function can be represented by a linear equation in two variables, $y = mx + b$, where m is the slope and b is the y-intercept. A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. The points may be distinct or connected.

Discrete Domain	Continuous Domain
A **discrete domain** is a set of input values that consists of only certain numbers in an interval. **Example:** Integers from 1 to 5 	A **continuous domain** is a set of input values that consists of all numbers in an interval. **Example:** All numbers from 1 to 5 

Example 1 **The linear function $y = 29.8x$ represents the number y of kilometers Earth travels in orbit around the Sun in x seconds. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain.**

a. Earth can travel in orbit for part of a second. The number x of seconds Earth travels in orbit can be any value greater than or equal to 0.

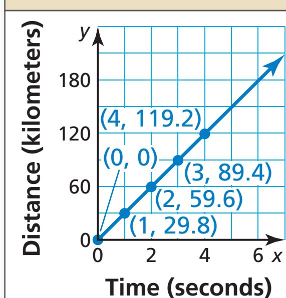
▶ So, the domain is $x \geq 0$, and it is continuous.

b. Make an input-output table to find ordered pairs.

x	0	1	2	3	4
$y = 29.8x$	0	29.8	59.6	89.4	119.2

Plot the ordered pairs. Draw a line through the points starting at (0, 0). Use an arrow to indicate that the line continues without end.

Earth's Orbit Around the Sun



Practice

Check your answers at BigIdeasMath.com.

Copy and complete the table.

1.

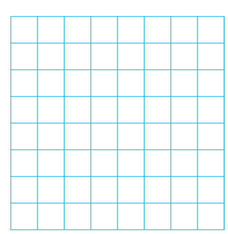
x	−2	−1	0	1	2
$y = x + 2$					

2.

x	−2	−1	0	1	2
$y = x - 7$					

3. **BOATING** A speed boat tour costs \$60 per ticket. There are 5 tickets left. The total cost y of the tickets is a function of the number t of tickets you buy.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.



b. Graph the function using its domain.

Name ______________________________ Date ______________

Function Notation

A linear function can be written in the form $y = mx - b$. By naming a linear function f, you can also write the function using **function notation**.

$$f(x) = mx + b \qquad \text{Function notation}$$

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

Example 1 **Evaluate the function for the given value of *x*.**

a. $f(x) = 2x + 5;\ x = 7$

$f(7) = 2(7) + 5$ Substitute 7 for x.

$= 14 + 5$ Multiply.

$= 19$ Add.

When $x = 7$, $f(x) = 19$.

b. $g(x) = 4x - x^2;\ x = -3$

$g(-3) = 4(-3) - (-3)^2$ Substitute -3 for x.

$= -12 - 9$ Multiply.

$= -21$ Subtract.

When $x = -3$, $g(x) = -21$.

Example 2 **Determine whether the ordered pair is a solution of the equation.**

a. $h(x) = 8 + x;\ (-6, 2)$

$2 \stackrel{?}{=} 8 + (-6)$ Substitute -6 for x and 2 for $h(x)$.

$2 = 2$ ✓ Add.

So, $(-6, 2)$ is a solution.

b. $p(x) = |3x - 1|;\ (-2, -7)$

$-7 \stackrel{?}{=} |3(-2) - 1|$ Substitute -2 for x and -7 for $p(x)$.

$-7 \stackrel{?}{=} |-7|$ Evaluate.

$-7 \neq 7$ ✗ Evaluate.

So, $(-2, -7)$ is *not* a solution.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the function for the given value of *x*.

1. $f(x) = x + 9;\ x = 8$

2. $g(x) = 6 - 5x;\ x = -1$

3. $h(x) = 4x + 3;\ x = 10$

4. $n(x) = -x - 4;\ x = -2$

5. $p(x) = -\frac{3}{4}x^2;\ x = 6$

6. $q(x) = x^2 - 11x;\ x = 4$

7. $k(x) = x^2 + 7x - 1;\ x = -3$

8. $h(x) = |3x - 8|;\ x = 1$

9. $f(x) = |x| + 2;\ x = -15$

Determine whether the ordered pair is a solution of the equation.

10. $f(x) = 3x + 5;\ (-1, 2)$

11. $h(x) = 7x - 2;\ (-3, -19)$

12. $g(x) = -x^2 + x + 5;\ (-5, 25)$

13. $n(x) = x^2 - 6x - 1;\ (4, -7)$

14. $h(x) = |x| - 14;\ (-4, 10)$

15. $p(x) = |-9x - 2|;\ (0, 2)$

16. **TICKETS** The function $C(x) = 49.5x + 19.5$ represents the cost (in dollars) of buying x concert tickets. How much does it cost to buy four tickets? How many tickets can you buy with \$465?

Name ______________________________ Date ______________

Zeros of Quadratic Functions

A **zero of a function** f is an x-value for which $f(x) = 0$. If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x-intercept of the graph of the function.

Example 1 **Find the zeros of each function.**

a. $f(x) = 9x^2 - 1$

Set $f(x)$ equal to 0. Then use square roots to solve for x.

$$9x^2 - 36 = 0$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

▶ The zeros of the function are $x = -2$ and $x = 2$.

b. $f(x) = x^2 - 2x - 8$

Set $f(x)$ equal to 0. Then use factoring to solve for x.

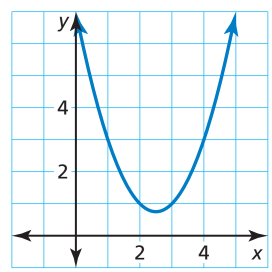
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \quad or \quad x + 2 = 0$$
$$x = 4 \quad or \quad x = -2$$

▶ The zeros of the function are $x = -2$ and $x = 4$.

Example 2 **Find the zeros of $f(x) = x^2 - 5x + 7$.**

Set $f(x)$ equal to 0. Then use the Quadratic Formula to solve for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{-3}}{2}$$
$$= \frac{5 \pm i\sqrt{3}}{2}$$



▶ The zeros of the function are $x = \frac{5}{2} + \frac{\sqrt{3}}{2}i$ and $x = \frac{5}{2} - \frac{\sqrt{3}}{2}i$.

Notice that the graph of f does not intersect the x-axis.

Practice

Check your answers at BigIdeasMath.com.

Find the zero(s) of the function.

1. $f(x) = 8x^2 + 32$

2. $f(x) = -5x^2 + 40$

3. $f(x) = x^2 - 8x + 16$

4. $f(x) = 4x^2 + 12x + 9$

5. $f(x) = 4(x + 5)(x - 1)$

6. $f(x) = -\frac{1}{2}x(x + 3)$

7. $f(x) = 3x^2 + 12x + 15$

8. $f(x) = 2x^2 - x - 15$

9. $f(x) = -(x + 1)^2 + 18$

10. $f(x) = (x - 7)^2 + 9$

Name ______________________ Date ______________

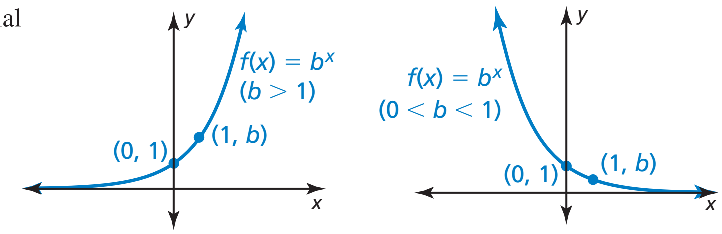
Exponential Functions

Graphing Exponential Functions

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

- When $a > 0$ and $b > 1$, the function is an exponential growth function.
- When $a > 0$ and $0 < b < 1$, the function is an exponential decay function.

The graphs of the parent exponential functions $y = b^x$ are shown.

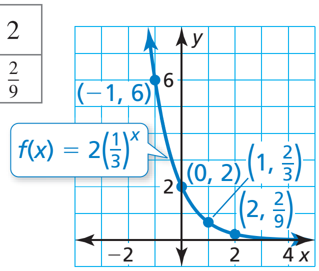


Example 1 **Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.**

a. $f(x) = 2\left(\frac{1}{3}\right)^x$

Because $a = 2$ is positive and $b = \frac{1}{3}$ is greater than 0 and less than 1, the function is an exponential decay function. Use a table to graph the function.

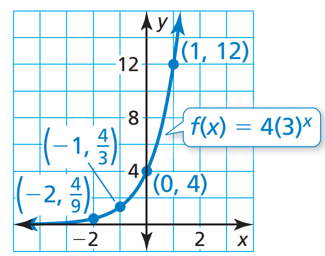
x	−1	0	1	2
y	6	2	$\frac{2}{3}$	$\frac{2}{9}$



b. $f(x) = 4(3)^x$

Because $a = 4$ is positive and $b = 3$ is greater than 1, the function is an exponential growth function. Use a table to graph the function.

x	−2	−1	0	1
y	$\frac{4}{9}$	$\frac{4}{3}$	4	12



Practice

Check your answers at BigIdeasMath.com.

Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

1. $f(x) = \left(\frac{1}{4}\right)^x$

2. $f(x) = \left(\frac{4}{3}\right)^x$

3. $f(x) = 0.5(4)^x$

4. $f(x) = 3(0.75)^x$

5. $f(x) = 2(0.8)^x$

6. $f(x) = 5(2)^x$

Name ____ Date ________________

Exponential Functions

Rewriting Exponential Functions

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time, whereas **exponential decay** occurs when a quantity decreases by the same factor over equal intervals of time.

Exponential Growth Model

initial amount   rate of growth (in decimal form)

final amount $\rightarrow y = a\underbrace{(1 + r)}_{\text{growth factor}}{}^{t} \leftarrow$ time

Exponential Decay Model

initial amount   rate of decay (in decimal form)

final amount $\rightarrow y = a\underbrace{(1 - r)}_{\text{decay factor}}{}^{t} \leftarrow$ time

Example 1 Rewrite the function $y = 120(1.25)^{t/12}$ to determine whether it represents *exponential growth* or *exponential decay*. Then find the percent rate of change.

$y = 120(1.25)^{t/12}$	Write the function.
$= 120[(1.25)^{1/12}]^t$	Power of a Power Property
$\approx 120(1.02)^t$	Evaluate the power.
$= 120(1 + 0.02)^t$	Rewrite in the form $y = a(1 + r)^t$.

▶ So, the function represents exponential growth and the growth rate is about 0.02, or 2%.

Practice

Check your answers at BigIdeasMath.com.

Rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then find the percent rate of change.

1. $y = 80(0.85)^{2t}$

2. $y = 67(1.13)^{t/4}$

3. $y = 5\left(\frac{3}{2}\right)^{-8t}$

4. $y = 17\left(\frac{2}{5}\right)^{0.65t}$

5. $y = 4(0.5)^{t/88}$

6. $y = 31(1.02)^{4t}$

7. $y = 9(1.12)^{0.3t}$

8. $y = 750(0.88)^{t/3}$

9. $y = (0.64)^{5t}$

10. $y = 6(0.82)^{-0.25t}$

Name ______________________________ Date ______________

Measures of Center

A **measure of center** is a measure that represents the center, or typical value, of a data set. The *mean*, *median*, and *mode* are measures of center.

Mean	Median	Mode
The **mean** of a numerical data set is the sum of the data divided by the number of data values. The symbol $\bar{x}$ represents the mean. It is read as "x-bar."	The **median** of a numerical data set is the middle number when the values are written in numerical order. When a data set has an even number of values, the median is the mean of the two middle values.	The **mode** of a data set is the value or values that occur most often. There may be one mode, no mode, or more than one mode. Mode is the only measure of center that can represent a nonnumerical data set.

Example 1 **The table shows the sizes (in kilobytes) of emails in your inbox.**

a. Find the mean, median, and model of the email sizes.

b. Which measure of center best represent the data? Explain.

Email Sizes (kilobytes)				
1.5	13	1.8	1.9	9.1
2.4	2.8	9.2	2	11
5.6	5	4.9	5.5	11

a. Mean $\bar{x} = \dfrac{1.5 + 13 + 1.8 + \cdots + 5.5 + 11}{15} = 5.78$

Median 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, 5, 5.5, 5.6, 9.1, 9.2, 11, 11, 13 Order the data.

middle value

Mode 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, 5, 5.5, 5.6, 9.1, 9.2, 11, 11, 13 11 occurs most often.

▶ The mean is 5.78 kilobytes, the median is 5 kilobytes, and the mode is 11 kilobytes.

b. The median best represents the data. The mean and mode are both greater than most of the data.

Practice

Check your answers at BigIdeasMath.com.

Find the mean, median, and mode of the data set.

1. 35, 44, 40, 35, 54, 50

2. 14, 8, 10, 12, 13, 18, 6, 11, 16

3. 834, 654, 711, 590, 578, 861, 525

4. 4, 8, 5, 6, 4, 5, 4, 2, 6, 5, 4, 3, 5, 4, 6, 5

5. 0.6, 1.4, 0.7, 2, 1.5, 1.2, 1.4, 0.9, 0.7, 1.8

6. $7\frac{3}{4}, 8\frac{1}{2}, 8, 6\frac{3}{4}, 7\frac{3}{4}, 8, 8\frac{1}{4}, 8$

7. **APARTMENTS** The table shows the monthly rental prices for apartments in a city. Find the mean, median, and mode of the prices. Which measure of center best represents the data? Explain.

Monthly Rental Prices			
\$535	\$625	\$850	\$480
\$895	\$420	\$500	\$485
\$1175	\$490	\$510	\$550

Name ______________________________ Date ______________

Mean Absolute Deviation

A **mean absolute deviation** is an average of how much data values differ from the mean.

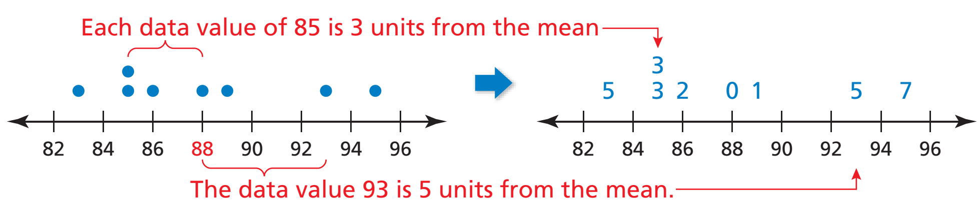
Finding the Mean Absolute Deviation (MAD)
Step 1 Find the mean of the data.
Step 2 Find the distance between each data value and the mean.
Step 3 Find the sum of the distances in Step 2.
Step 4 Divide the sum in Step 3 by the total number of data values.

Example 1 **The scores on a science test are shown below. Find and interpret the mean absolute deviation of the data.**

86, 93, 88, 85, 89, 95, 85, 83

Step 1 Mean $= \dfrac{86 + 93 + 88 + 85 + 89 + 95 + 85 + 83}{8} = \dfrac{704}{8} = 88$

Step 2 You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



Step 3 The sum of the distances is $5 + 3 + 3 + 2 + 0 + 1 + 5 + 7 = 26$.

Step 4 The mean absolute deviation is $\dfrac{26}{8} = 3.25$.

So, the data values differ from the mean by an average of 3.25 points.

Practice

Check your answers at BigIdeasMath.com.

Find and interpret the mean absolute deviation of the data.

1. 9, 10, 11, 11, 12, 12, 13, 13, 14, 15

2. 2, 4, 4, 5, 6, 7, 7, 7

3. 24, 26, 27, 27, 28, 28, 30, 32

4. 8, 28, 29, 31, 32, 35, 38, 41, 43, 44

5. **TEMPERATURES** The table shows the high temperatures for several July days in a city. Find the interpret the mean absolute deviation of the data.

Temperatures (°F)				
85	79	82	80	90
79	83	83	78	78
80	82	82	86	83

Name ______________________________ Date ______________

Standard Deviation

Standard Deviation
The **standard deviation** of a numerical data set is a measure of how much a typical value in the data set differs from the mean. The symbol σ represents the standard deviation. It is read as "sigma." It is given by $\sigma = \sqrt{\dfrac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$ where n is the number of values in the data set. The deviation of a data value x is the difference of the data value and the mean of the data set, $x - \bar{x}$. **Step 1** Find the mean, $\bar{x}$. **Step 2** Find the deviation of each data value, $x - \bar{x}$. **Step 3** Square each deviation, $(x - \bar{x})^2$. **Step 4** Find the mean of the squared deviations. This is called the *variance*. **Step 5** Take the square root of the variance.

Example 1 **The data set shows the hourly pay rates for several employees at a restaurant. Find and interpret the standard deviation of the data set.**

\$9.30, \$8.00, \$7.50, \$8.50, \$10.40, \$9.30, \$9.70, \$9.30

Step 1 Find the mean, $\bar{x}$. $\bar{x} = \dfrac{9.3 + 8 + \cdots + 9.7 + 9.3}{8} = \dfrac{72}{8} = 9$

Step 2 Find the deviation of each data value, $x - \bar{x}$, as shown.

Step 3 Square each deviation, $(x - \bar{x})^2$, as shown.

Step 4 Find the mean of the squared deviations, or variance.

$$\frac{0.09 + 1 + \cdots + 0.49 + 0.09}{8} = \frac{6.22}{8} \approx 0.8$$

Step 5 Use a calculator to take the square root of the variance.

$$\sqrt{\frac{6.22}{8}} \approx 0.9$$

x	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$
9.3	9	0.3	0.09
8	9	−1	1
7.5	9	−1.5	2.25
8.5	9	−0.5	0.25
10.4	9	1.4	1.96
9.3	9	0.3	0.09
9.7	9	0.7	0.49
9.3	9	0.3	0.09

The standard deviation is about 0.9. This means that the typical hourly pay rate of an employee differs from the mean by about \$0.90.

Practice

Check your answers at BigIdeasMath.com.

Find and interpret the standard deviation of the data set.

1. Exam scores: 98, 95, 82, 85, 77, 85, 91, 93, 75, 80, 81, 90

2. Stock prices per share: \$9.70, \$13.50, \$9.50, \$7, \$7.80, \$16.40, \$10.20, \$9, \$14.90, \$12

Name ______________________ Date ______________

Venn Diagrams

A **Venn diagram** uses shapes to describe relationships between two or more sets.

Example 1 **Draw a Venn diagram of the positive integers less than 19, where set *A* consists of factors of 18 and set *B* consists of even numbers.**

Positive integers less than 19:

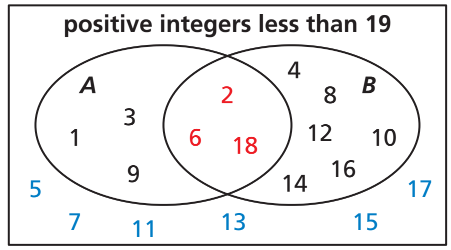
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Set *A* (factors of 18): 1, 2, 3 6, 9, 18

Set *B* (even numbers): 2, 4, 6, 8, 10, 12, 14, 16, 18

Both set *A* and set *B*: 2, 6, 18

Neither set *A* nor set *B*: 5, 7, 11, 13, 15, 17



Example 2 **Use the Venn diagram above to decide whether the statement is *true* or *false*. Explain your reasoning.**

a. If a positive integer less than 19 is not even, then it is not a factor of 18.

▶ False. 1, 3, and 9 are not even, but they are factors of 18.

b. All positive integers less than 19 that are even are factors of 18.

▶ False. 4, 8, 10, 12, 14, and 16 are even, but they are not factors of 18.

Practice

Check your answers at BigIdeasMath.com.

Draw a Venn diagram of the sets described.

1. Of the positive integers less than 13, set *A* consists of the factors of 12 and set *B* consists of even numbers.

2. Of the positive integers less than 11, set *A* consists of prime numbers and set *B* consists of odd numbers.

3. Of the positive integers less than 25, set *A* consists of the multiples of 3 and set *B* consists of the multiples of 4.

Use the Venn diagrams you drew in Exercises 1–3 to decide whether the statement is *true* or *false*. Explain you reasoning.

4. The only positive factors of 12 less than 13 that are not even are 1 and 3.

5. All positive odd numbers less than 11 are prime.

6. All prime numbers less than 11 are odd.

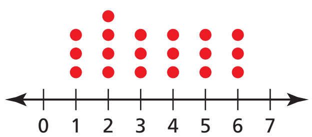
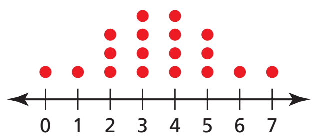
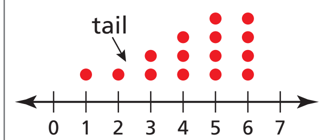
7. There are 2 positive integers less than 25 that are both a multiple of 3 and a multiple of 4.

8. The number of positive integers less than 25 that are multiples of either 3 or 4 is equal to the number of positive integers less than 25 that are not multiples of either 3 or 4.

Name ______________________________ Date ______________

Dot Plots

The **dot plot** uses a number line to show the number of times each value in a data set occurs. Dot plots (or *line plots*) show clusters, peaks, and gaps in a data set. You can also use a dot plot to identify the shape of a distribution.

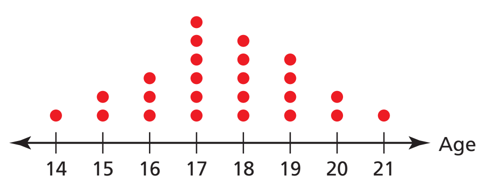
Uniform	Symmetric	Skewed
 All the dots are about the same height. A uniform distribution is also symmetric.	 The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution.	 The "tail" extends either left or right. A distribution is *skewed left* when most of the data are on the right and *skewed right* when most of the data are on the left.

Example 1 **The table shows the ages of volunteers participating in a park cleanup. Draw a dot plot that represents the data. Describe the distribution.**

Ages							
19	17	18	17	18	18	20	19
15	17	16	17	16	17	18	19
16	18	19	20	15	17	21	14

Draw a number line that includes the least value, 14, and the greatest value, 21. Then place a dot above the number line for each data value.

► A peak occurs at 17. The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution. So, the distribution is symmetric.



Practice

Check your answers at BigIdeasMath.com.

Draw a dot plot that represents the data. Describe the distribution.

1. Televisions in households:
2, 4, 3, 6, 2, 1, 3, 4, 0, 1, 3, 3, 2, 2, 5, 2, 1, 7, 5, 4

2. Ages of new drivers:
15, 16, 16, 17, 16, 15, 15, 18, 16, 17, 18, 18, 15, 17, 17, 18

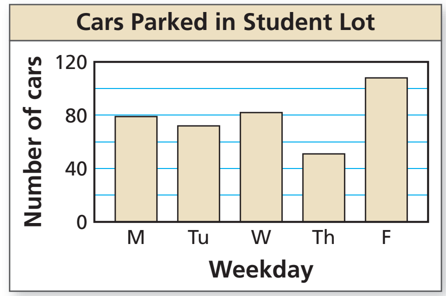
3. Heights of basketball players (in inches):
74, 79, 80, 81, 71, 73, 73, 72, 78, 79, 80, 79, 72, 73

Name ______________________________ Date ______________

Bar Graphs and Line Graphs

A **bar graph** shows data in specific categories. A **line graph** shows how data change over time.

Example 1 **Use the bar graph to answer each question.**



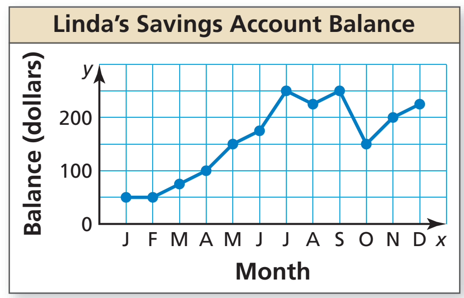
a. On which day of the week were the greatest number of cars parked in the student lot?

▶ The tallest bar on the graph is the bar for Friday. So, the answer is Friday.

b. How many cars were parked in the student lot on Monday?

▶ The bar for Monday shows that about 80 cars were parked in the student lot.

Example 2 **Use the line graph to answer each question.**



a. In which month(s) was Linda's account balance $150?

▶ From the graph, Linda's account balance was $150 in May and October.

b. Between which two consecutive months did Linda's account balance increase the most?

▶ Of the graph's line segments that have positive slopes, the graph is steepest from June to July. So, Linda's account balance increased the most between June and July.

Practice

Check your answers at BigIdeasMath.com.

Use the bar graph in Example 1 to answer the question.

1. On which day of the week were the least number of cars parked in the student lot?
2. On which day(s) of the week were there about 70 cars parked in the student lot?
3. About how many more cars were parked in the student lot Friday than on Thursday?
4. About how many more cars were parked in the student lot on Friday than on Monday?

Use the line graph in Example 2 to answer the question.

5. In which month(s) was Linda's account balance $250?
6. Between which two consecutive months did Linda's account balance decrease the most?
7. How much less was Linda's account balance in October than in July?
8. How much more was Linda's account balance in September than in April?

Name ______________________________ Date ______________

Circle Graphs

A **circle graph** displays data as sections of a circle. The entire circle represents all of the data. Each section represents part of the data and can be labeled using the actual data or the data expressed as fractions, decimals, or percents. When the data are expressed as fractions or decimals, the sum of the data is 1. When the data are expressed as percents, the sum of the data is 100%.

The sum of the angle measures in a circle graph is 360°. When the data are given as percents, multiply the decimal form of each percent by 360° to find the angle measure for each section.

Example 1 **The table shows the results of a survey. Display the data in a circle graph.**

Favorite Season	Students
Winter	16
Spring	29
Summer	67
Fall	48

Step 1 Find the total number of students in the survey.

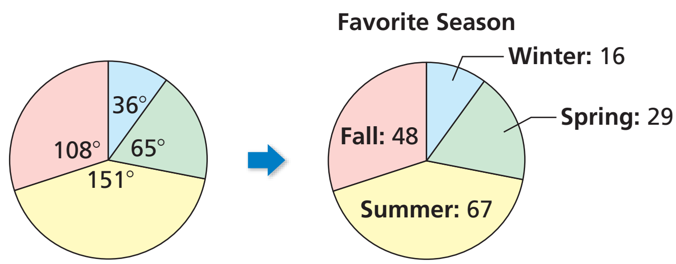
$$16 + 29 + 67 + 48 = 160$$

Step 2 Find the angle measure for each section of the circle graph. Multiply the fraction of students that chose each season by 360°.

Winter: $\frac{16}{160} \cdot 360° = 36°$ **Spring:** $\frac{29}{160} \cdot 360° \approx 65°$

Summer: $\frac{67}{160} \cdot 360° \approx 151°$ **Fall:** $\frac{46}{160} \cdot 360° = 108°$

Step 3 Use a protractor to draw the angle measures found in Step 2 on a circle. Then label the sections.



Practice

Check your answers at BigIdeasMath.com.

Display the data in a circle graph.

1.

Candidate	Ben	Emma	Elle	Ryan	Li
Votes	94	172	113	161	60

2.

Transportation	Percent of Students
Car	24%
Bus	20%
Bike	13%
Walk	43%

Name ______________________________ Date ______________

Stem-and-Leaf Plots

A **stem-and-leaf plot** uses the digits of data values to organize a data set. Each data value is broken into a **stem** (digit or digits on the left) and a **leaf** (digit or digits on the right).

A stem-and-leaf plot shows how data are distributed.

Stem	Leaf
1	0 0 1 3 8 9
2	1 5 8
3	6
4	1 7

Key: 1 | 0 = 10

The *key* explains what the stems and leaves represent.

Example 1 **Make a stem-and-leaf plot of the data.**

Ages of Actors in a Play					
30	24	24	35	12	15
18	31	30	30	19	32
18	36	16	28	39	16
68	29	34	23	25	14

Step 1 Order the data.

12, 14, 15, 16, 16, 18, 18, 19, 23, 24, 24, 25, 28, 29, 30, 30, 30, 31, 32, 34, 35, 36, 39, 68

Step 2 Choose the stems and the leaves. Because the data values range from 12 to 68, use the *tens* digits for the stems and the *ones* digits for the leaves. Be sure to include the key.

Step 3 Write the stems to the *left* of the vertical line.

Step 4 Write the leaves for each stem to the *right* of the vertical line.

Ages of Actors in a Play

Stem	Leaf
1	2 4 5 6 6 8 8 9
2	3 4 4 5 8 9
3	0 0 0 1 2 4 5 6 9
4	
5	
6	8

Key: 1 | 2 = 12

Order the stems vertically.

Write the leaves horizontally.

Include stems without leaves.

Practice

Check your answers at BigIdeasMath.com.

Make a stem-and-leaf plot of the data.

1.

Weights of Dogs (pounds)					
33	33	55	44	39	26
34	42	52	34	48	58
28	39	42	40	45	62
56	26	49	37	64	23

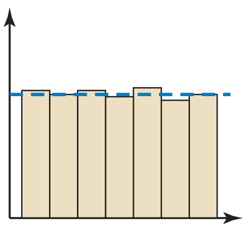
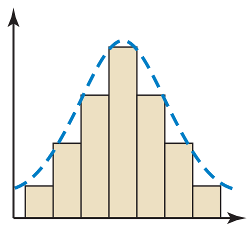
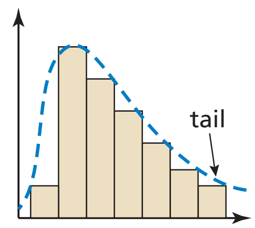
2.

Test Scores (%)					
85	82	100	82	93	76
84	59	71	89	79	87
91	100	89	78	90	85
75	96	99	86	84	92

Name ______________________________ Date ______________

Histograms

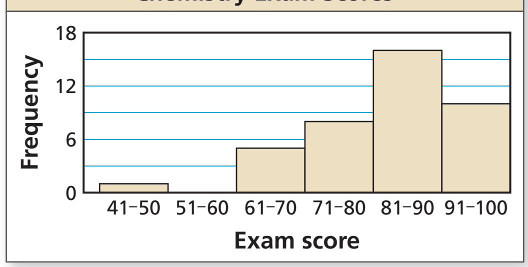
A **histogram** is a bar graph that shows the frequency of data values in intervals of the same size. The height of a bar represents the frequency of the values in the interval. You can use a histogram to identify the shape of a distribution.

Uniform	Symmetric	Skewed
		
All the bars are about the same height. A uniform distribution is also symmetric	The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution.	The "tail" extends either left or right. A distribution is *skewed left* when most of the data are on the right and *skewed right* when most of the data are on the left.

Example 1 **The frequency table shows the chemistry exam scores for a class. Display the data in a histogram. Describe the distribution.**

Score	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	1	0	5	8	16	10

Draw and label the axes. Then draw a bar to represent the frequency of each interval. There is no space between the bars of a histogram. Be sure to include the interval 51–60 with a frequency of 0. The bar height is 0.



▶ Most of the data are on the right and the tail of the graph extends to the left. So, the distribution is skewed left.

Practice

Check your answers at BigIdeasMath.com.

Display the data in a histogram.
Describe the distribution.

1.

	Speeds of Vehicles On a Highway				
Speed (mi/h)	35–44	45–54	55–64	65–74	75–84
Frequency	2	15	30	18	3

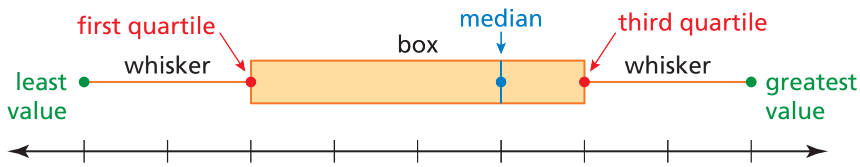
2.

	Ages of People at a Family Picnic				
Age	0–19	20–39	40–59	60–79	80–99
Frequency	10	14	9	6	3

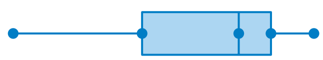
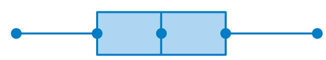
Name ______________________________ Date ______________

Box-and-Whisker Plots

A **box-and-whisker plot** represents a data set along a number line by using the least value, the greatest value, and the quartiles of the data. The five numbers that make up the box-and-whisker plot are called the **five-number summary** of the data set.



You can use a box-and-whisker plot to identify the shape of a distribution.

Skewed Left	Symmetric	Skewed Right
		
The left whisker is longer than the right whisker. Most of the data are on the right side of the plot.	The whiskers are about the same length. The median is in the middle of the plot.	The right whisker is longer than the left whisker. Most of the data are on the left side of the plot.

Example 1 **The data set shows the prices (in dollars) of lacrosse helmets. Make a box-and-whisker plot that represents the data. Describe the distribution.**

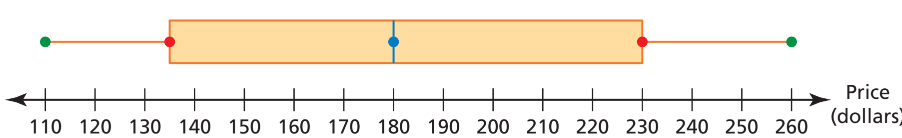
125, 120, 250, 110, 190, 220, 145, 260, 240, 150, 170, 200

Step 1 Order the data. 110 120 125 145 150 170 190 200 220 240 250 260

Find the median and the quartiles. first quartile, 135 median, 180 third quartile, 230

Step 2 Draw a number line that includes the least value, 110, and the greatest value, 260. Graph points above the number line that represent the five-number summary.

Step 3 Draw a box using the quartiles. Draw a line through the median. Draw whiskers from the box to the least and the greatest values.



▶ The whiskers are about the same length and the median is in the middle of the plot. So, the distribution is symmetric.

Practice

Check your answers at BigIdeasMath.com.

Make a box-and-whisker plot that represents the data. Describe the distribution.

1. Video game prices (in dollars):
45, 40, 50, 35, 30, 40, 40, 30, 45, 60

2. Exam scores: 79, 86, 100, 82, 94, 98, 96,
86, 90, 92, 62, 84

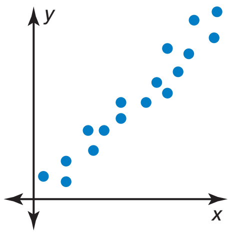
Name ______________________________ Date ______________

Scatter Plots

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane. Scatter plots can show trends in the data.

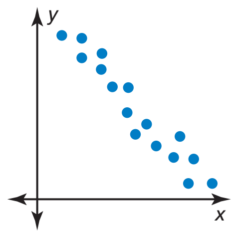
A **correlation** is a relationship between data sets. You can use a scatter plot to describe the correlation between data.

Positive Correlation



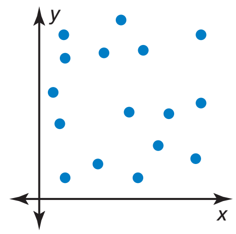
As x increases, y increases.

Negative Correlation



As x increases y decreases.

No Correlation



The points show no pattern.

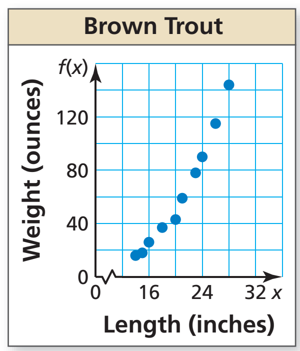
Example 1 **The table shows the lengths and weights of 10 brown trout. Create a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.**

Length (inches), x	16	20	18	15	14	26	21	28	23	24
Weight (ounces), $f(x)$	26	43	37	18	16	115	59	144	78	90

Plot the ordered pairs in a coordinate plane.

As the length increases, the weight increases.

So, the scatter plot shows a positive correlation.



Practice

Check your answers at BigIdeasMath.com.

Create a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

1.

Height (inches), x	72	68	64	70	67	65
Weight (pounds), $f(x)$	180	162	118	145	143	174

2.

Temperature (°F), x	38	80	54	76	31	46
Bowls of warm soup, $f(x)$	44	20	32	22	46	38